



Optimized vs. Hypergaussian Occulters A Comparative Sensitivity Analysis

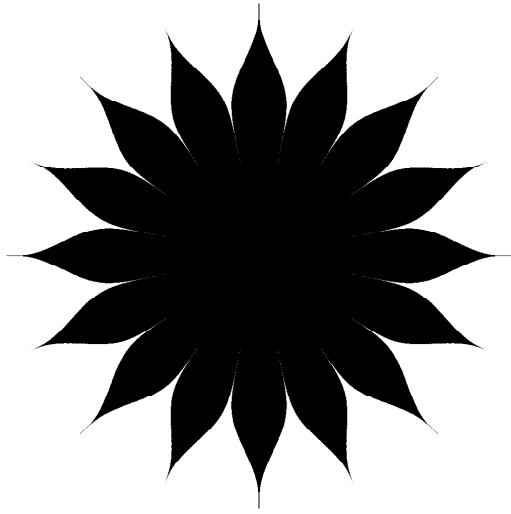
Robert J. Vanderbei

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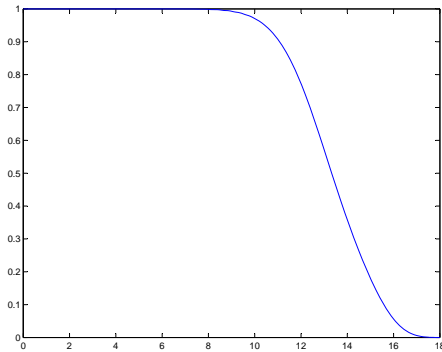
TPF Navigator Program Forum
NASA Ames Research Center, Moffett Field

<http://www.princeton.edu/~rvdb>

Petal-Shaped Occulters



16-Petal Occulter $A(r, \theta)$



Radial Attenuation $A(r)$

- Babinet's principle plus Fresnel propagation:

$$E(\rho, \phi) = 1 - \frac{1}{i\lambda z} \int_0^\infty \int_0^{2\pi} e^{\frac{i\pi}{\lambda z}(r^2 + \rho^2 - 2r\rho \cos(\theta - \phi))} A(r, \theta) r d\theta dr.$$

- From Jacobi-Anger expansion we get:

$$E(\rho, \phi) = 1 - \frac{2\pi}{i\lambda z} \int_0^R e^{\frac{i\pi}{\lambda z}(r^2 + \rho^2)} J_0\left(\frac{2\pi r\rho}{\lambda z}\right) A(r) r dr - \sum_{k=1}^{\infty} \frac{2\pi(-1)^k}{i\lambda z} \left(\int_0^R e^{\frac{i\pi}{\lambda z}(r^2 + \rho^2)} J_{kN}\left(\frac{2\pi r\rho}{\lambda z}\right) \frac{\sin(\pi k A(r))}{\pi k} r dr \right) \times \left(2 \cos(kN(\phi - \frac{\pi}{2})) \right)$$

where N is the number of petals.

- For small ρ , truncated summation well-approximates full sum.
- Truncated after 10 terms.
- $\lambda \in [0.4, 1.1]$ microns.
- $z = 72,000$ km, $R = 25$ m.
- In angular terms, $R/z = 73$ mas.

Hypergaussian Profile

$$A(r) = \begin{cases} 1 & \text{if } r \leq a \\ e^{-\left(\frac{r-a}{b}\right)^n} & \text{if } a < r \leq R \\ 0 & \text{if } R < r. \end{cases}$$

Specific choice:

$$a = 9.5, \quad b = 9.5, \quad R = 25, \quad n = 6.$$

Profile Optimization

$$\begin{aligned} &\text{minimize} && \int_0^R A(r)rdr \\ &\text{subject to} && -10^{-c} \leq \Re(E(\rho)) \leq 10^{-c} && \text{for } \rho \in \mathcal{S}, \lambda \in \mathcal{L} \\ &&& -10^{-c} \leq \Im(E(\rho)) \leq 10^{-c} && \text{for } \rho \in \mathcal{S}, \lambda \in \mathcal{L} \\ &&& A(r) = 1 && \text{for } r \leq a \\ &&& A'(r) \leq 0 && \text{for all } r \\ &&& -d \leq A''(r) \leq d && \text{for all } r \end{aligned}$$

Specific choice:

$$a = 6.25, \quad R = 25, \quad c = 5.25, \quad d = 0.08, \quad \mathcal{S} = [0, 3], \quad \mathcal{L} = [0.4, 1.1] \times 10^{-6}$$

where all metric quantities are in meters.

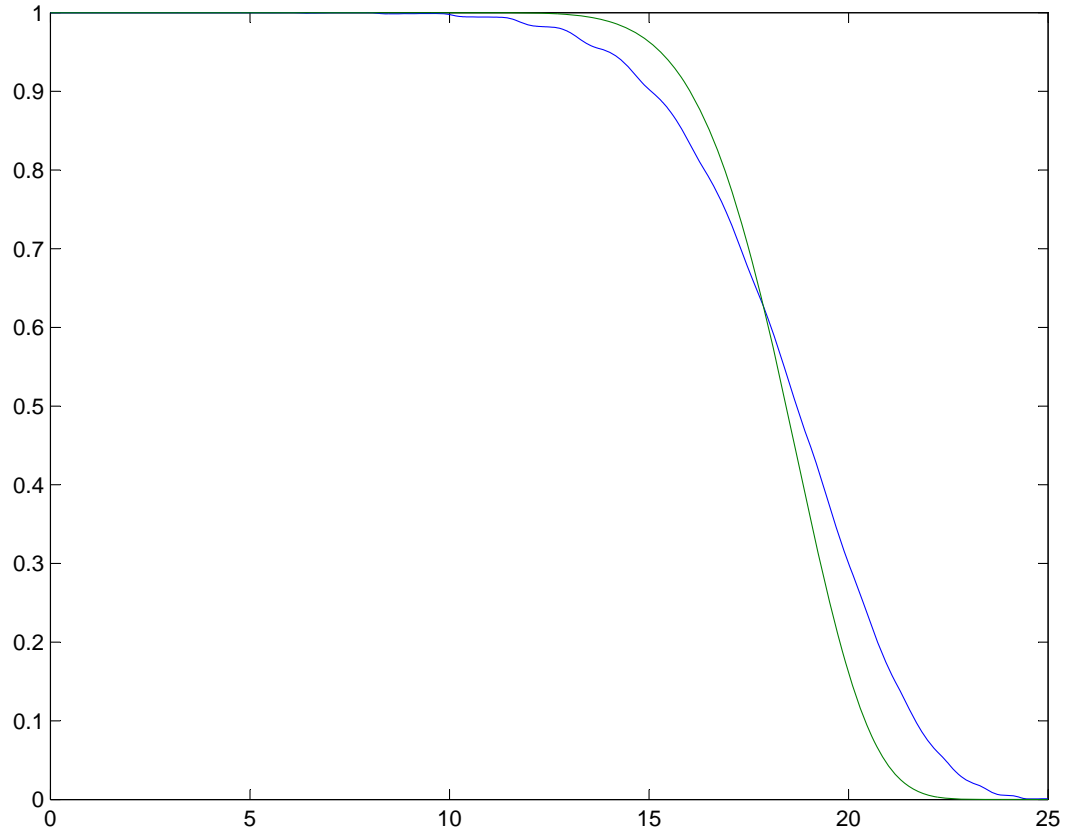
An infinite dimensional linear programming problem.

Discretize:

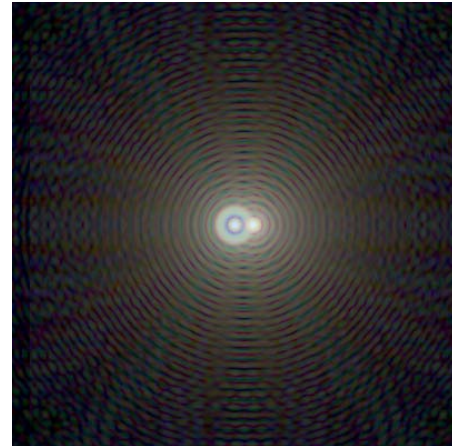
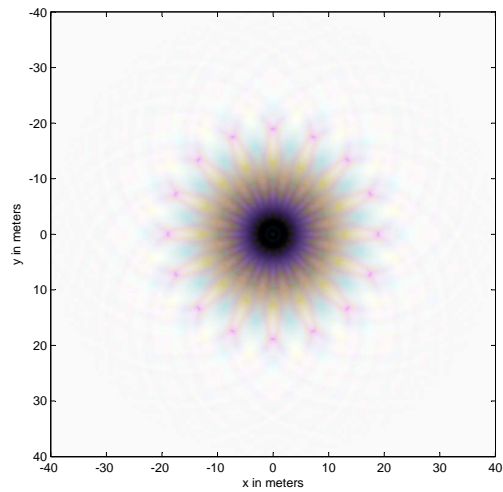
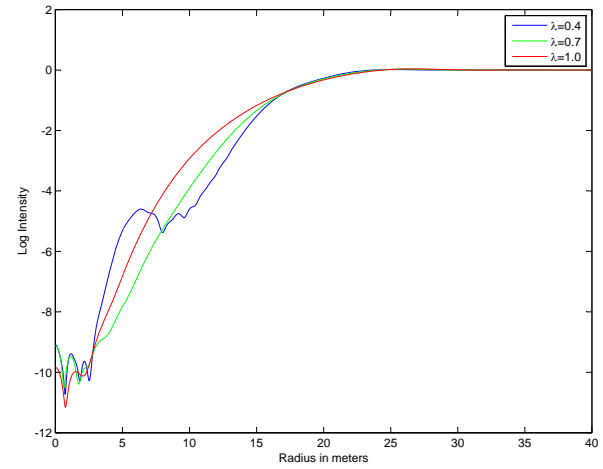
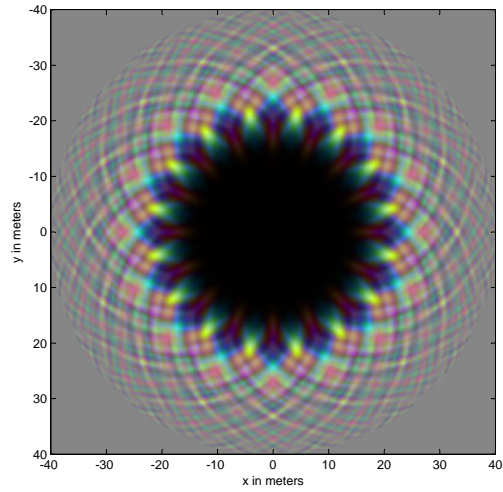
- $[0, R]$ into 3000 evenly space points.
- \mathcal{S} into 18 evenly spaced points.
- \mathcal{L} into increments of 100 nm.

Attenuation Profiles

Optimized vs. Hypergaussian



Optimized 16-Petal Occulter



Hypergaussian 16-Petal Occulter

