Extinguishing Poisson's Spot with Linear Programming

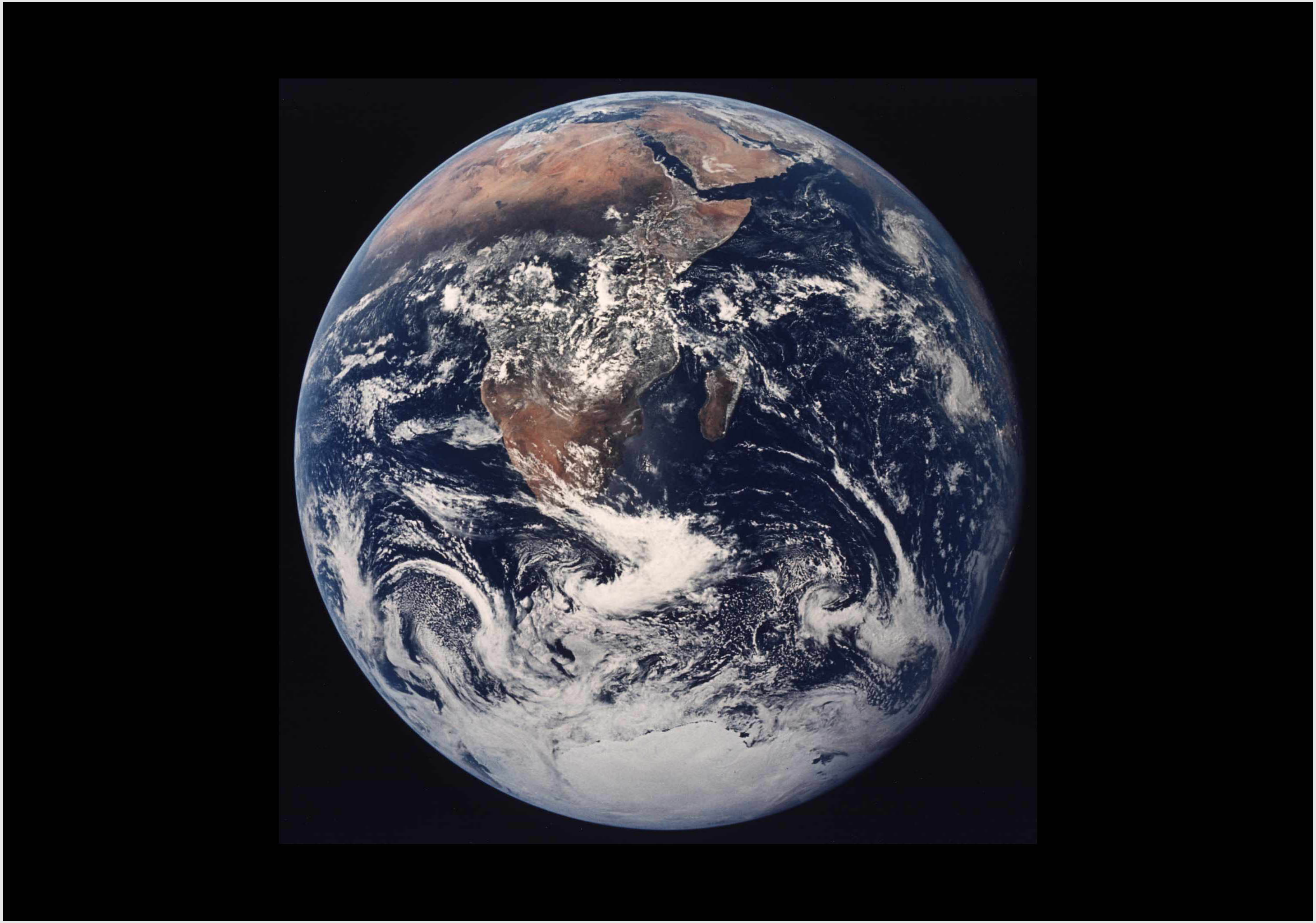
Robert J. Vanderbei

2008 May 13

SIAG/OPT 2008
Boston, MA

http://www.princeton.edu/∼rvdb

Are We Alone?



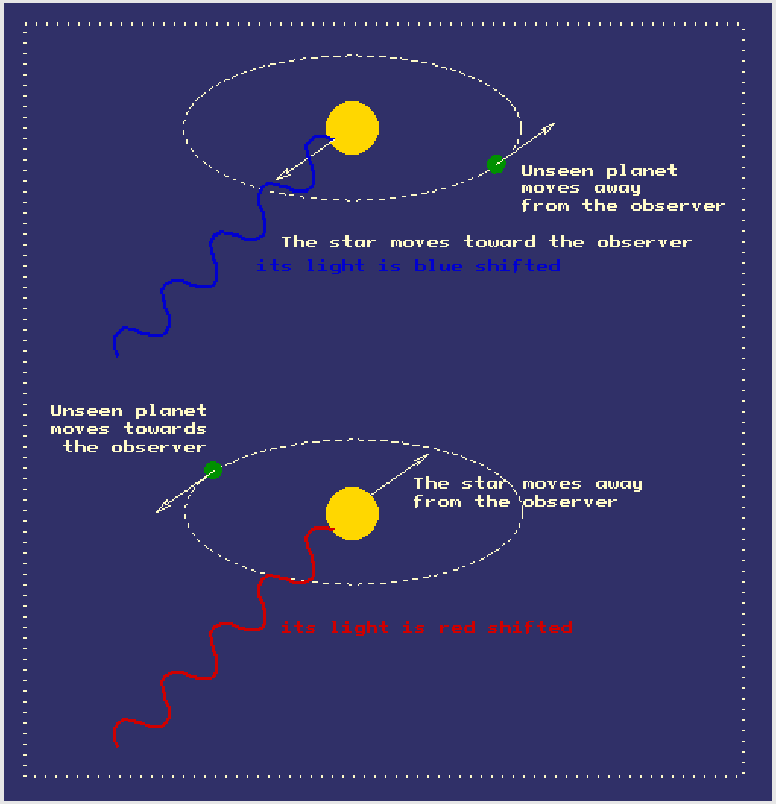
Indirect Detection Methods

Almost 300 planets found so far

Wobble Methods

Radial Velocity.
For edge-on systems.
Measure periodic doppler shift.

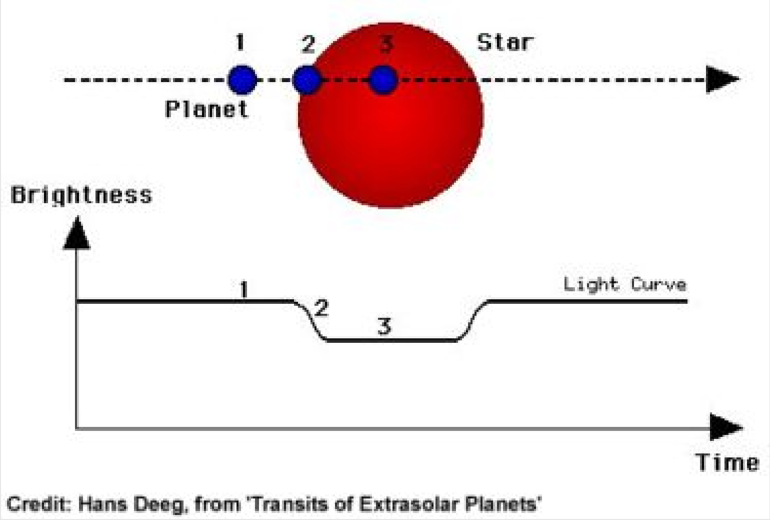
Astrometry.
Best for face-on systems.
Measure circular wobble against background stars.



Transit Method

- HD209458b confirmed both via RV and transit.
- Period: 3.5 days
- Separation: 0.045 AU (0.001 arcsecs)
- Radius: $1.3 R_J$
- Intensity Dip: $\sim 1.7\%$
- Venus Dip $= 0.01\%$, Jupiter Dip: 1%

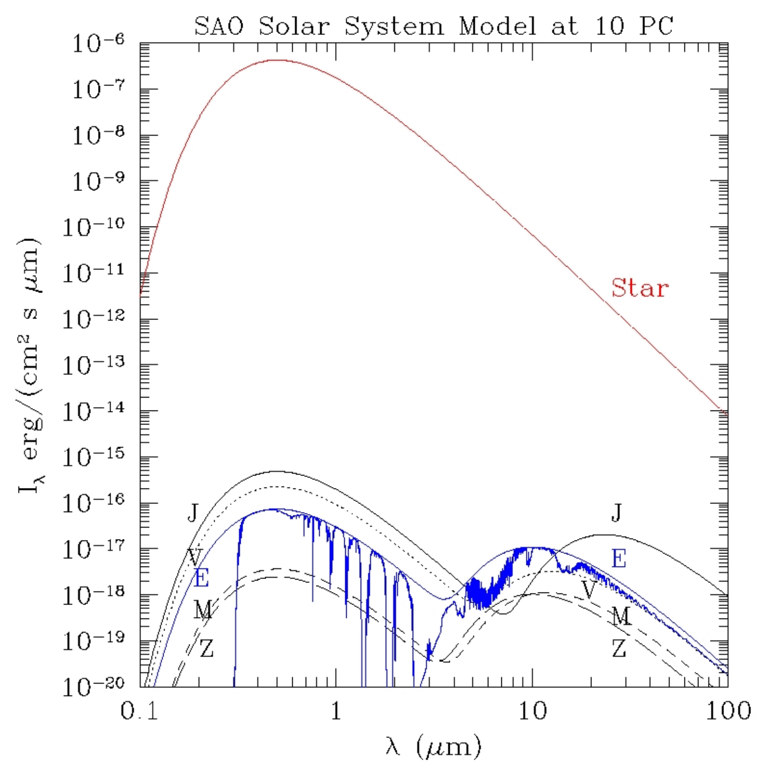
Venus Transit (R.J. Vanderbei)



Direct Detection

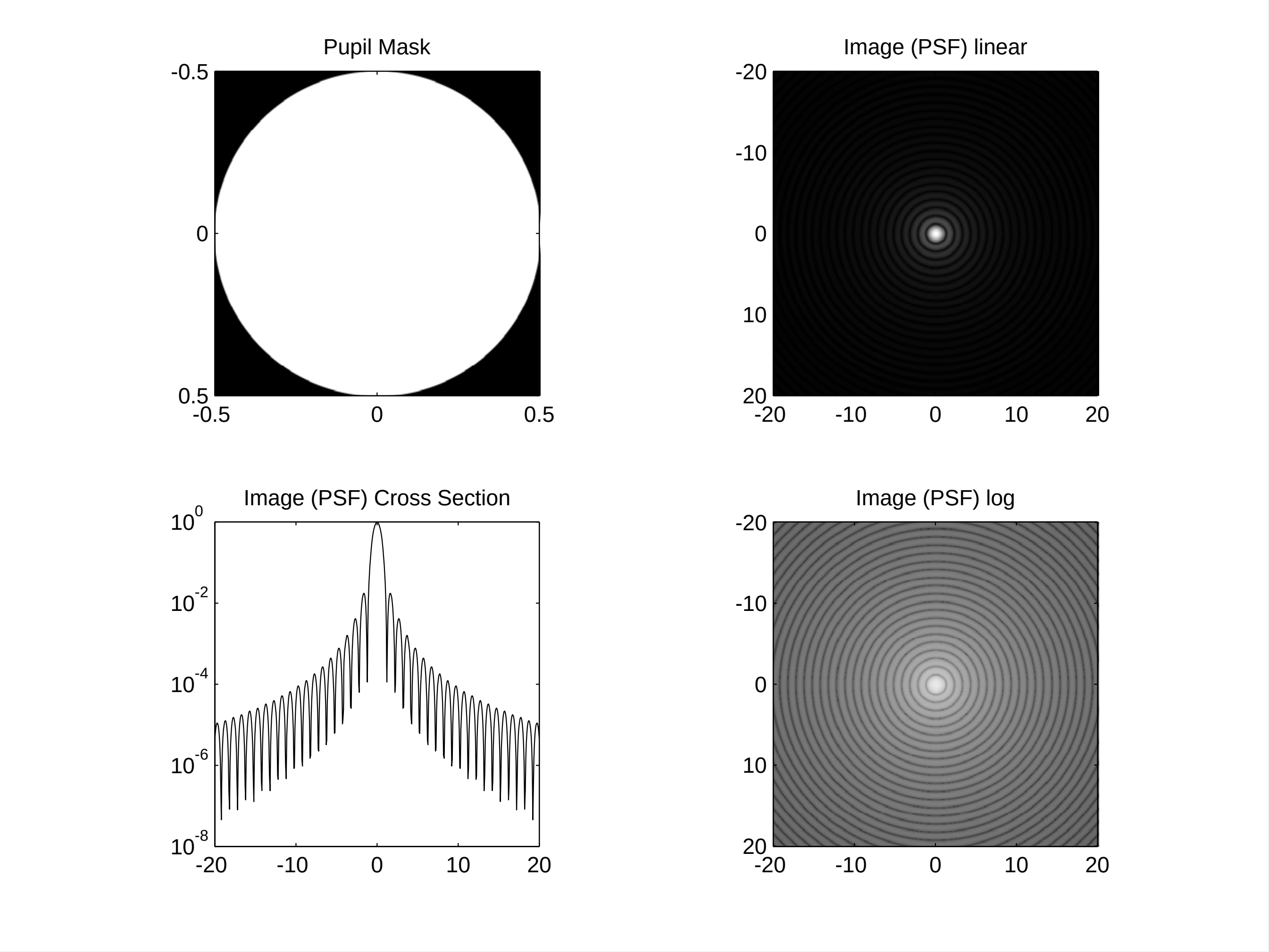
Why It's Hard

- *Bright Star/Faint Planet:* In visible light, our Sun is 10^{10} times brighter than Earth. That's 25 mags.
- *Close to Each Other:* A planet at 1 AU from a star at 10 parsecs can appear at most 0.1 arcseconds in separation.
- *Far from Us:* There are less than 100 Sun-like stars within 10 parsecs.



Telescope w/ Unobstructed Aperture

Doesn't Work! Requires an aperture measured in kilometers to mitigate diffraction effects.



Space-based Occulter (TPF-O)

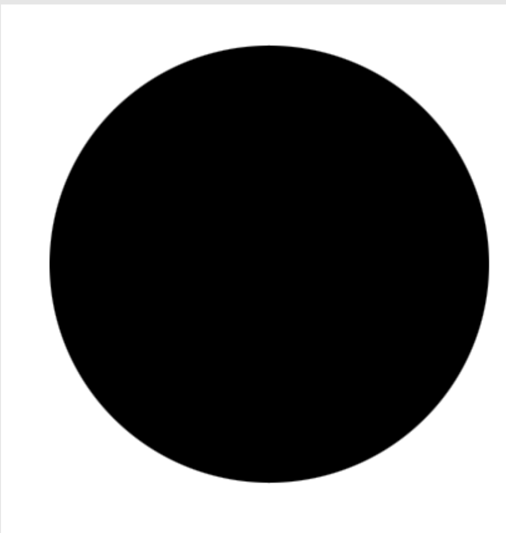
Telescope Aperture: 4m, Occulter Diameter: 50m, Occulter Distance: $72,000$km

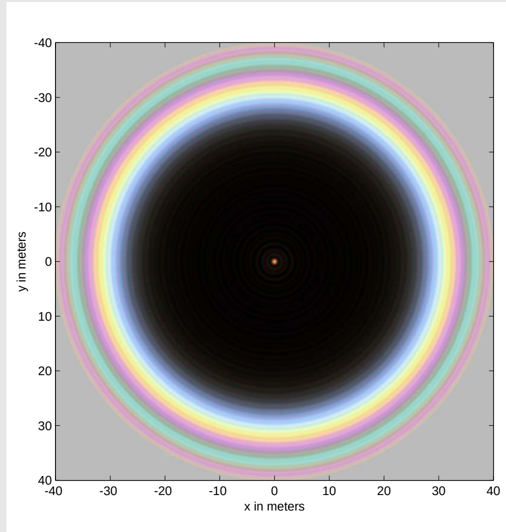
Plain External Occulter (Doesn't Work!)

Shadow ⇒

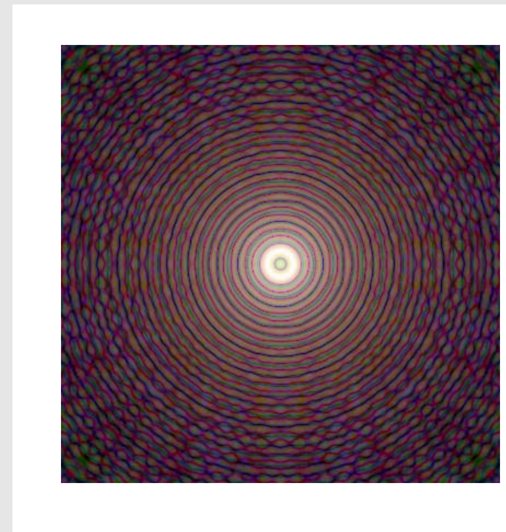
Circular Occulter

⇓



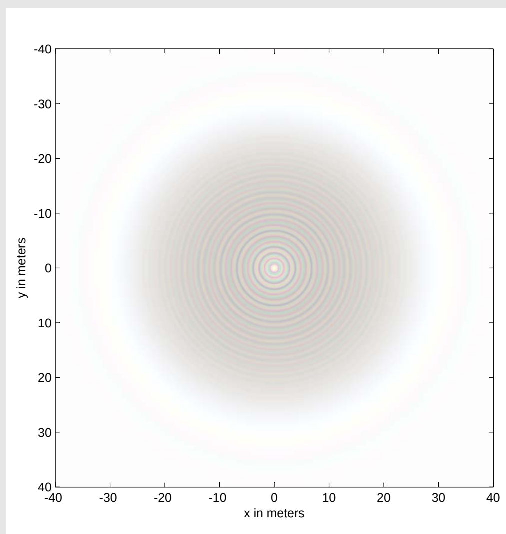


⇐ Note bright spot at center (Poisson's spot)



⇑

Telescope Image

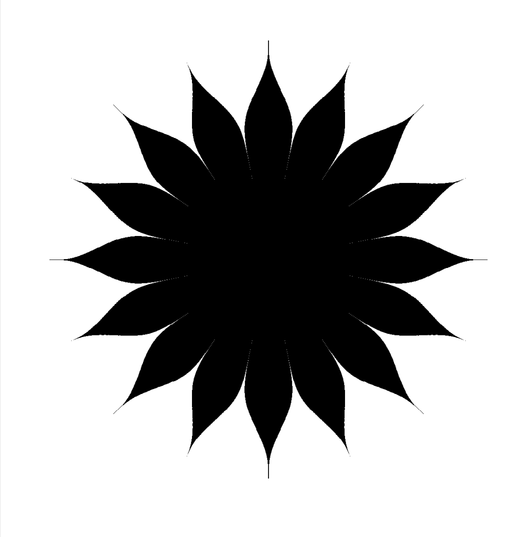


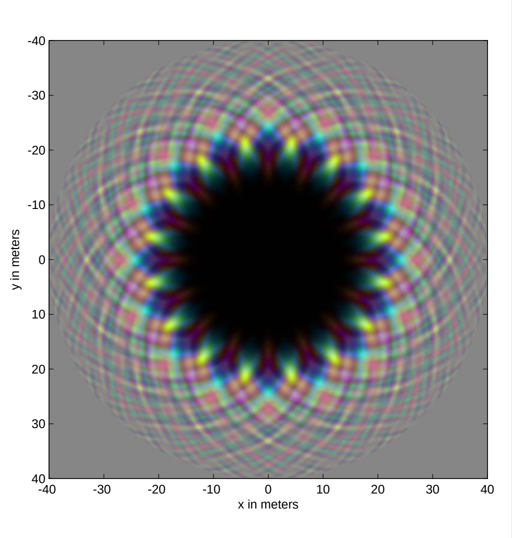
⇐ Shadow (Log Stretch)

Shaped Occulter—Eliminates Poisson's Spot

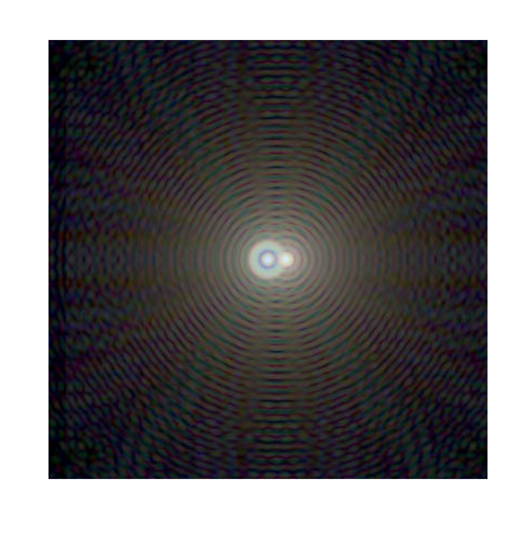
Shadow ⇒

Shaped Occulter
⇓

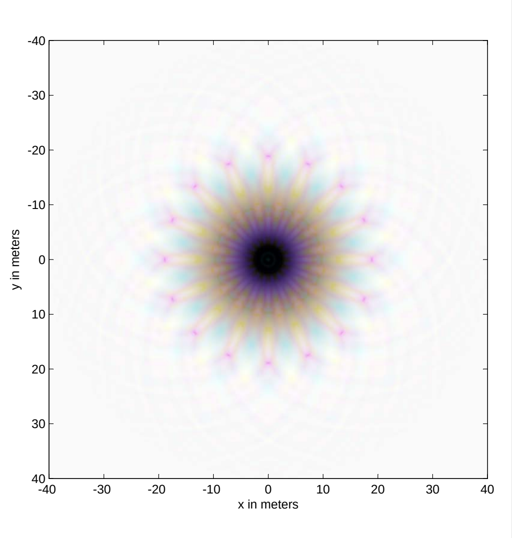




⇐ Bright spot is gone

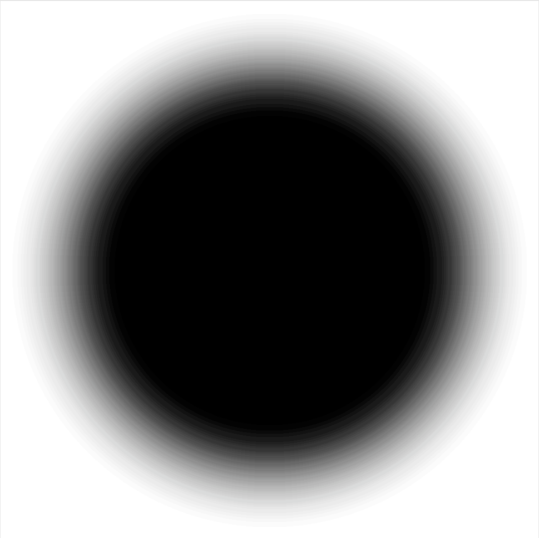


⇑
Telescope image shows planet

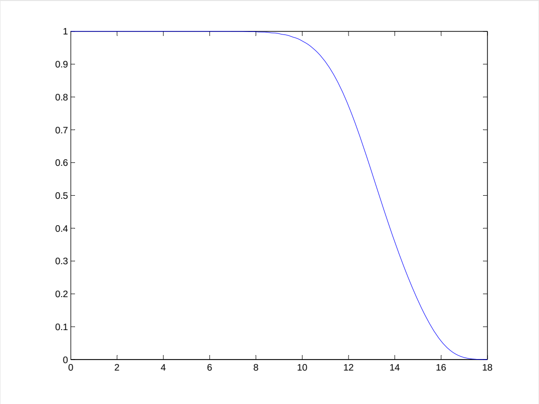


⇐ Shadow is dark
(Log Stretch)

Apodized Occulters



Apodized Occulter



Radial Attenuation $A(r)$

- The problem is *diffraction*.
- Abrupt edges create unwanted diffraction.
- *Solution:* Soften the edges with a partially transmitting material—an *apodizer*.
- Let $A(r, \theta)$ denote *attenuation* at location (r, θ) on the occulter.
- The *intensity* of the downstream light is given by the *square of the magnitude of the electric field* $E(\rho, \phi)$.
- *Babinet's principle* plus *Fresnel propagation* gives a formula for the downstream electric field:

$$E(\rho, \phi) = 1 - \frac{1}{i\lambda z} \int_0^\infty \int_0^{2\pi} e^{\frac{i\pi}{\lambda z}(r^2+\rho^2-2r\rho\cos(\theta-\phi))} A(r, \theta) r d\theta dr.$$

where

  - z is distance "downstream" and
  - λ is wavelength of light.

Attenuation Profile Optimization

$$\begin{array}{llll} \text{minimize} & \gamma & & \\ \text{subject to} & -\gamma \leq \Re(E(\rho)) \leq \gamma & & \text{for } \rho \in \mathcal{R}, \lambda \in \mathcal{L} \\ & -\gamma \leq \Im(E(\rho)) \leq \gamma & & \text{for } \rho \in \mathcal{R}, \lambda \in \mathcal{L} \\ & A'(r) \leq 0 & & \text{for } 0 \leq r \leq R \\ & -d \leq A''(r) \leq d & & \text{for } 0 \leq r \leq R \end{array}$$

Specific choice:

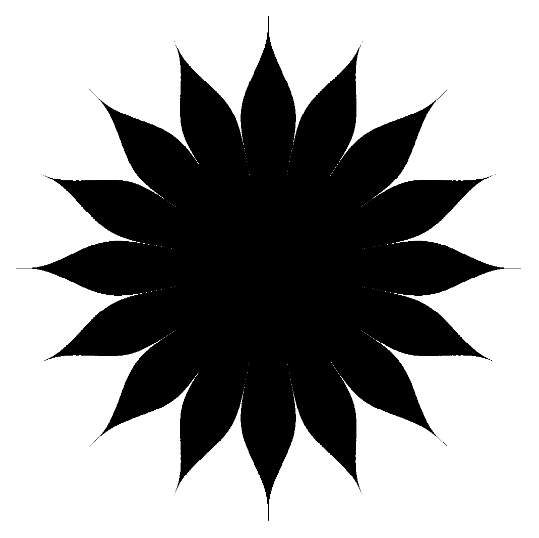
$$R = 25, \quad d = 0.04, \quad \mathcal{R} = [0, 3], \quad \mathcal{L} = [0.4, 1.1] \times 10^{-6}$$

where all metric quantities are in meters.

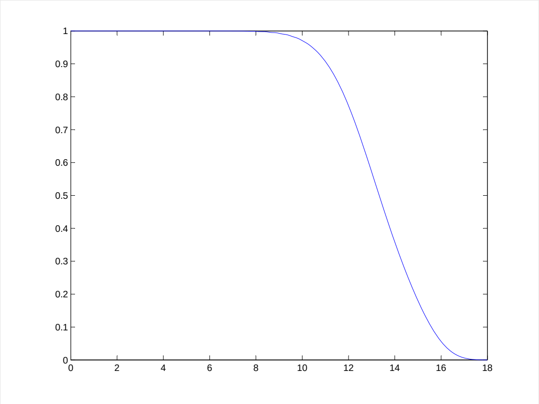
An infinite dimensional linear programming problem.
Discretize:

- $[0, R]$ into 5000 evenly space points.
- $\mathcal{R}$ into 150 evenly spaced points.
- $\mathcal{L}$ into increments of 0.1×10^{-6}.

Petal-Shaped Occulters



16-Petal Occulter $A(r,\theta)$



Radial Attenuation $A(r)$

- From Jacobi-Anger expansion we get:

$$\begin{aligned} E(\rho,\phi) &= 1 - \frac{2\pi}{i\lambda z}\int_0^R e^{\frac{i\pi}{\lambda z}(r^2+\rho^2)} J_0\left(\frac{2\pi r\rho}{\lambda z}\right) A(r) r dr \\ &\quad - \sum_{k=1}^{\infty} \frac{2\pi(-1)^k}{i\lambda z}\left(\int_0^R e^{\frac{i\pi}{\lambda z}(r^2+\rho^2)} J_{kN}\left(\frac{2\pi r\rho}{\lambda z}\right)\frac{\sin(\pi k A(r))}{\pi k} r dr\right) \\ &\quad \times \left(2\cos(kN(\phi - \frac{\pi}{2}))\right) \end{aligned}$$

  where N is the number of petals.

- For small ρ, truncated summation well-approximates full sum.
- Truncated after 10 terms.
- $\lambda \in [0.4, 1.1]$ microns.
- $z = 72,000$ km, $R = 25$ m.
- In angular terms, $R/z = 73$ mas.