# Detecting ExtraSolar Planets: A Survey of Methods and Results

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2007 Dec 10

PICASso

http://www.princeton.edu/∼rvdb

# Are We Alone?

# Indirect Detection Methods

$\geq 250$ planets found so far

# Wobble Methods

**Radial Velocity.**

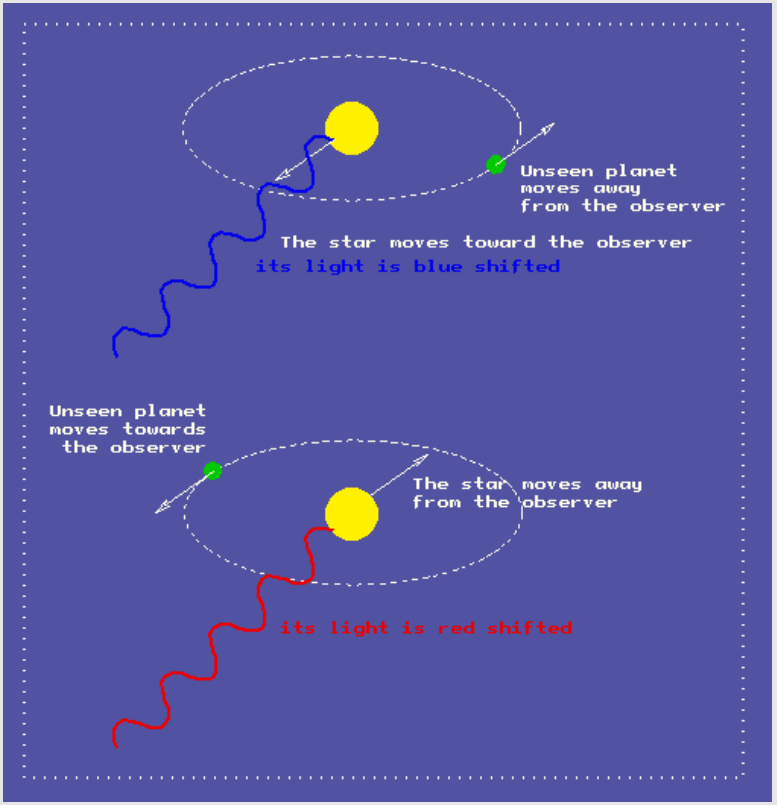
For edge-on systems.

Measure periodic doppler shift.

**Astrometry.**

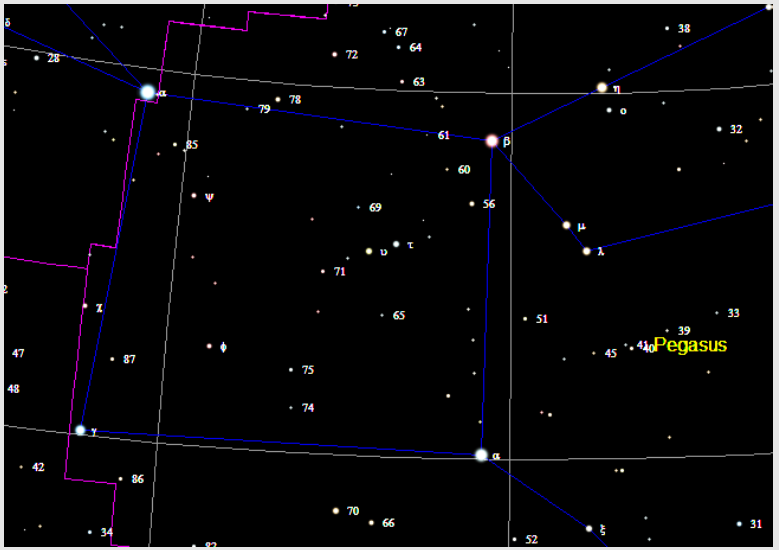
Best for face-on systems.

Measure circular wobble against background stars.



# First Discovery: 51 Pegasi b

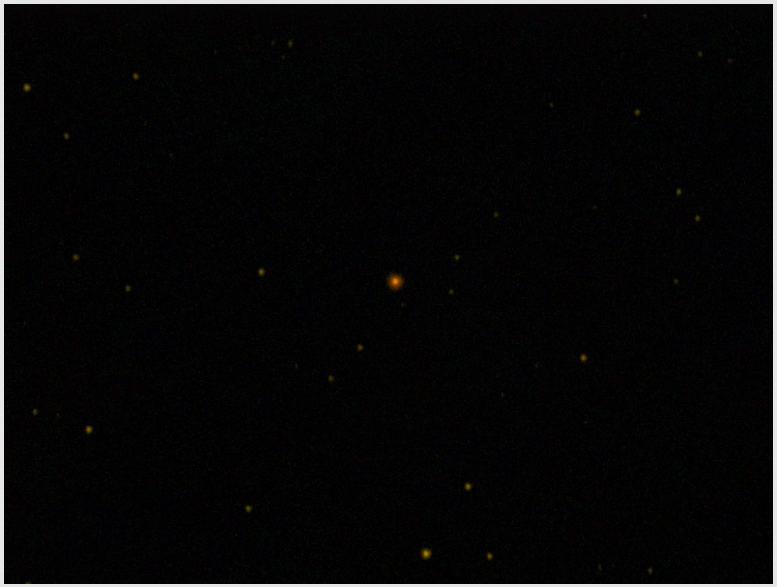
- Mayor and Queloz (1995)
- Mag. $5.5$ main sequence star
- Detected by *radial velocity* method
- Velocity difference: $70$ m/s $= 160$ mph
- Period: $4.2$ days
- Separation: $0.05$ AU
- Angular separation: 0.0035 arcseconds
- Mass: $> 0.47 M_{\mathrm{J}}$
- Hot Jupiter



# Notable Recent Discovery: Gliese 581c

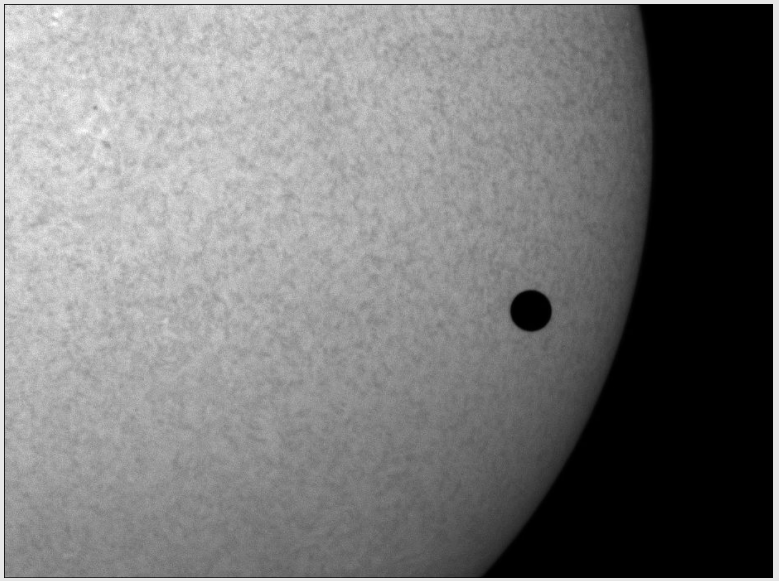
## Possibly Terrestrial

- Mag. $10.5$ red dwarf
- Detected by radial velocity method
- Period: $13$ days
- Separation: $0.07$ AU
- Angular Separation: $0.012$ arcseconds
- Mass: $> 5M_{\mathsf{E}}$

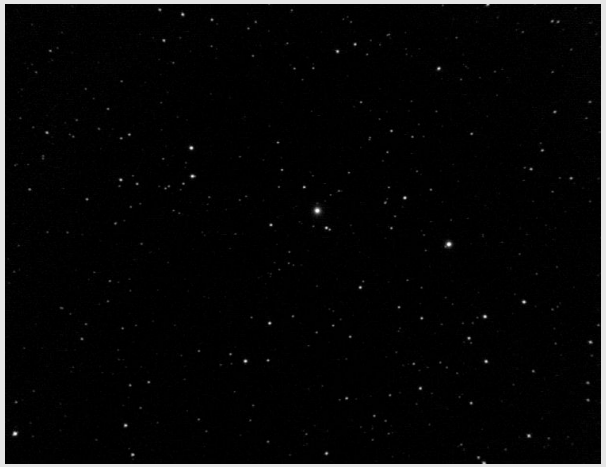


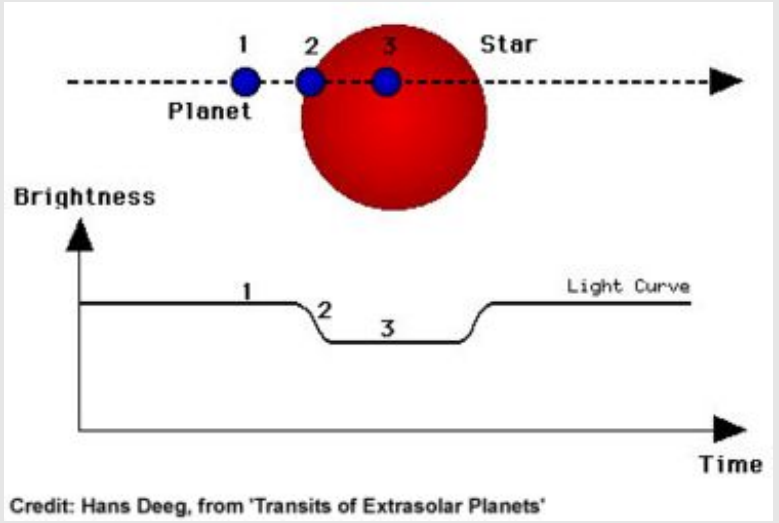
# Transit Method

- HD209458b confirmed both via RV and transit.
- Period: $3.5$ days
- Separation: $0.045$ AU ($0.001$ arcsecs)
- Radius: $1.3R_{\mathrm{J}}$
- Intensity Dip: $\sim 1.7\%$
- Venus Dip $= 0.01\%$, Jupiter Dip: $1\%$
- Kepler and Corot



Venus Transit (R.J. Vanderbei)

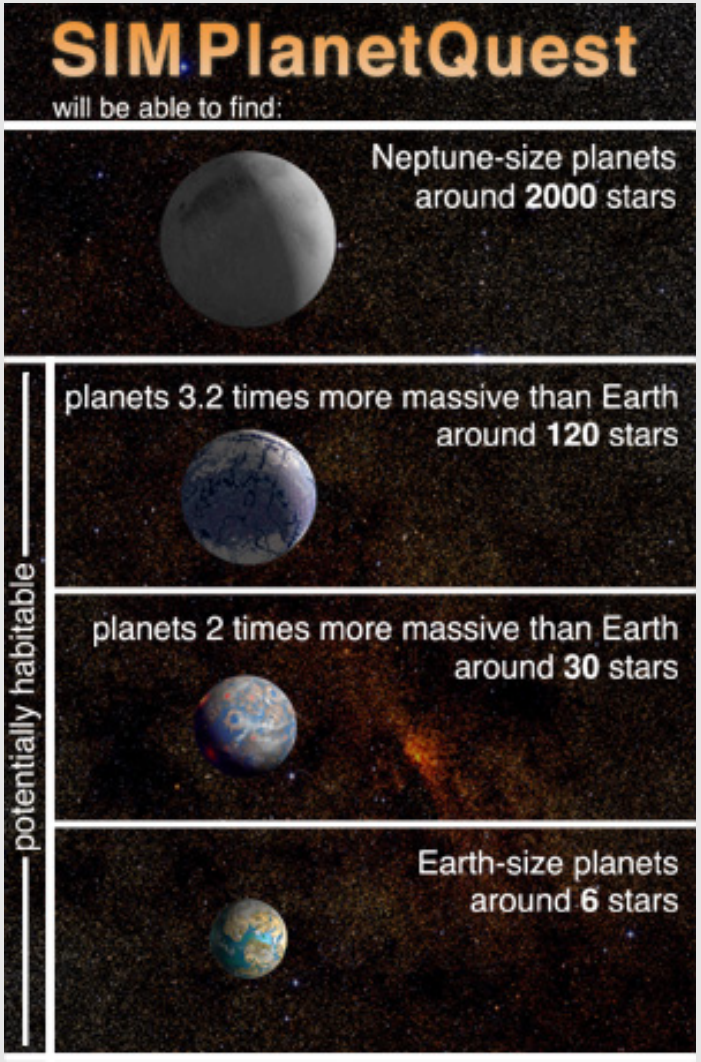





Credit: Hans Deeg, from 'Transits of Extrasolar Planets'

# Astrometry

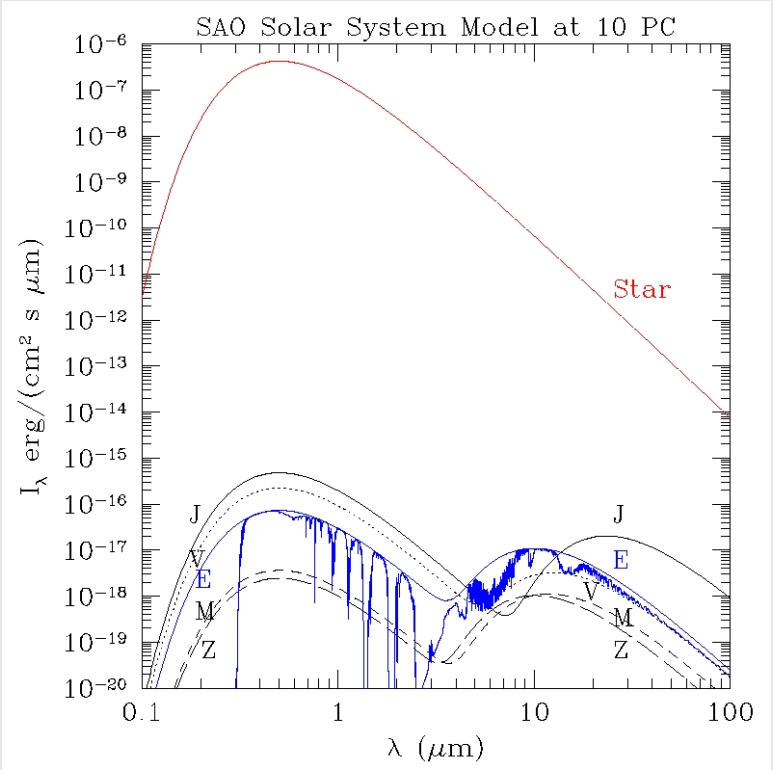
- *Space Interferometry Mission (SIM)*
- Wobbles as small as $0.000001$ arcsecs (the thickness of a nickel viewed from the distance of the moon).
- Mission Cancelled



# Direct Detection

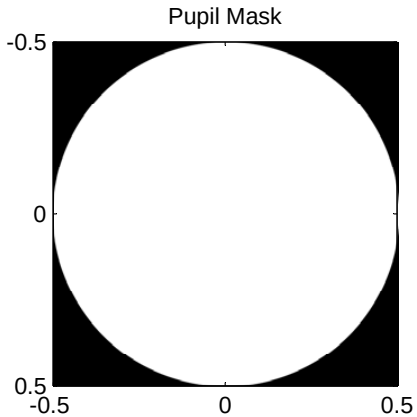
# Why It's Hard

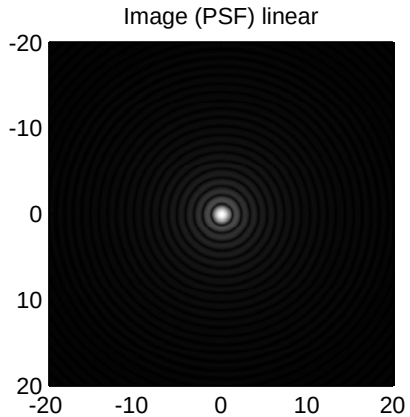
- *Bright Star/Faint Planet:* In visible light, our Sun is $10^{10}$ times brighter than Earth. That's 25 mags.
- *Close to Each Other:* A planet at $1$ AU from a star at $10$ parsecs can appear at most $0.1$ arcseconds in separation.
- *Far from Us:* There are less than $100$ Sun-like stars within $10$ parsecs.

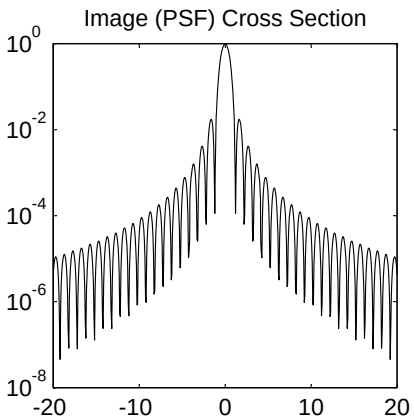


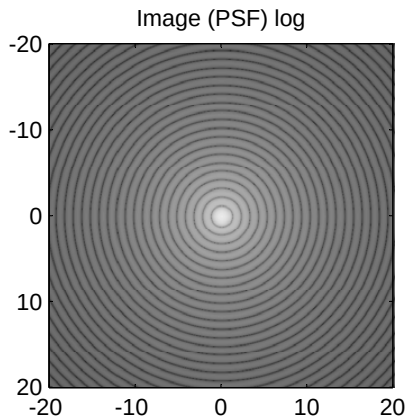
# Telescope w/ Unobstructed Aperture

Doesn't Work! Requires an aperture measured in kilometers to mitigate diffraction effects.



# Apodized Pupil Coronagraph (Unmanufacturable)

Apodized pupil

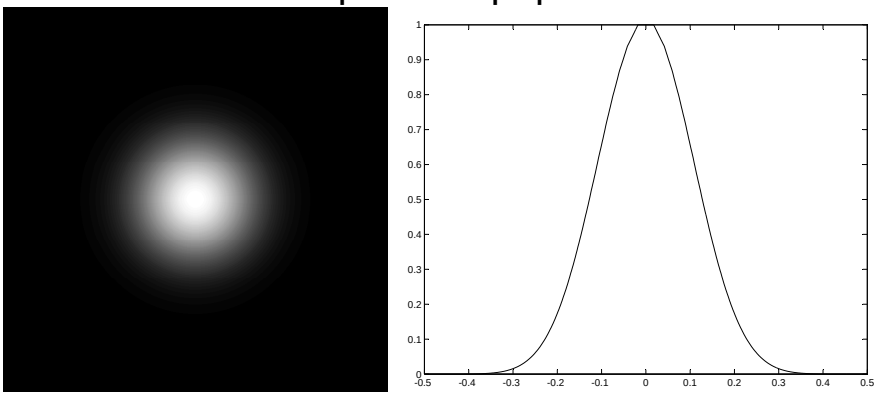
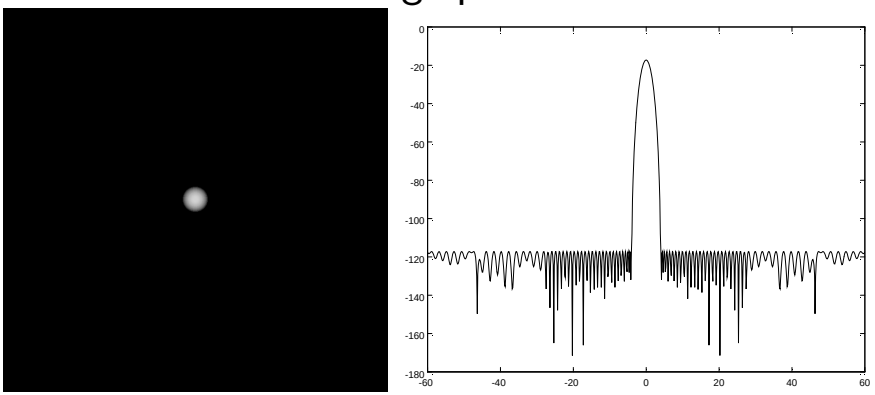
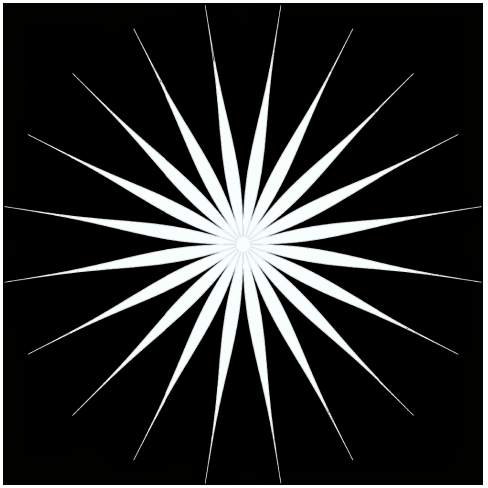


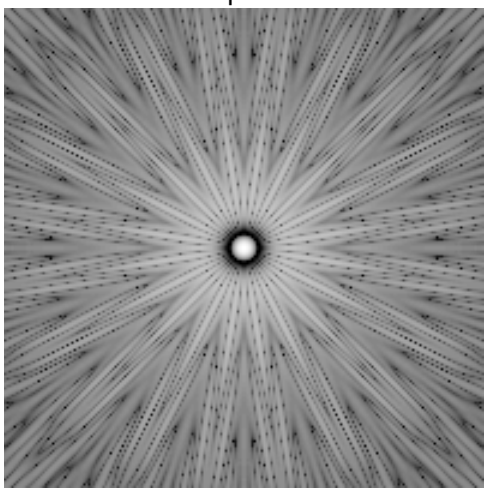

Image plane



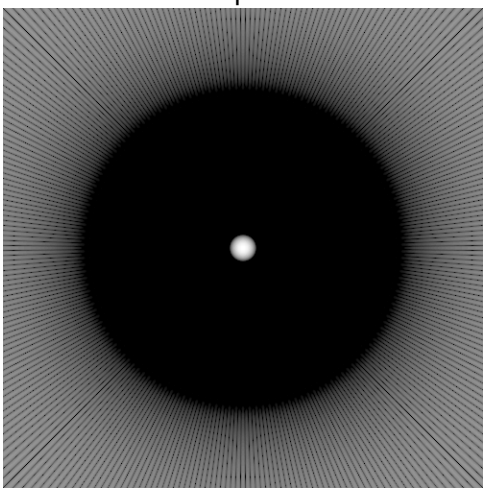
# Shaped Pupil Coronagraph (TPF-C)



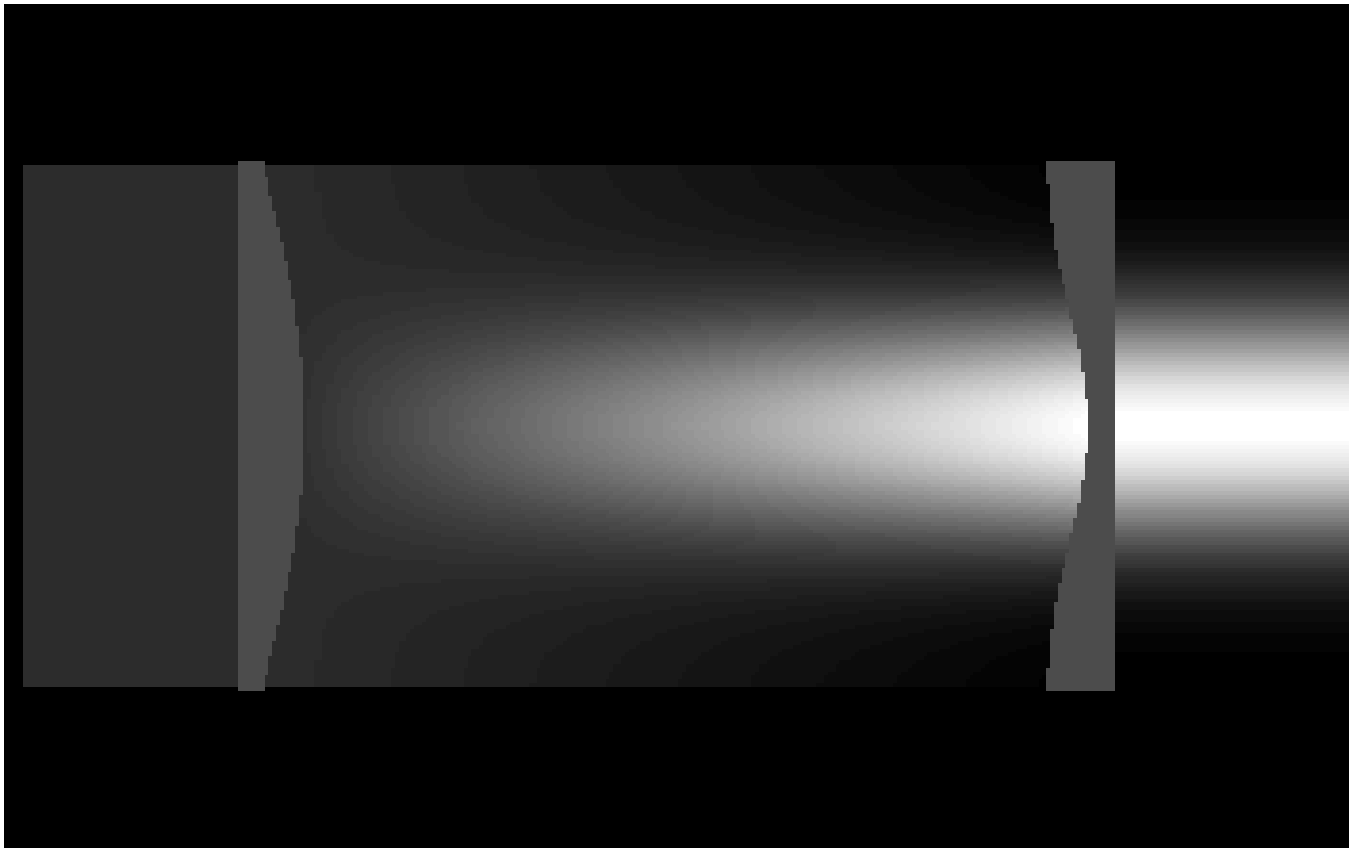
20 petals



150 petals



# Pupil Mapping (TPF-C)

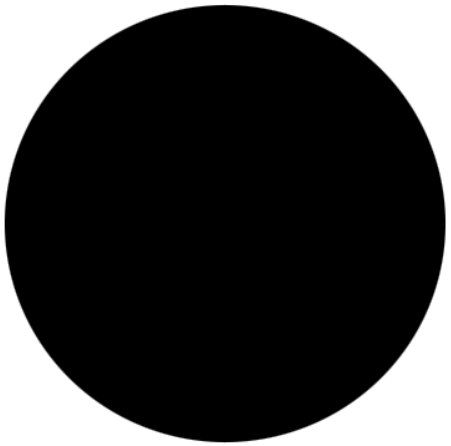


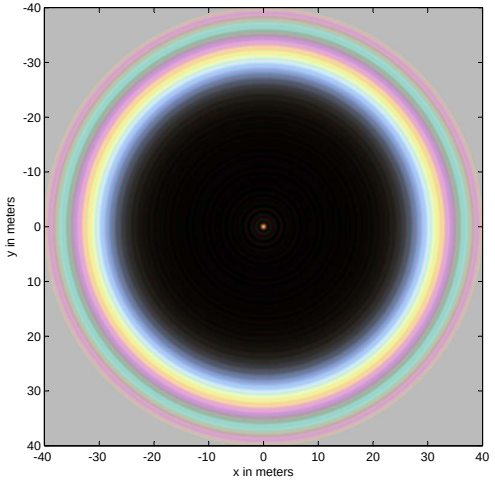
All above methods require optics of extraordinary quality: $1/100,000$ wave precision.

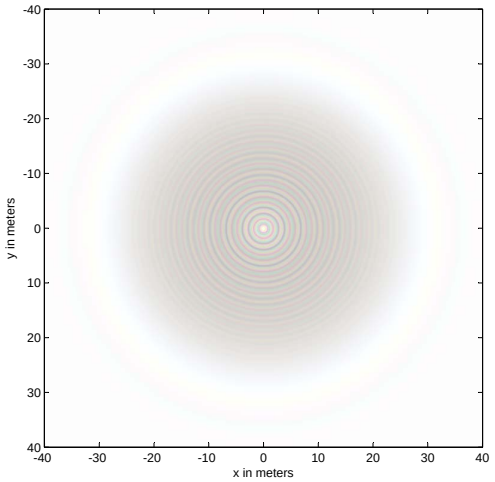
# Space-based Occulter (TPF-O)

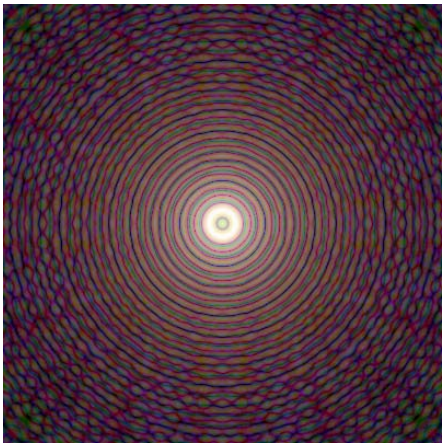
Telescope Aperture: $4$m, Occulter Diameter: $50$m, Occulter Distance: $72,000$km

# Plain External Occulter (Doesn't Work!)

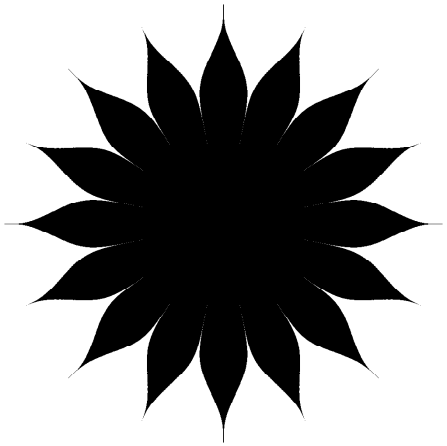


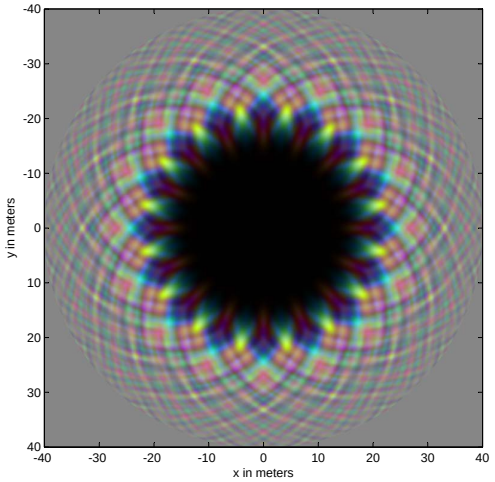


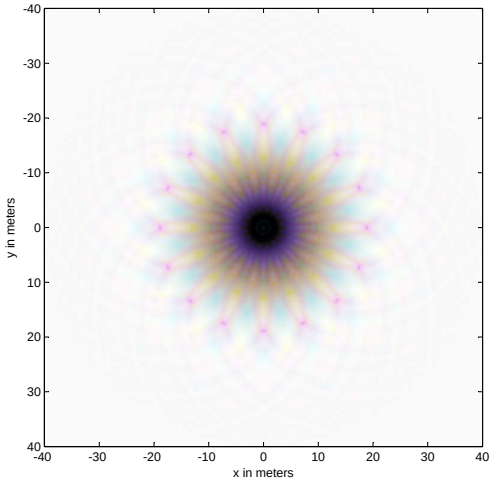



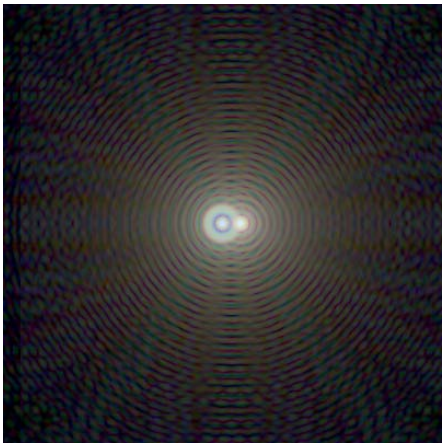



# Shaped Occulter

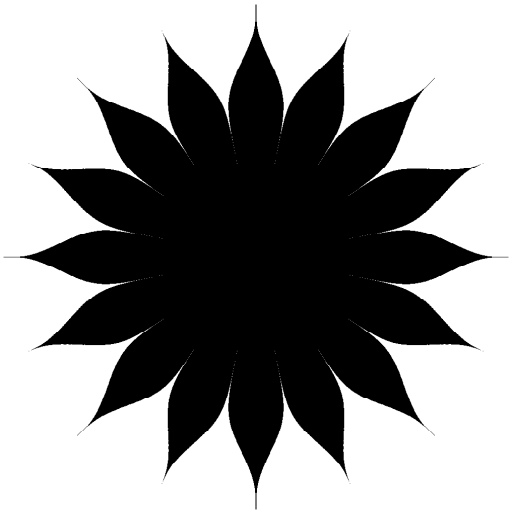




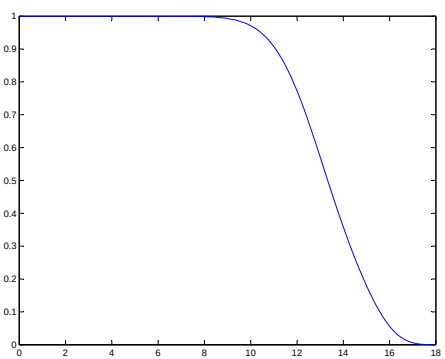



# Petal-Shaped Occulters



16-Petal Occulter $A(r, \theta)$




Radial Attenuation $A(r)$

- Babinet's principle plus Fresnel propagation:

$$E(\rho, \phi) = 1 - \frac{1}{i\lambda z} \int_0^{\infty} \int_0^{2\pi} e^{\frac{i\pi}{\lambda z}(r^2+\rho^2-2r\rho\cos(\theta-\phi))} A(r,\theta) r d\theta dr.$$

- From Jacobi-Anger expansion we get:

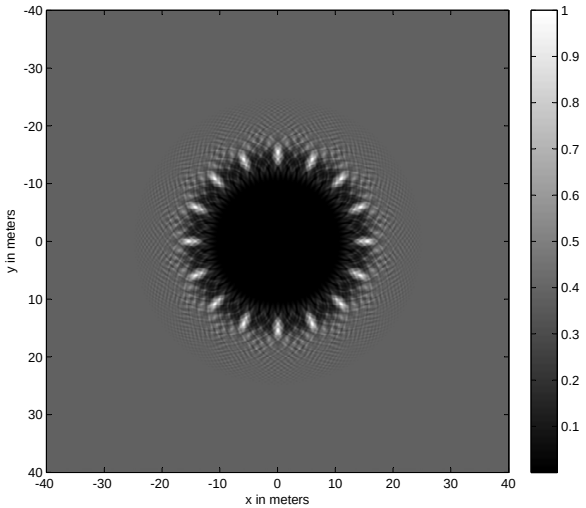
$$\begin{aligned} E(\rho,\phi) &= 1 - \frac{2\pi}{i\lambda z} \int_0^R e^{\frac{i\pi}{\lambda z}(r^2+\rho^2)} J_0\left(\frac{2\pi r\rho}{\lambda z}\right) A(r) r dr \\ &\quad - \sum_{k=1}^{\infty} \frac{2\pi(-1)^k}{i\lambda z} \left( \int_0^R e^{\frac{i\pi}{\lambda z}(r^2+\rho^2)} J_{kN}\left(\frac{2\pi r\rho}{\lambda z}\right) \frac{\sin(\pi k A(r))}{\pi k} r dr \right) \\ &\quad \times \left(2\cos(kN(\phi - \frac{\pi}{2}))\right) \end{aligned}$$

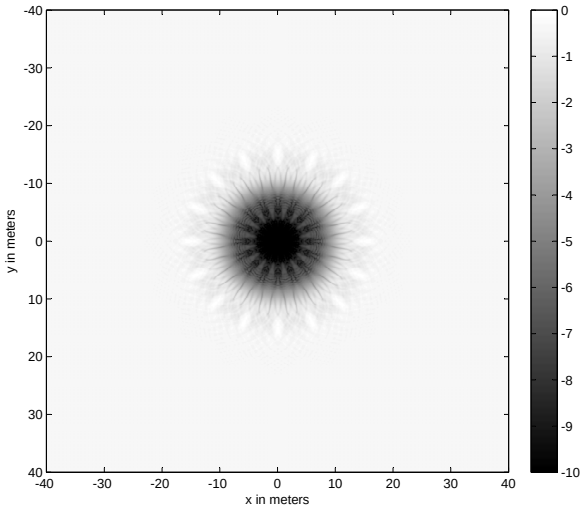
  where $N$ is the number of petals.

- For small $\rho$, truncated summation well-approximates full sum.
- Truncated after 10 terms.
- Old Baseline: $\lambda \in [0.4, 1.1]$ microns, $z = 18,000$ km, $R = 18$ m.

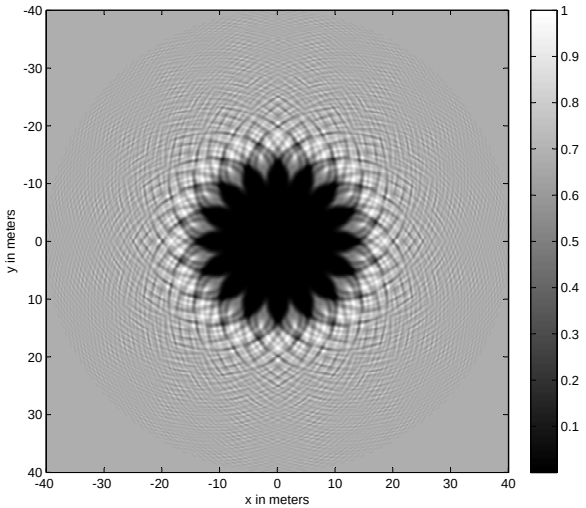
# Intensity at Telescope Pupil—16 Petals

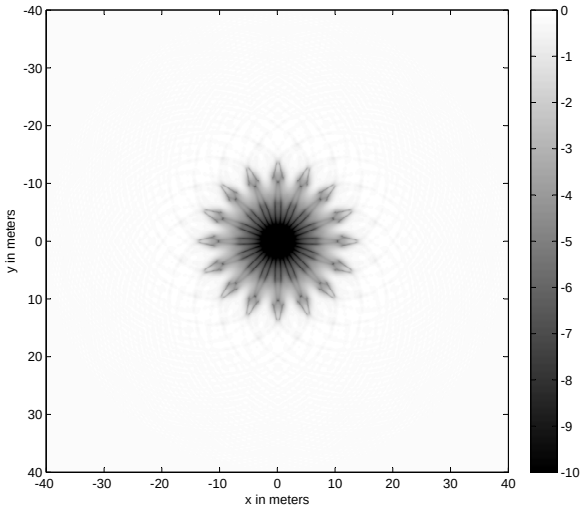
$\lambda = 0.4$ microns



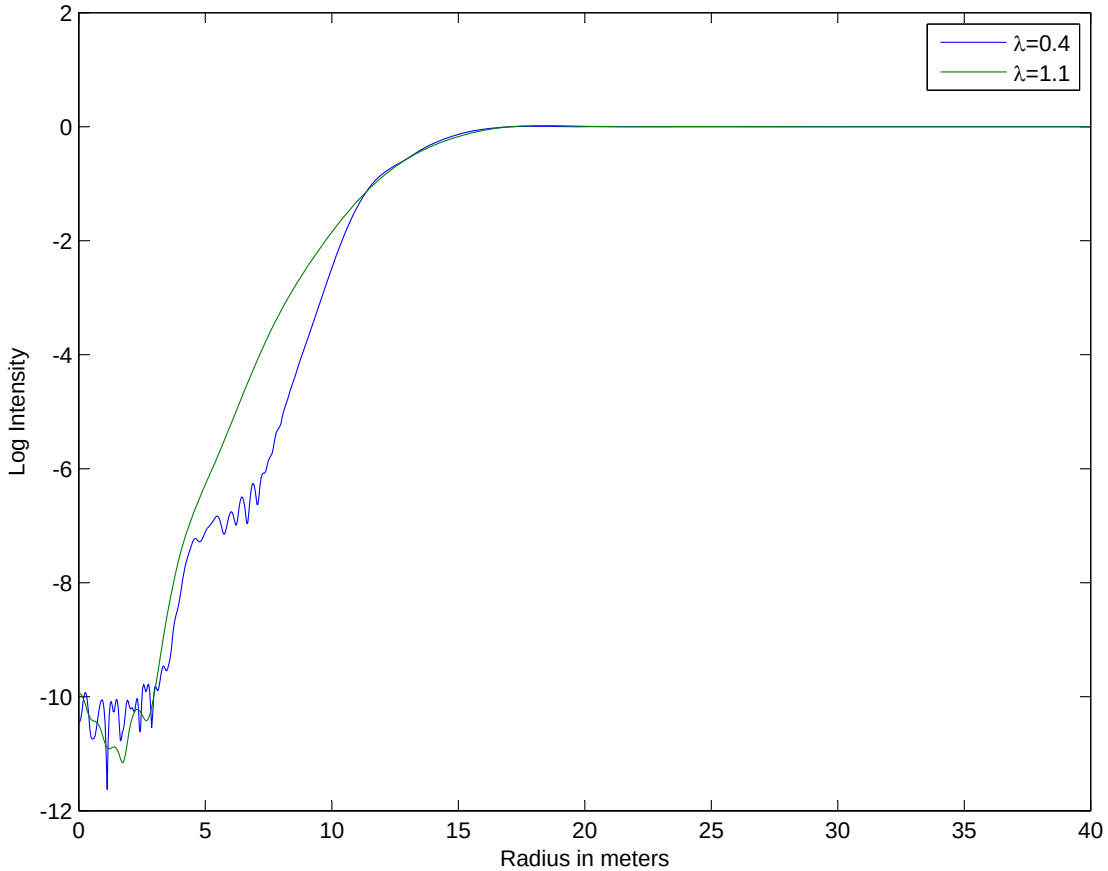




$\lambda = 1.0$ microns



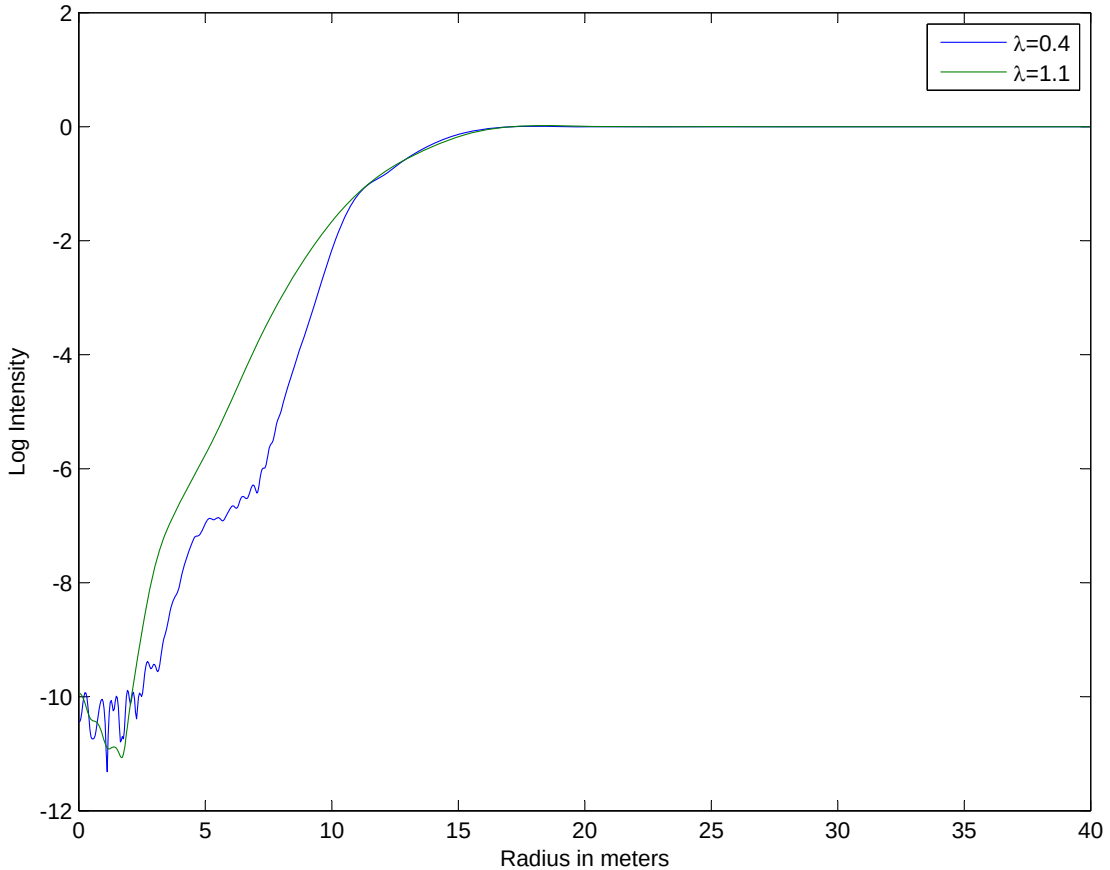



# Radial Plots—16 Petals




GOOD ENOUGH!

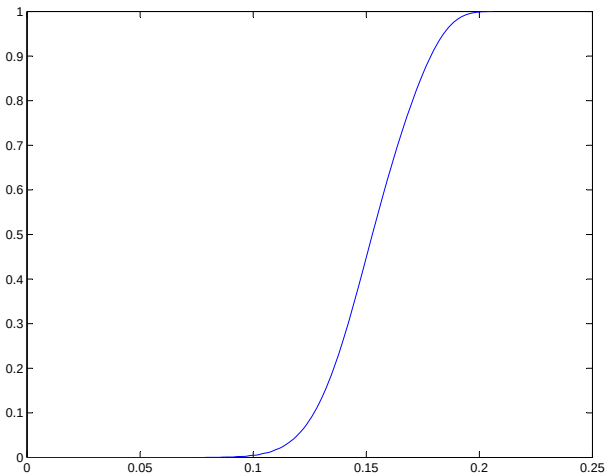
# Radial Plots—12 Petals




NOT GOOD ENOUGH!

# Throughput vs. Angular Separation

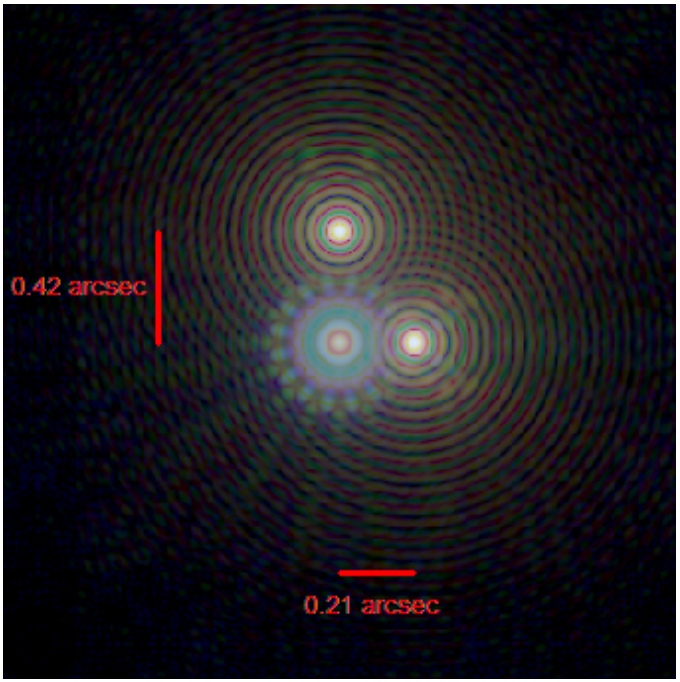
- Radius at tip: $18$ m.
- Occulter distance: $18,000$ km.
- Angle between tip and on-axis: $0.2$ arcseconds.
- Planet virtually unattenuated at tip.
- Throughput quickly drops to zero as separations decrease.
- $50\%$ throughput at $0.15$ arcseconds.




IS IT GOOD ENOUGH?

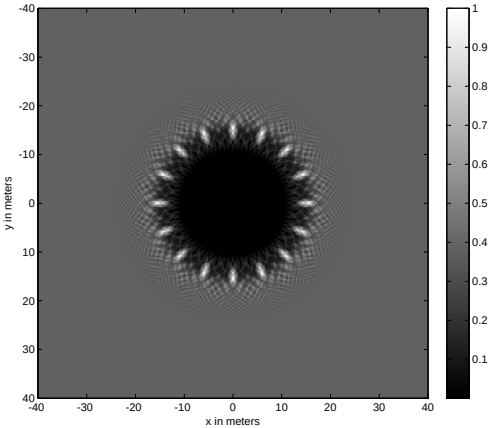
# Image Plane

- Residual starlight dispersed but concentrated in center.
- Three $10^{-10}$ planets.
  - $0.4$ arcsecond separation at 12 o'clock.
  - $0.2$ arcsecond separation at 3 o'clock.
  - $0.1$ arcsecond separation at 6 o'clock (INVISIBLE).
- RGB log-stretched image with
  - R $= 1.0$ microns
  - G $= 0.7$ microns
  - B $= 0.4$ microns

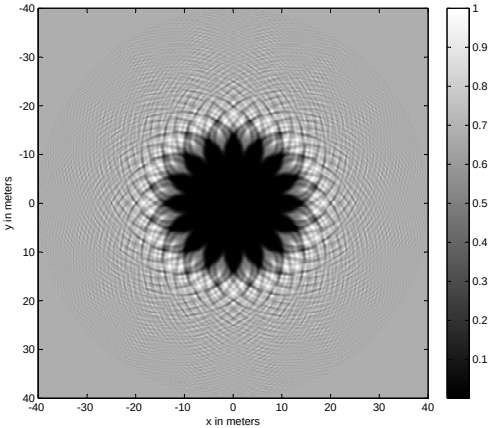


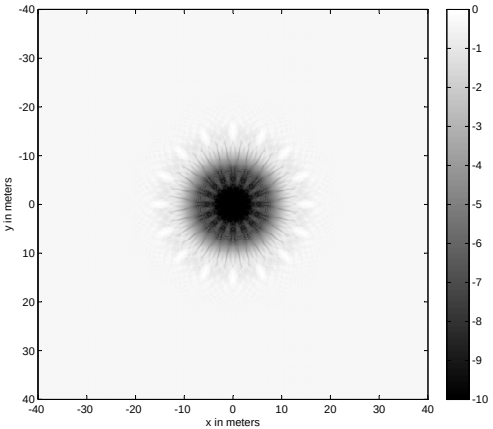
# $z = 18000$ **km,** $R = 18$ **m**

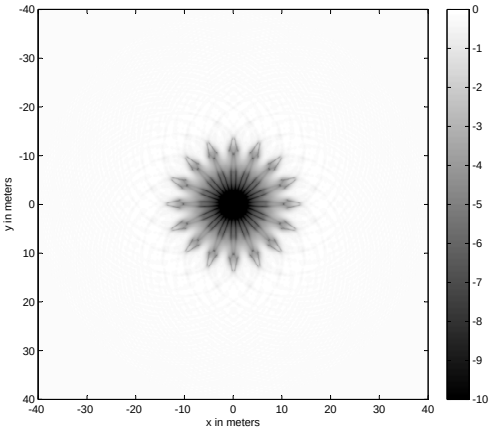
$\lambda = 0.4$ microns

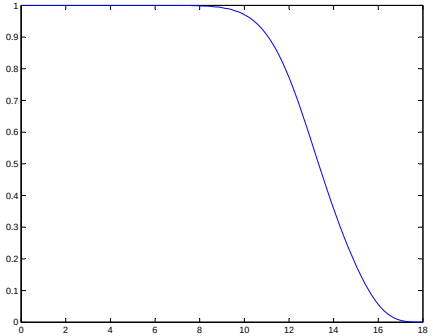


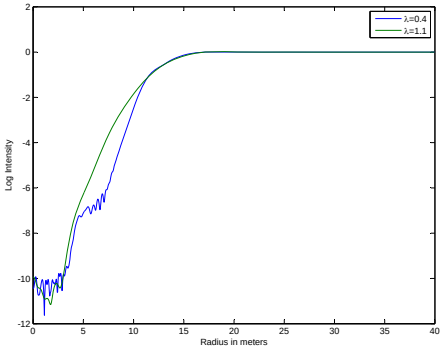

$\lambda = 1.0$ microns

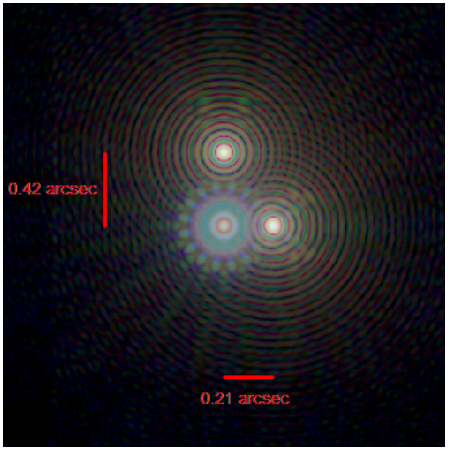


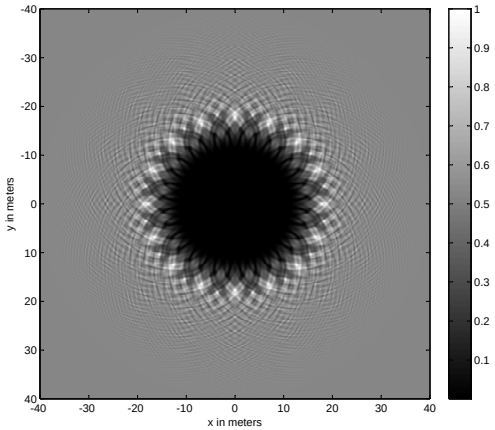





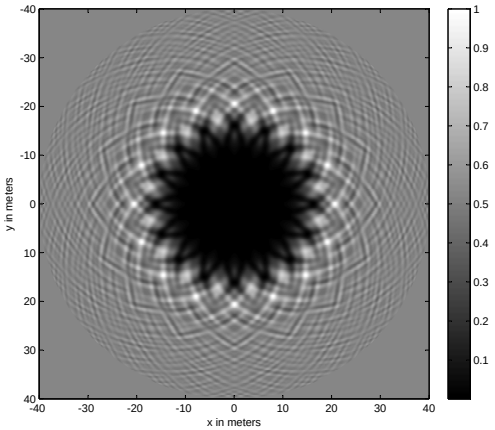



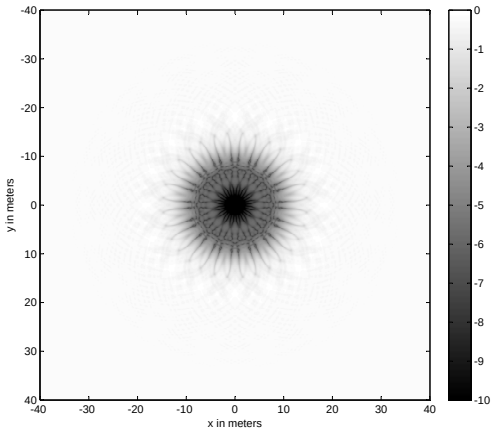
# $z = 36000$ **km,** $R = 20$ **m**

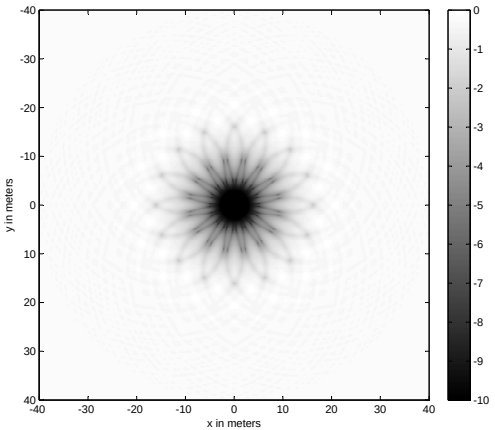
$\lambda = 0.4$ microns

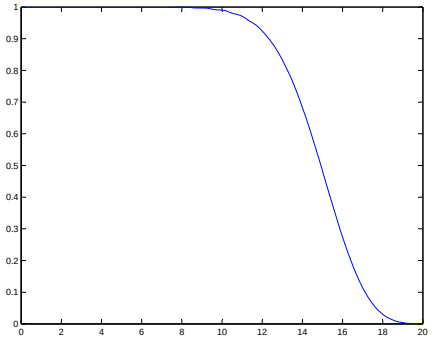


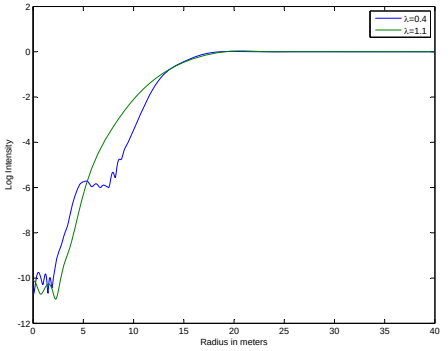

$\lambda = 1.0$ microns

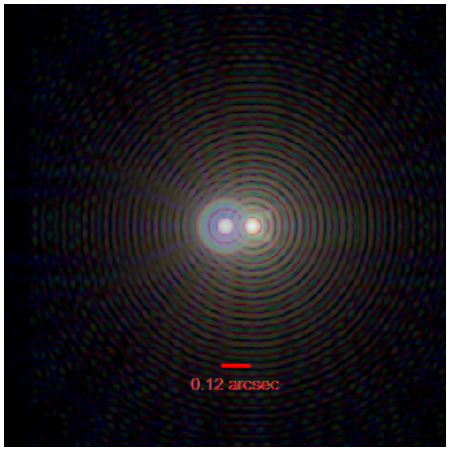


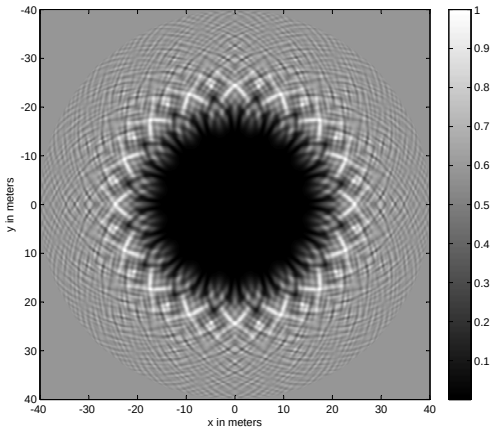





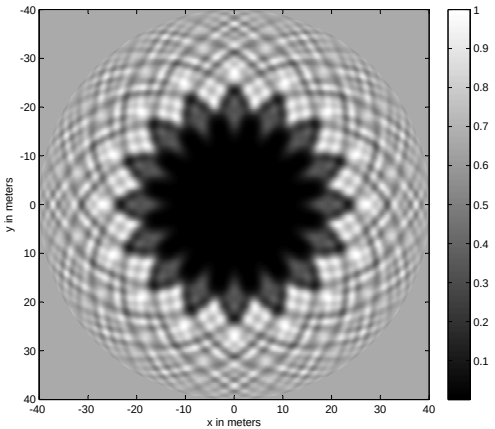



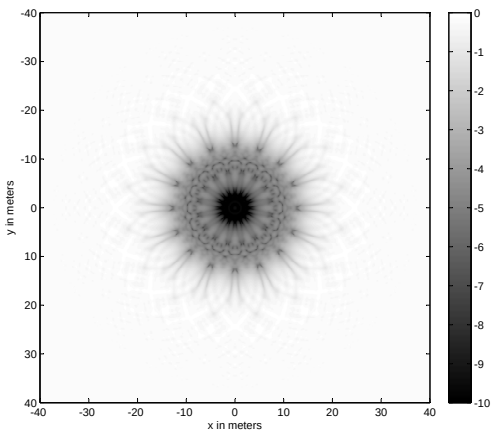
# $z = 72000$ **km**, $R = 25$ **m**

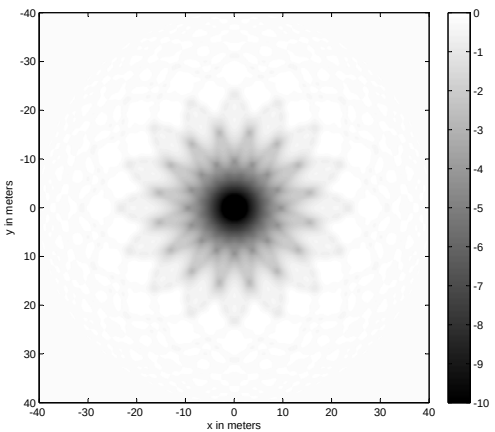
$\lambda = 0.4$ microns

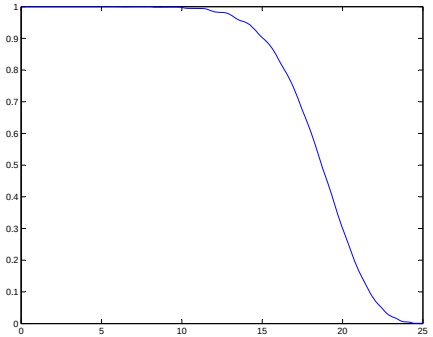


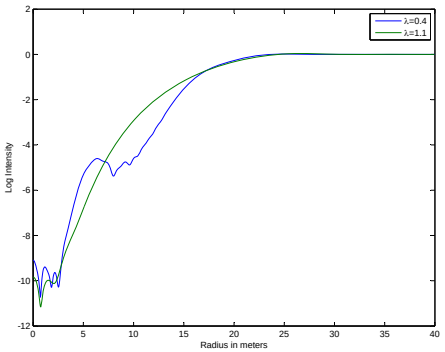

$\lambda = 1.0$ microns

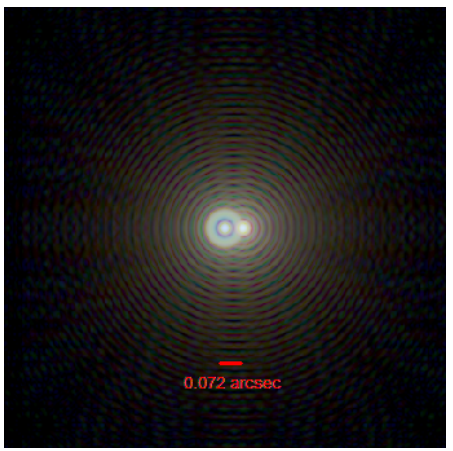


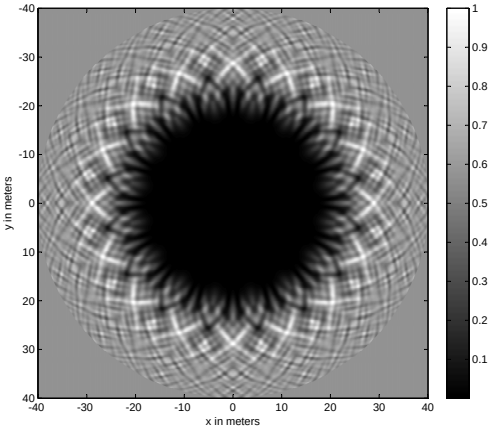





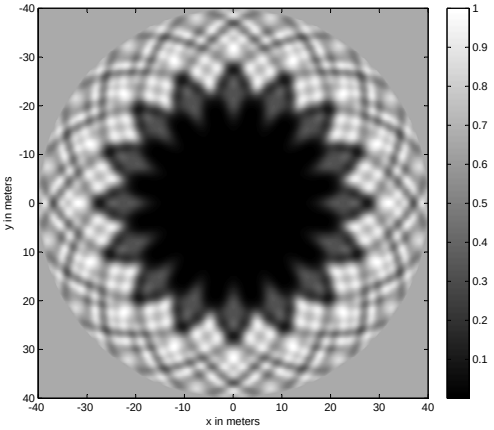



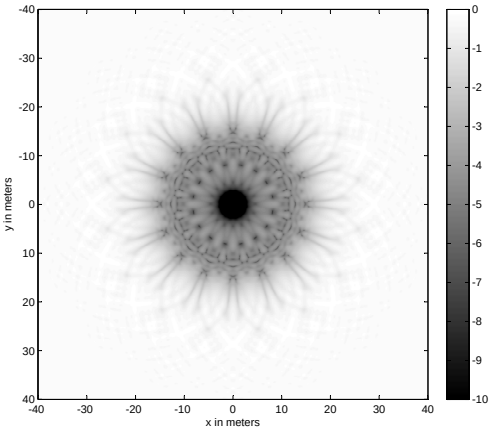
# $z = 100000$ **km,** $R = 30$ **m**

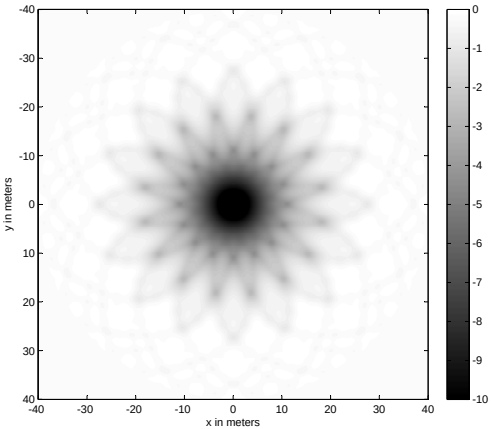
$\lambda = 0.4$ microns

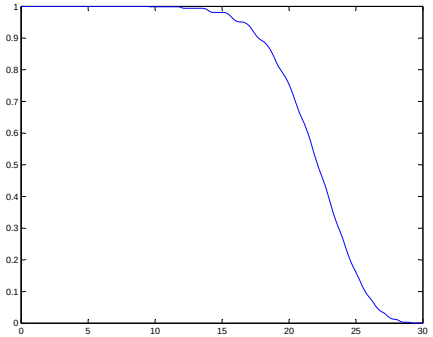


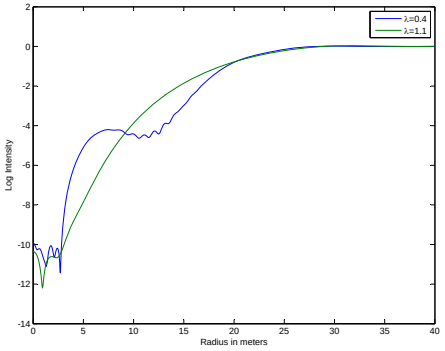

$\lambda = 1.0$ microns

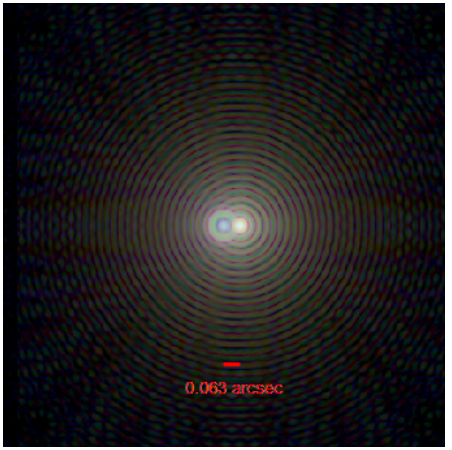


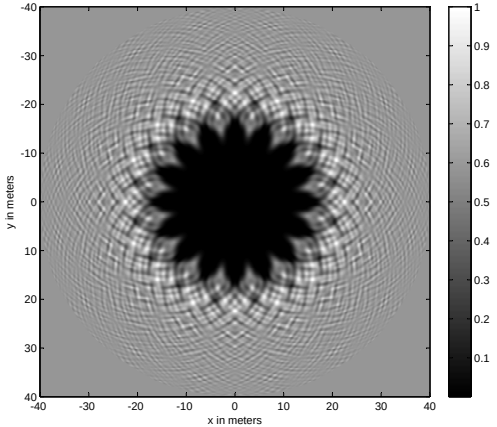





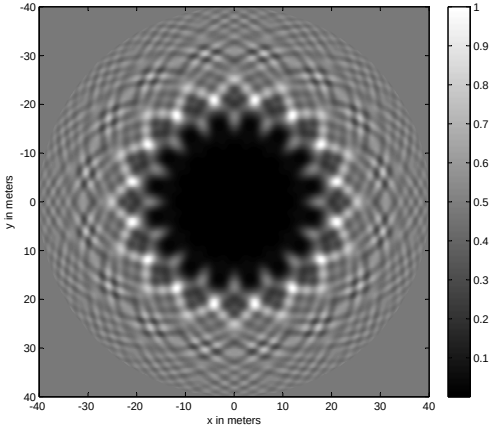



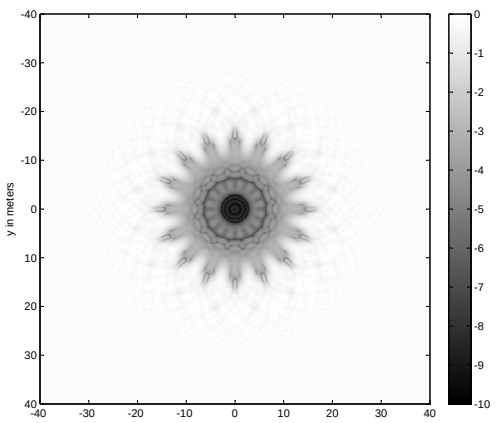
# $z = 66000$ **km**, $R = 22$ **m**, $Q = 1$

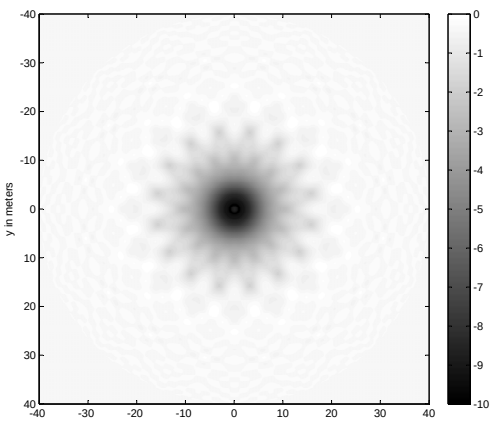
$\lambda = 0.4$ microns

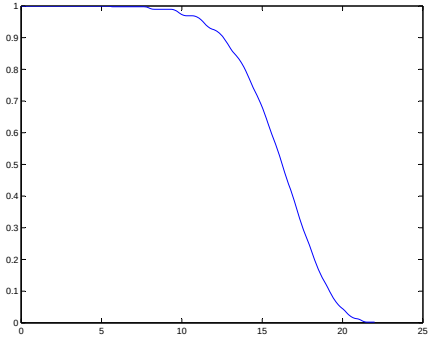


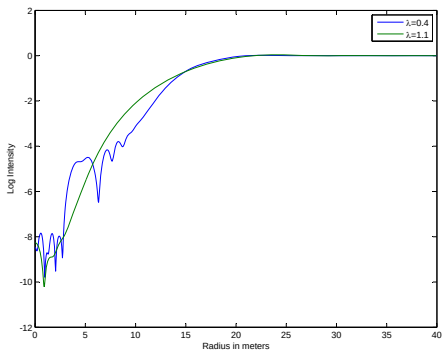

$\lambda = 1.0$ microns

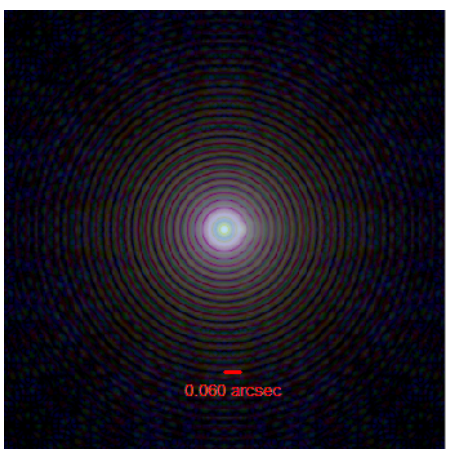




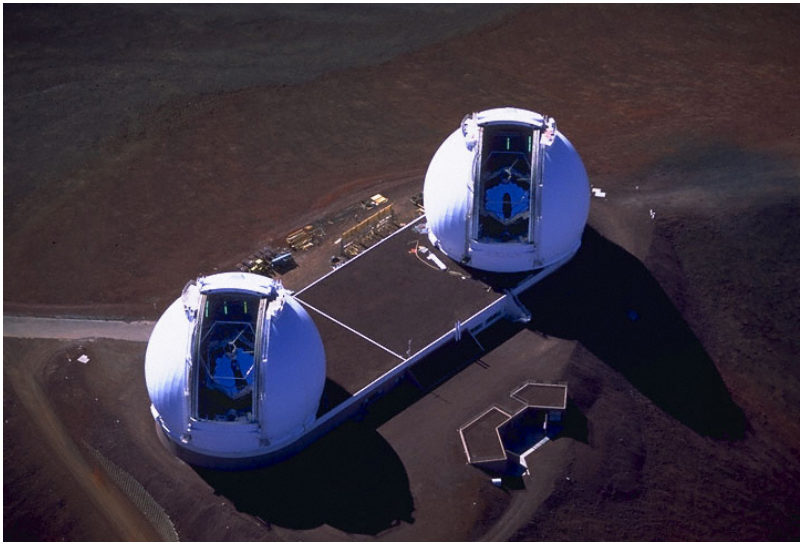



# Mission Comparisons

For 30 stars, 75 visits, 2 week travel time.

| | Separation/Mask radius | |
|---|---|---|
| | $18000/15$ | $72000/25$ |
| Retargeting $\Delta v$ (m/s) | 700 | 2500 |
| Total $\Delta v$ (m/s) | 1600 | 4250 |
| Mass (kg) | 2111 | 3300 |

# Ground-Based Possibilities



- Atmospheric *seeing* limits resolution to about $1$ arcsec.
- Large aperture with adaptive optics.
- Interferometry.