Optimization in Engineering Design

Robert J. Vanderbei

2009 October 5

Computational Engineering and Science/HPC

Lehigh University

http://www.princeton.edu/∼rvdb

Collaborators

Optimization:

- Dave Shanno, Rutgers
- Hande Benson, Drexel

Exoplanets:

- Jeremy Kasdin, Mechanical & Aerospace Eng., PU
- David Spergel, Astrophysics, PU
- and many others at JPL, Goddard, Arizona, Cornell, MIT, ...

Celestial Mechanics:

- J. Richard Gott, Astrophysics, PU
- Ege Kolemen, Princeton Plasma Physics Lab

What is High Performance Computing?

- Computing that requires a supercomputer (whatever that is).
- Computing that requires multiple processors.
- Computing that requires a large number of processors computing in parallel.
- Computing that requires high precision (quad and beyond)?
- Computing that consumes enough energy to require air-conditioning.
- Computing that makes a laptop computer too hot for your lap.

Engineering = Optimization

Make something.

Make it better.

Make it better yet.

Make it still better.

⋮

Types of Optimization Problems

Constrained vs. unconstrained.

Convex vs. nonconvex.

Smooth vs. nonsmooth.

Continuous vs. discrete (integer).

Local vs. global.

My main interest: finding *locally optimal* solutions to *nonconvex, constrained, smooth* optimization problems in which all variables are assumed to be *continuous* (based on LOQO).

Types of Algorithms

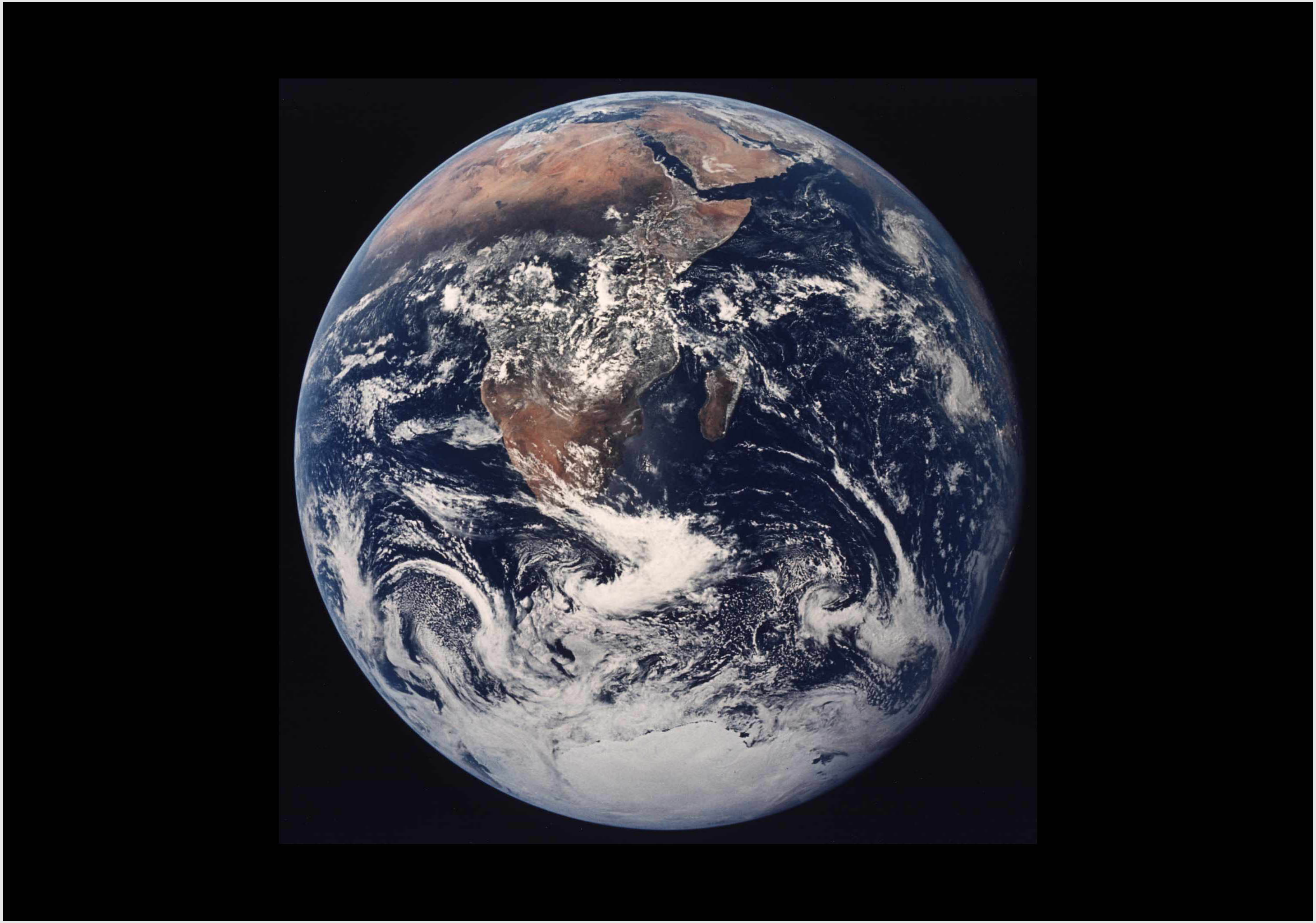
Black-box-around-legacy-code vs. start-from-scratch (Dennis vs. Betts).

My main interest: the *start-from-scratch* approach (based on AMPL).

Telescope Design

for Planet-Finding

The Fundamental Question: Are We Alone?

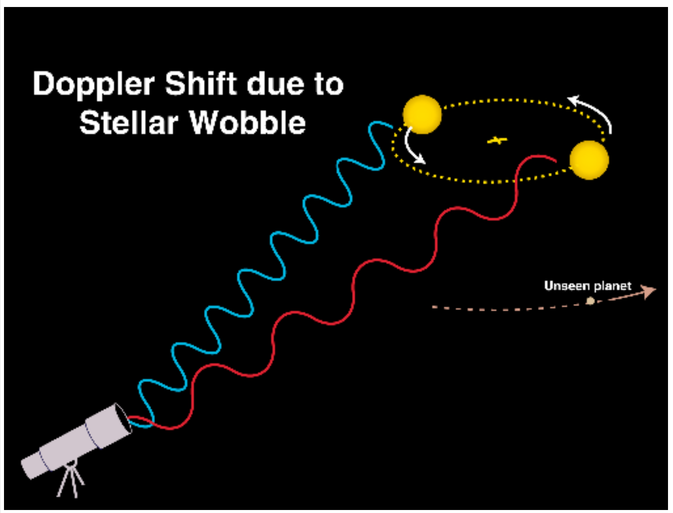


Why Search NOW?

Because we can: More than 300 planets found so far via Indirect Detection Methods

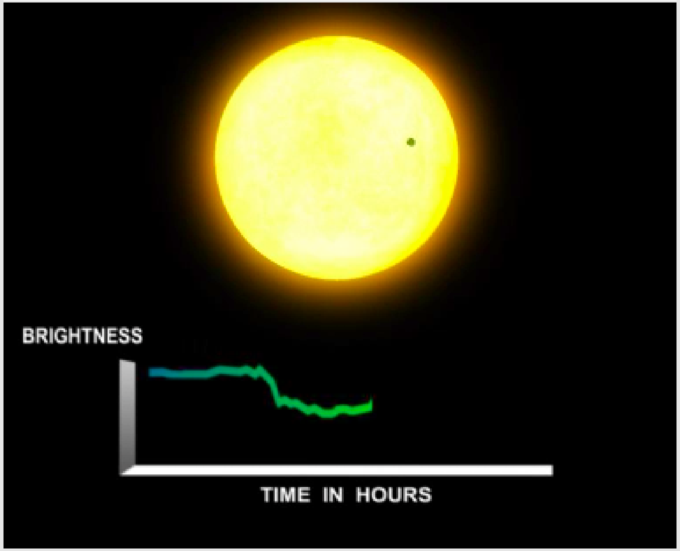
Because we must: ...

Indirect Methods



Radial Velocity.

For edge-on systems.
Measure periodic doppler shift.



Transits.

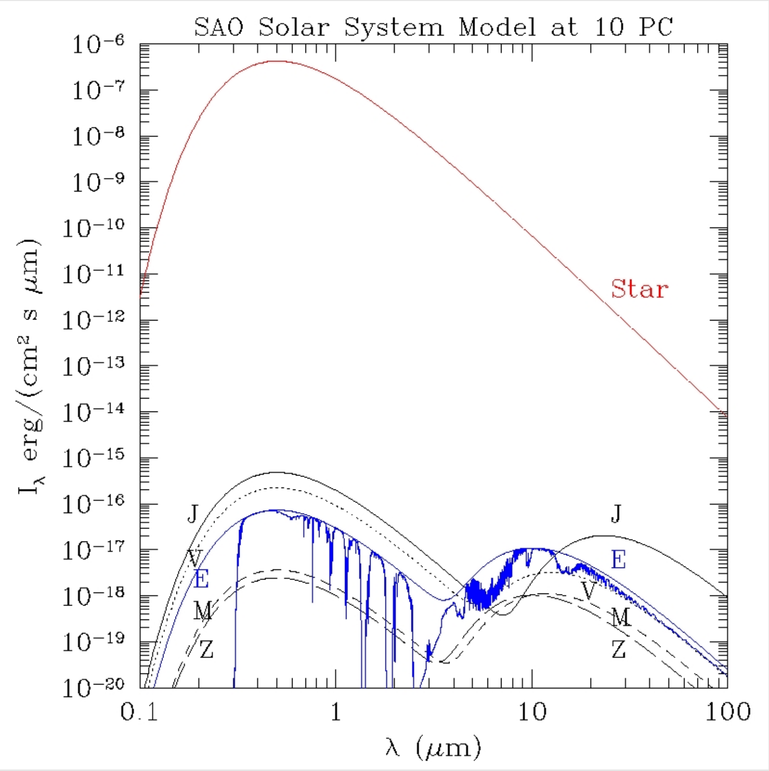
Only works for edge-on systems.
Watch star fade and rebrighten as planet moves across face.

Direct Detection

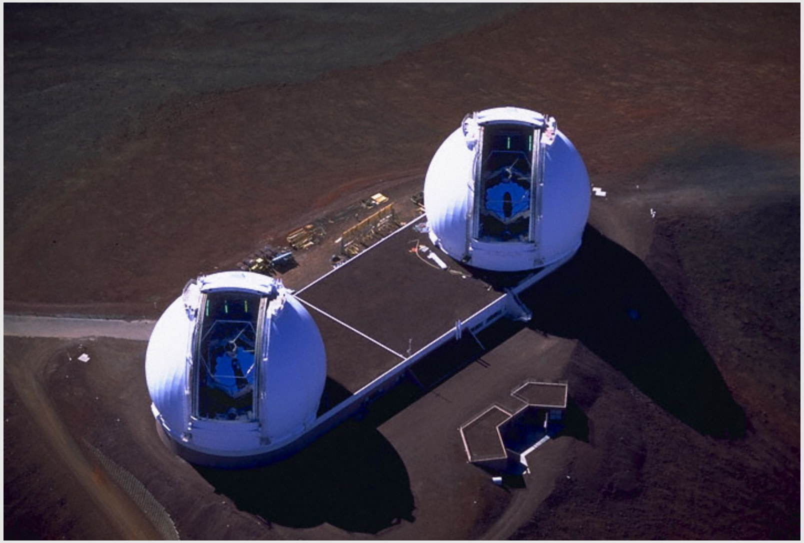
Why It's Hard

Premise: If there is intelligent life "out there", it probably is similar to life as we know it on Earth.

- *Bright Star/Faint Planet:* In visible light, our Sun is ten billion times brighter than Earth.
- *Close to Each Other:* A planet at 1 AU from a star at 33 light-years can appear at most 0.1 arcseconds in separation. (The full moon is 1800 arcseconds in diameter.)
- *Far from Us:* There are less than 100 Sun-like stars within 33 light-years.



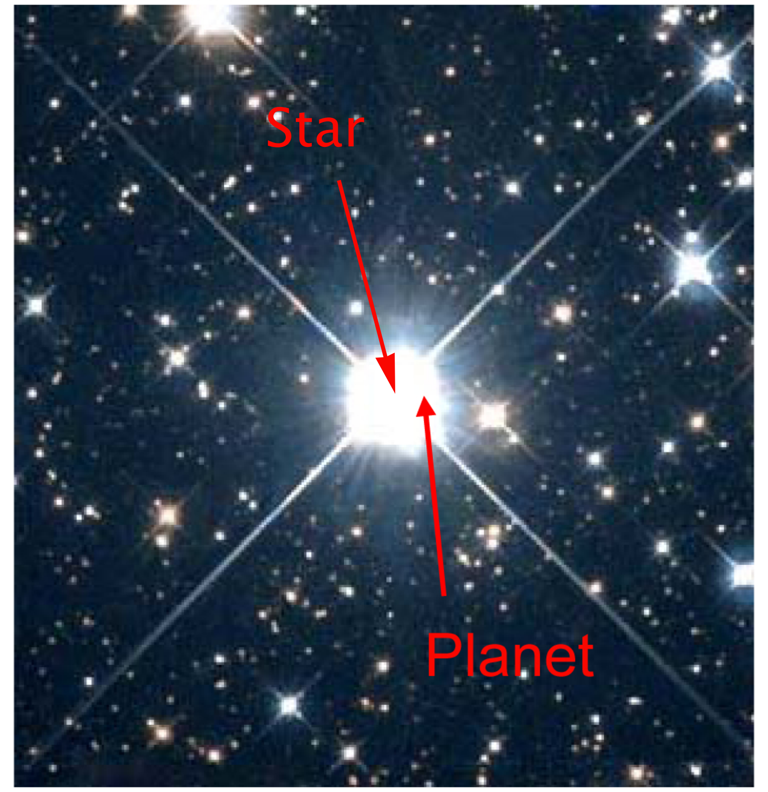
Can Ground-Based Telescopes Do It?

- Atmospheric distortion limits *resolution* to about 1 arcsec.
  Note: Resolution refers to equally bright objects.
  If one is much brighter than the other, then it is more difficult.
- Large aperture with adaptive optics.

No they can't!

Can Hubble Do It?

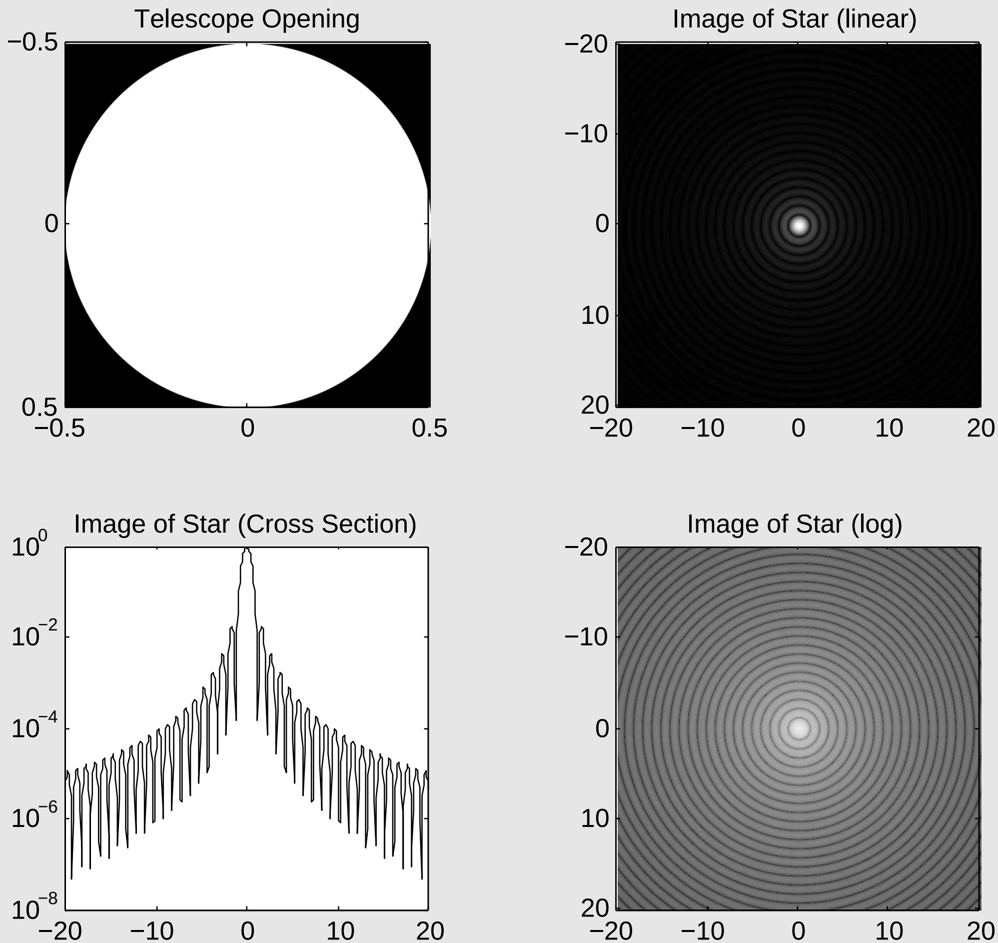


No it can't!

The problem is diffraction

The Problem is Diffraction

Requires a telescope with a mirror measured in *kilometers* to mitigate diffraction effects.



Diffraction Control via Tinting the Pupil

Physicists have understood for a long time that the abrupt edge of the telescope's "mirror" causes the bright diffraction rings.

Solution: Use tinted glass to ease the transition from transparent to opaque.

Some of the Math

The image-plane *electric field* $E()$ produced by an on-axis plane wave (i.e., starlight) and an apodized (i.e., tinted) aperture defined by an *apodization function* $A()$ is given by the *Fourier transform*:

$$E(\xi, \zeta) = \iint_{\bigcirc} e^{i(x\xi + y\zeta)} A(x, y) dy dx$$

$$\vdots$$

$$E(\rho) = 2\pi \int_0^{1/2} J_0(r\rho) A(r) r dr,$$

where J_0 denotes the 0-th order Bessel function of the first kind.

NOTE: The *electric field* depends *linearly* on the *apodization function*.

The *intensity* is the square of the electric field.

The unitless pupil-plane "length" r is given as a multiple of the aperture D.

The unitless image-plane "length" ρ is given as a multiple of focal-length times wavelength over aperture ($f\lambda/D$) or, equivalently, as an angular measure on the sky, in which case it is a multiple of just λ/D. (Example: $\lambda = 0.5\mu$m and $D = 10$m implies $\lambda/D = 10$mas.)

Optimization

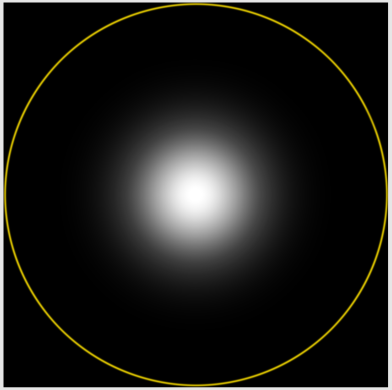
Find *apodization* function $A()$ that solves:

$$\begin{array}{lll} \text{maximize} & \displaystyle\int_0^{1/2} A(r) 2\pi r dr & \\ \text{subject to} & -10^{-5} E(0) \leq E(\rho) \leq 10^{-5} E(0), & \rho_{\text{iwa}} \leq \rho \leq \rho_{\text{owa}}, \\ & 0 \leq A(r) \leq 1, & 0 \leq r \leq 1/2, \\ & -50 \leq A''(r) \leq 50, & 0 \leq r \leq 1/2 \end{array}$$

An infinite dimensional *linear programming* problem.

Mirror with “Optimal” Tinting

Mirror with Softened Edge



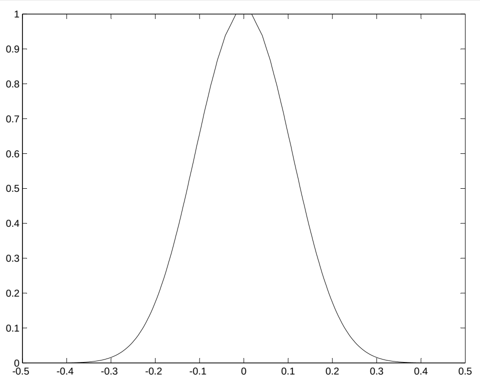
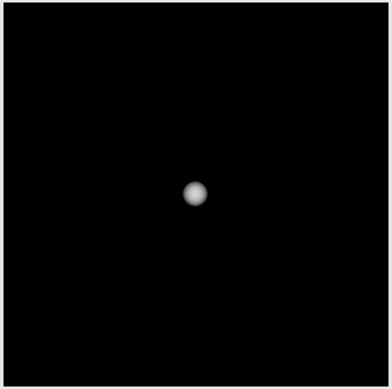
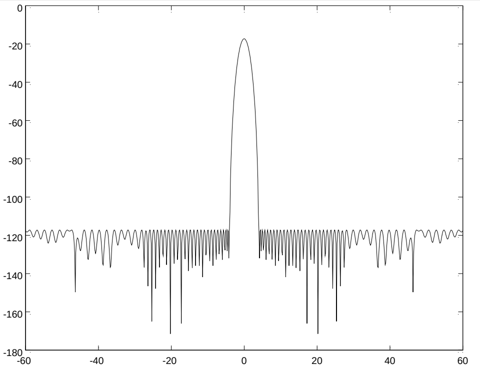


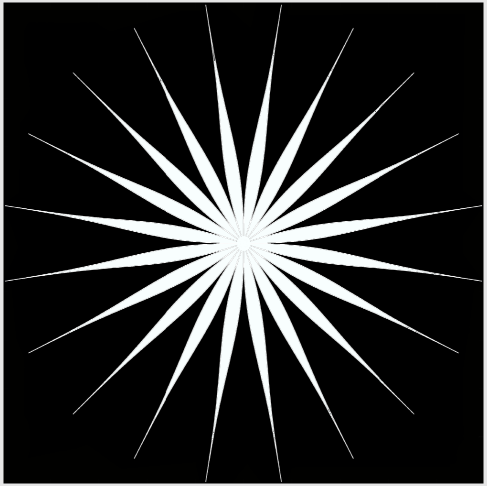
Image of Star



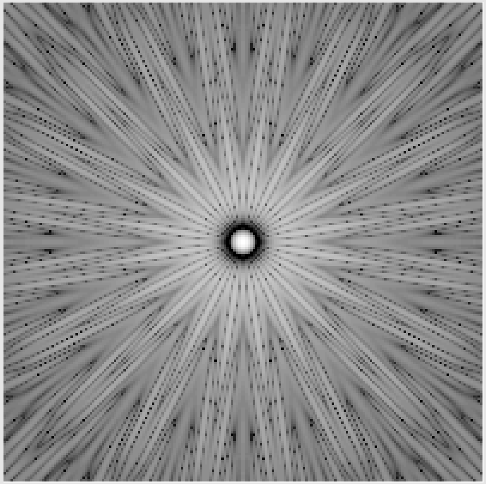


Mathematically Perfect... But Unmanufacturable!

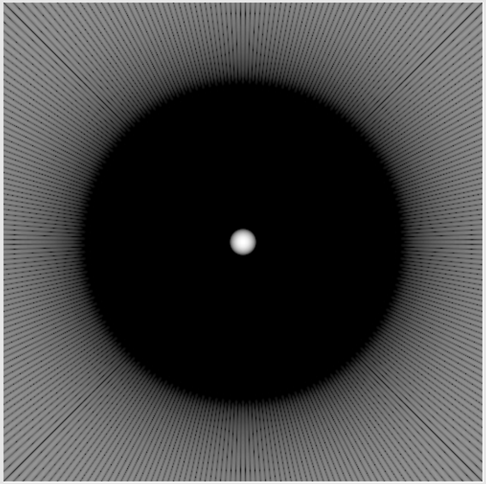
Shaped Pupil Coronagraph



20 petals



150 petals

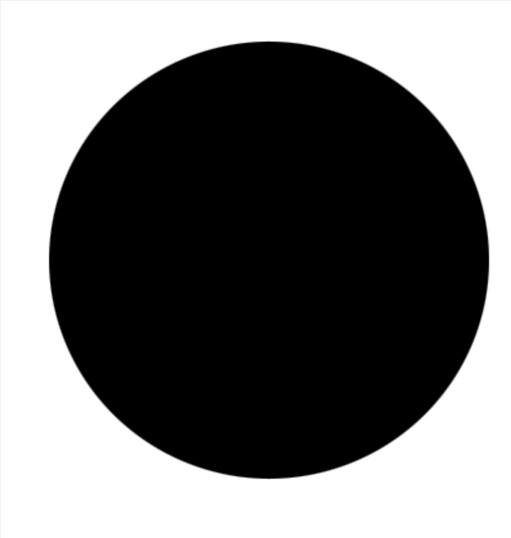


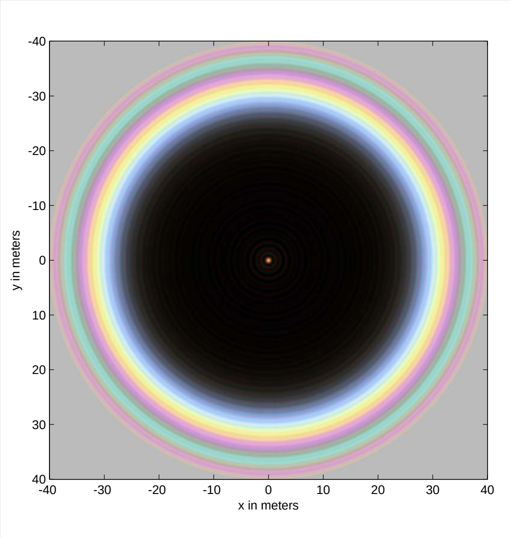
Maybe We Can!

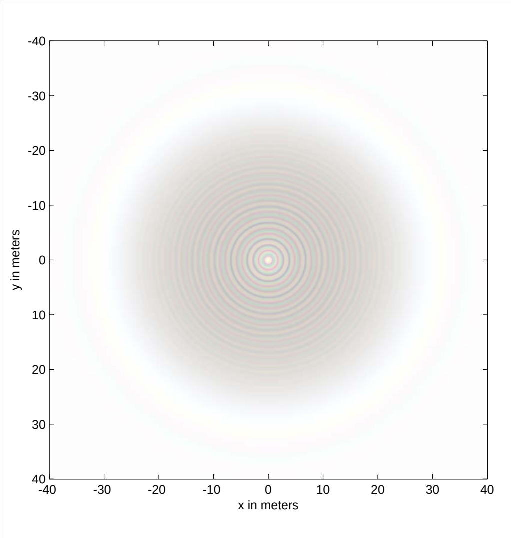
An Alternative Concept: Space-based Occulter

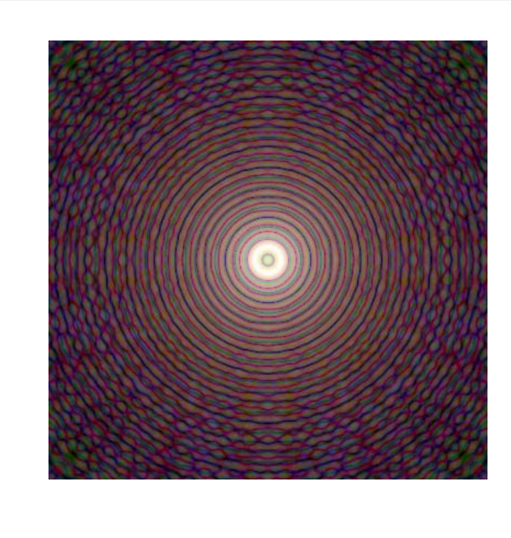
Telescope Aperture: 4m, Occulter Diameter: 50m, Occulter Distance: $72,000$km

Plain External Occulter (Doesn't Work!)

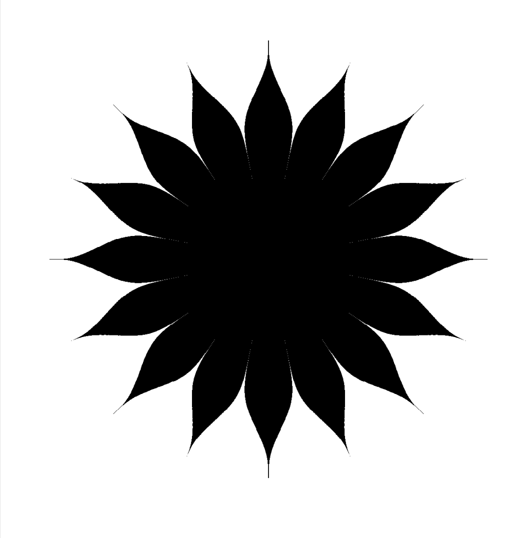


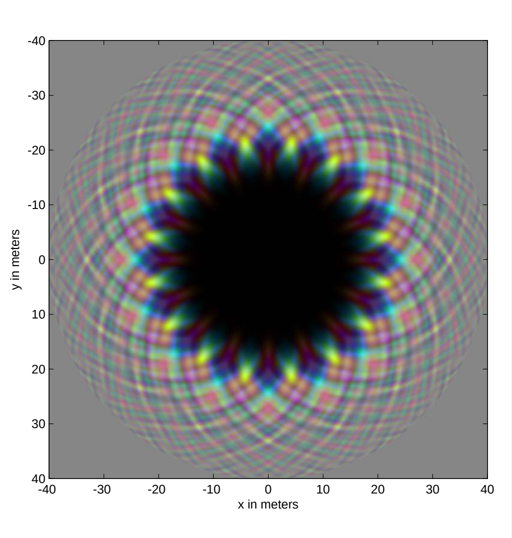


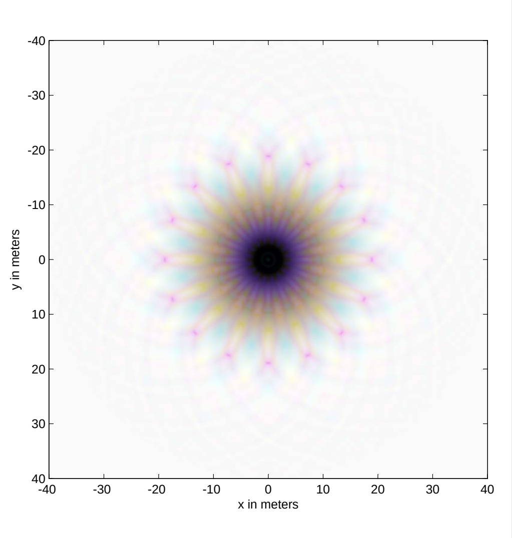


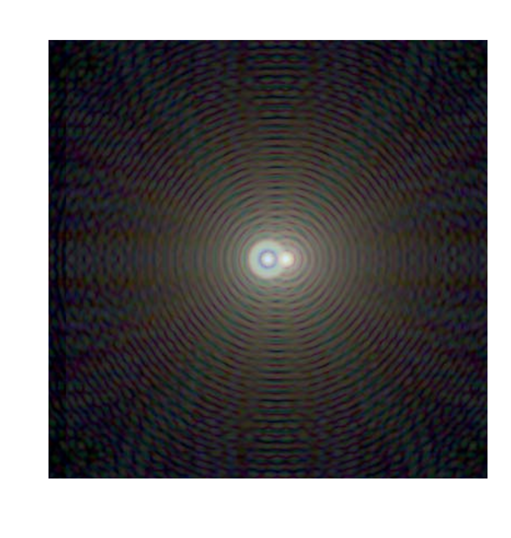


Shaped Occulter









Where does HPC fit in?

There are several mathematical approximations needed to reduce the physical problem (propagation of light according to Maxwell's laws) to the simple, linear, Fourier transform representation of the problem. Simulating/optimizing using a model based on Maxwell's equations definitely requires HPC.

The assumption that $A(x, y)$ depends only on a radius r was made to simplify the computation. It would be great to enlarge the optimization space to include all possible apodization functions.

Sensitivity analyses applied to the solutions found will be key in choosing which concepts move forward. Sensitivity analyses cannot assume that quantities are just functions of radius. Hence, HPC is key.

Celestial Mechanics

The N-Body Problem

Least Action Principle

Given: n bodies.

Let:

m_j denote the mass and
$z_j(t)$ denote the position in $\mathbb{R}^2 = \mathbb{C}$ of body j at time t.

Action Functional:

$$A = \int_0^{2\pi} \left(\sum_j \frac{m_j}{2} \|\dot{z}_j\|^2 + \sum_{j,k:k<j} \frac{m_j m_k}{\|z_j - z_k\|} \right) dt.$$

Critical points satisfy equations of motion.

Minimize!

Equations of Motion

First Variation:

$$\begin{aligned}\delta A &= \int_0^{2\pi} \sum_\alpha \left(\sum_j m_j \dot{z}_j^\alpha \dot{\delta z}_j^\alpha - \sum_{j,k:k<j} m_j m_k \frac{(z_j^\alpha - z_k^\alpha)(\delta z_j^\alpha - \delta z_k^\alpha)}{\|z_j - z_k\|^3} \right) dt \\ &= -\int_0^{2\pi} \sum_j \sum_\alpha \left(m_j \ddot{z}_j^\alpha + \sum_{k:k\neq j} m_j m_k \frac{z_j^\alpha - z_k^\alpha}{\|z_j - z_k\|^3} \right) \delta z_j^\alpha dt\end{aligned}$$

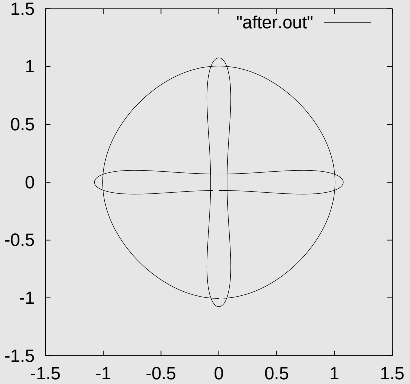
Setting first variation to zero, we get:

$$m_j \ddot{z}_j^\alpha = -\sum_{k:k\neq j} m_j m_k \frac{z_j^\alpha - z_k^\alpha}{\|z_j - z_k\|^3}, \qquad j = 1, 2, \ldots, n, \quad \alpha = 1, 2$$

Note: If $m_j = 0$ for some j, then the first order optimality condition reduces to $0 = 0$, which is *not* the equation of motion for a massless body.

Choreographies and the Ducati

The previous AMPL model was used to find many *choreographies* (a la Moore and Montgomery/Chencinier) in the equimass n-body problem and the stable *Ducati* solution to the 3-body problem.



http://www.princeton.edu/~rvdb/JAVA/astro/galaxy/Galaxy0.html