Extreme Optics
and
The Search for Earth-Like Planets

Robert J. Vanderbei

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http://www.princeton.edu/∼rvdb

ABSTRACT

- NASA/JPL plans to build and launch a space telescope to look for Earth-like planets.

- I will describe the detection problem and explain why it is hard.

- Optimization is key to several design concepts.

The Big Question: Are We Alone?

- **Are there Earth-like planets?**
- **Are they common?**
- **Is there life on some of them?**

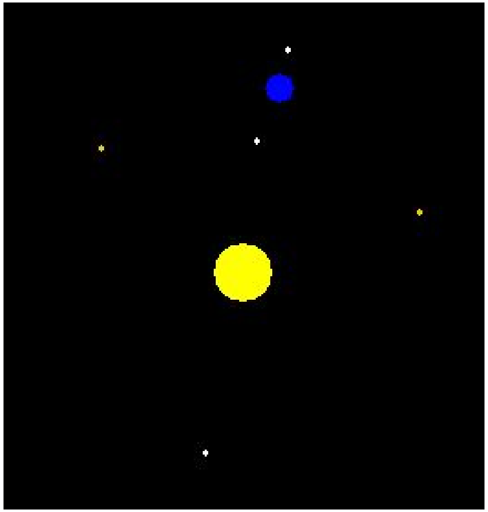
Indirect Discovery of Exosolar Planets

The Radial Velocity Method

About 200 exosolar planets known today.

Almost all were discovered by detecting a sinusoidal doppler shift in the parent star's spectrum due to gravitationally induced wobble.

This method works best for large Jupiter-sized planets with close-in orbits.



Our Earth generates only a tiny wobble in our Sun's position.

One of these planets, HD209458b, also transits its parent star once every 3.52 days. These transits have been detected photometrically as the star's light flux decreases by about 1.5% during a transit.

The Transit Method

A few planets have been discovered using the Transit Method.

On June 8, 2004, Venus transited in front of the Sun.

I took a picture of this event with my small telescope.

If we on Earth are lucky to be in the right position at the right time, we can detect similar transits of exosolar planets.

A few exosolar planets have been discovered this way.

Terrestrial Planet Finder Telescope

- **NASA/JPL space telescope.**

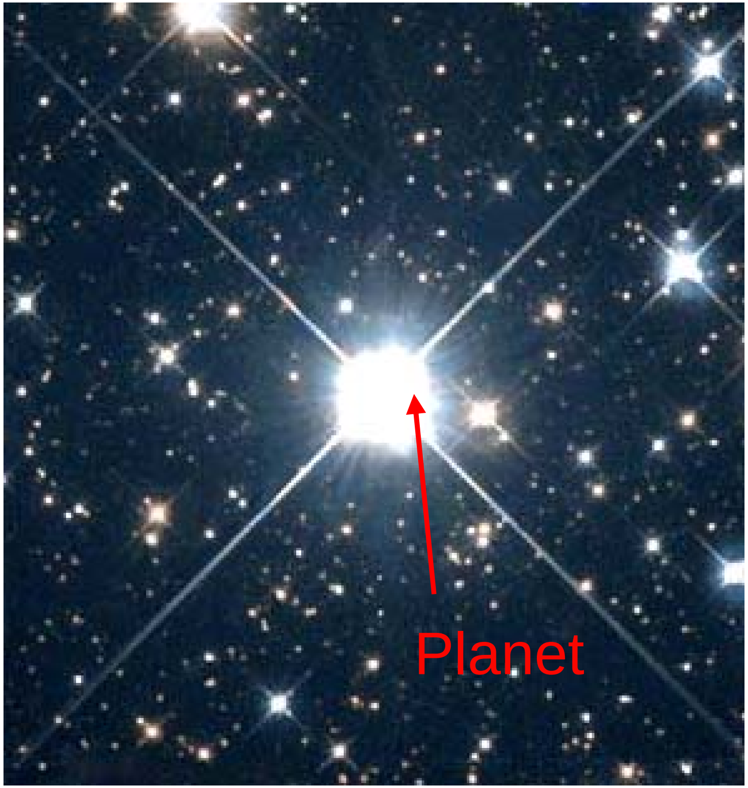
- **Launch date: ~~2014~~...well, sometime in our lifetime.**

- **DETECT: Search 150-500 nearby (5-15 pc distant) Sun-like stars for Earth-like planets.**

- **CHARACTERIZE: Determine basic physical properties and measure "bio-markers", indicators of life or conditions suitable to support it.**

Why Is It Hard?

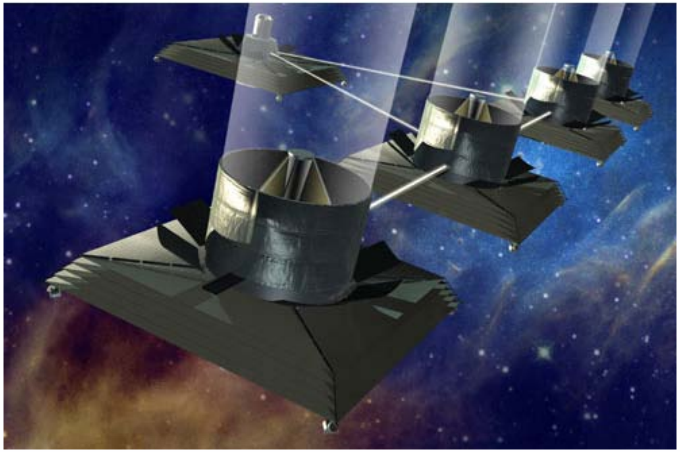
- **Contrast.** Star $= 10^{10} \times$ Planet
- **Angular Separation.** 0.1 arcseconds.



Early Design Concepts

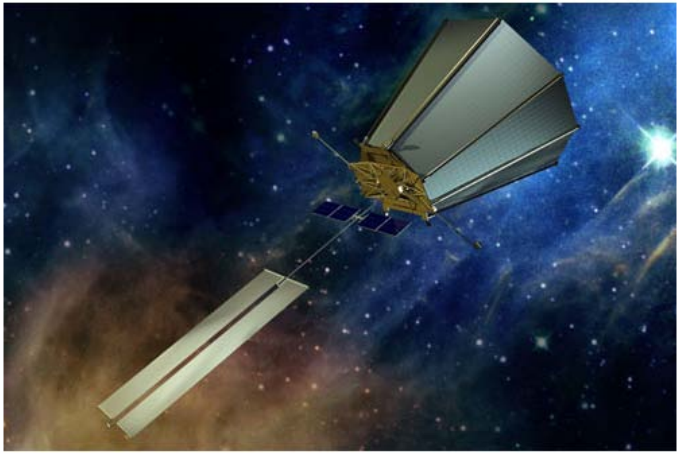
Space-based infrared nulling interferometer (TPF-I).

TPF-Interferometer



Visible-light telescope with an elliptical mirror (3.5 m x 8 m) and an optimized diffraction control system (TPF-C).

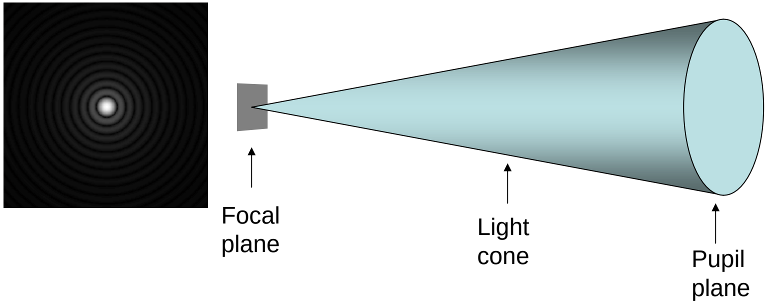
TPF-Coronagraph



Diffraction Control via Shaped Pupils

Consider a telescope. Light enters the front of the telescope—the pupil plane.

The telescope focuses the light passing through the pupil plane from a given direction at a certain point on the focal plane, say $(0, 0)$.



However, a point source produces not a point image but an Airy pattern consisting of an Airy disk surrounded by a system of diffraction rings.

These diffraction rings are too bright. The rings would completely hide the planet.

By placing a mask over the pupil, one can control the shape and strength of the diffraction rings. The problem is to find an optimal shape so as to put a very deep null very close to the Airy disk.

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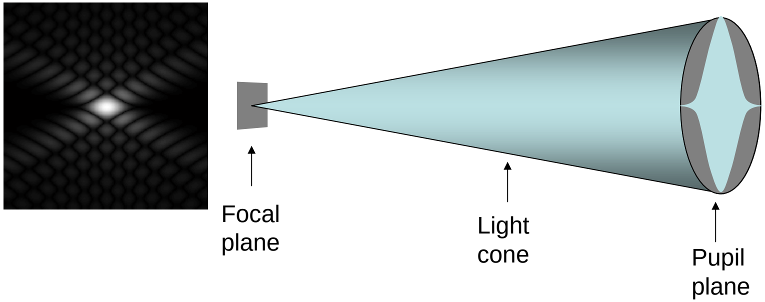

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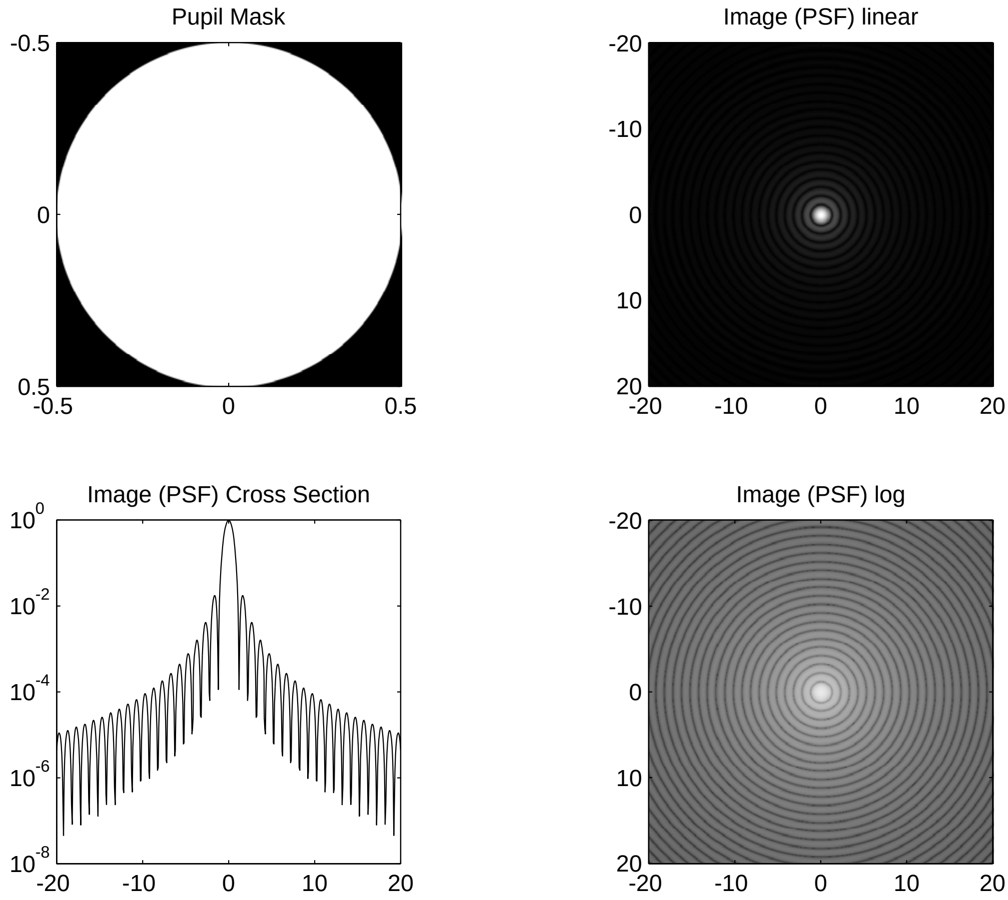
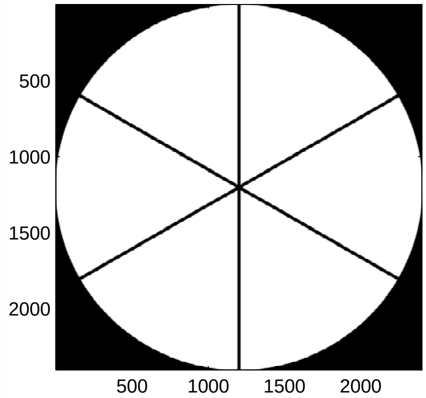
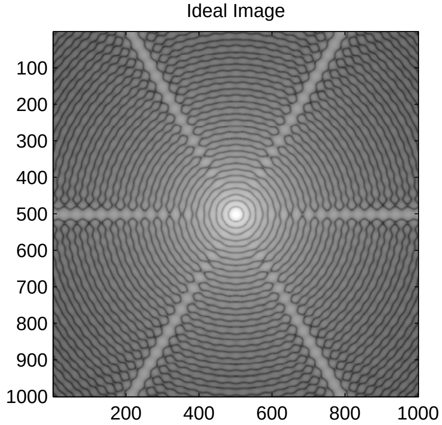
The Airy Pattern



Spiders are an Example of a Shaped Pupil





Note the six bright radial spikes

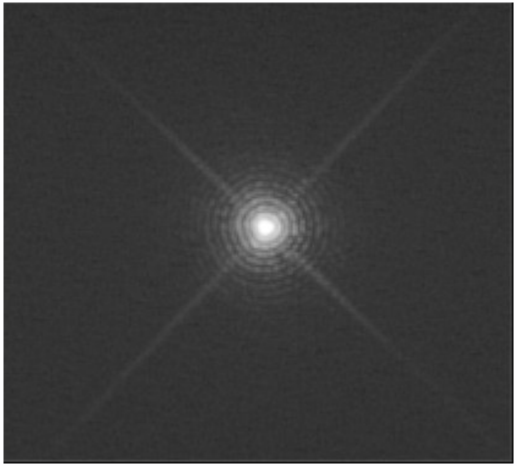
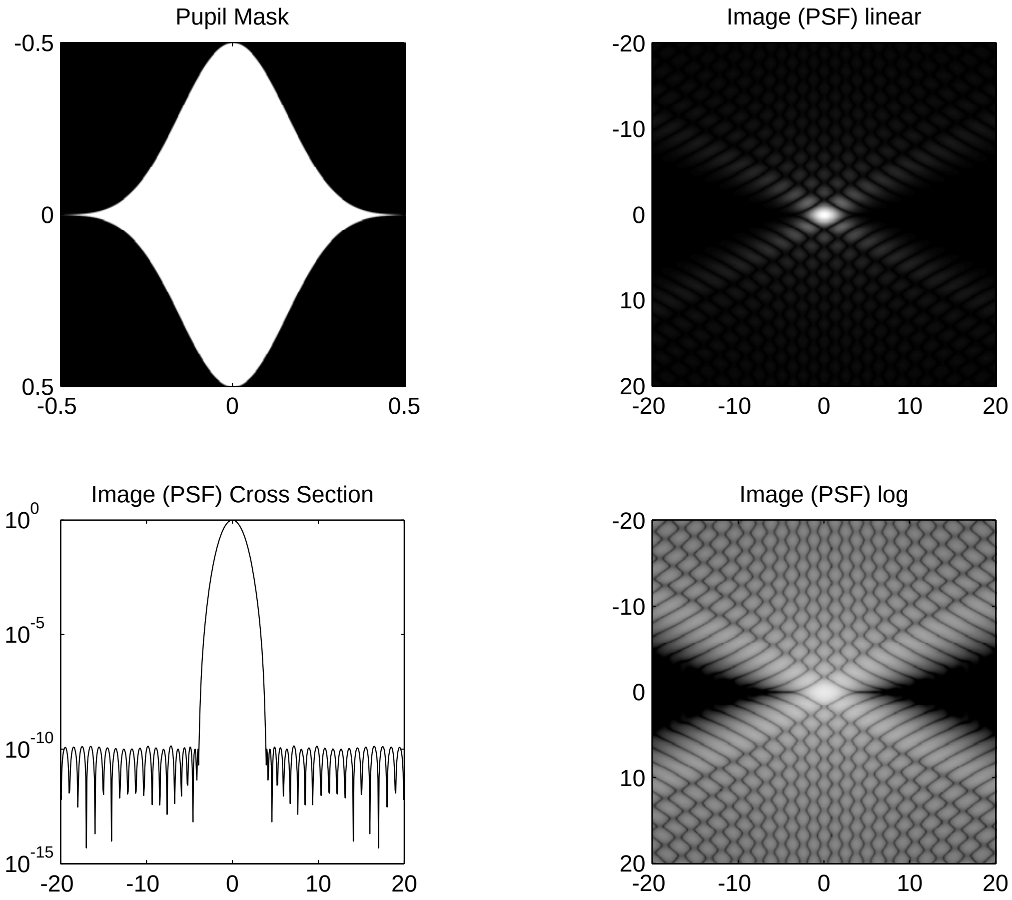


Image of Vega taken with my “big” 250mm telescope.

The Seven Sisters with Spikes

Pleiades image taken with small refractor equipped with dental floss spiders.

The Spergel-Kasdin-Vanderbei Pupil



Telescopes Designed for High Contrast

aka Coronagraphs

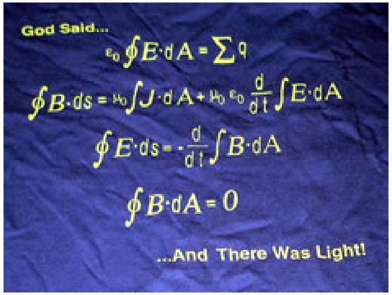
Problem Class: Maximize a linear functional of a "design" function given constraints on its Fourier transform.

The Physics of Light

Light consists of photons.

Photons are wave packets.

Diffraction is a wave property.



Maxwell's equations for the Electro-Magnetic field.

$\Downarrow$

Wave equation for electric field (and magnetic field).

$\Downarrow$

Huygens wavelet model

$\Downarrow$

Fresnel/Fraunhofer approximation (Fourier transform!)

$\Downarrow$

Ray optics

Electric Field—Fraunhofer Model

Input: Perfectly flat wavefront (electric field is unity).

Pupil: Described by a mask/tint function $A(x, y)$.

Output: Electric field $E()$:

$$E(\xi, \zeta) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i(x\xi + y\zeta)} A(x, y) dy dx$$

$$\vdots$$

$$E(\rho) = 2\pi \int_0^{1/2} J_0(r\rho) A(r) r dr,$$

where J_0 denotes the 0-th order Bessel function of the first kind.

The intensity is the magnitude of the electric field squared.

The unitless pupil-plane "length" r is given as a multiple of the aperture D.

The unitless image-plane "length" ρ is given as a multiple of focal-length times wavelength over aperture ($f\lambda/D$) or, equivalently, as an angular measure on the sky, in which case it is a multiple of just λ/D. (Example: $\lambda = 0.5\mu$m and $D = 10$m implies $\lambda/D = 10$mas.)

Performance Metrics

Inner and Outer Working Angles

$$\rho_{\mathsf{iwa}} \qquad \rho_{\mathsf{owa}}$$

Contrast:

$$E^2(\rho)/E^2(0)$$

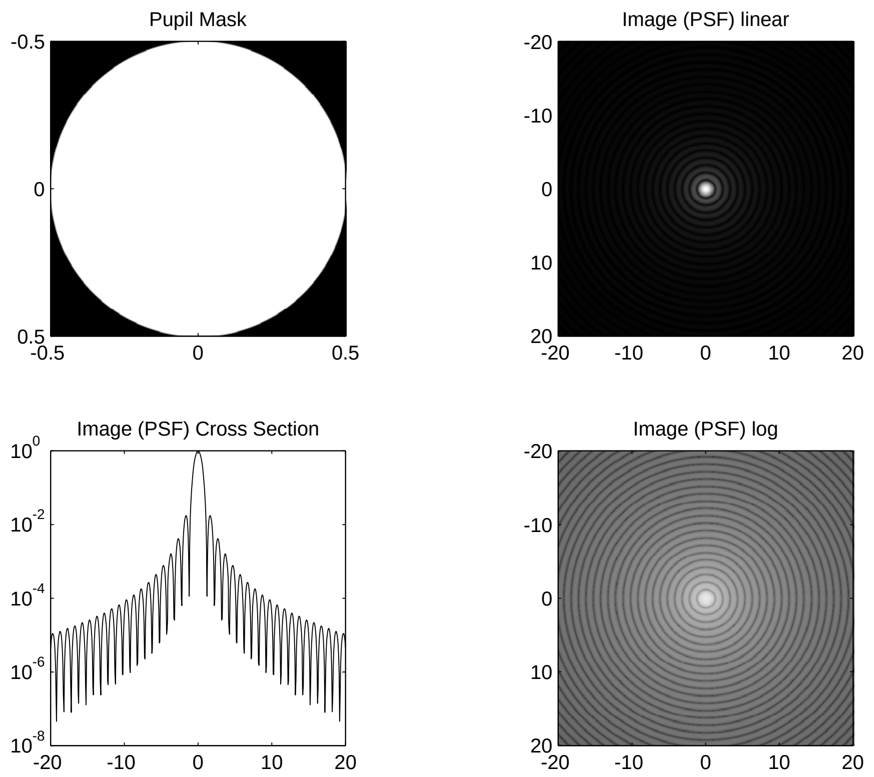
Useful Throughput:

$$\mathcal{T}_{\mathsf{Useful}} = \int_0^{\rho_{\mathsf{iwa}}} E^2(\rho)\rho d\rho.$$

Clear Aperture—Airy Pattern

$\rho_{\text{iwa}} = 1.24 \quad \mathcal{T}_{\text{Useful}} = 84.2\% \quad \textbf{Contrast} = 10^{-2}$

$\rho_{\text{iwa}} = 748 \quad \mathcal{T}_{\text{Useful}} = 100\% \quad \textbf{Contrast} = 10^{-10}$



Optimization

Simple First Case: Tinted Glass

Variably tinting glass is called apodization.

Find apodization function $A()$ that solves:

$$\begin{array}{lll} \textbf{maximize} & \displaystyle\int_0^{1/2} A(r) 2\pi r dr & \\ \textbf{subject to} & -10^{-5}E(0) \le E(\rho) \le 10^{-5}E(0), & \rho_{\text{iwa}} \le \rho \le \rho_{\text{owa}}, \\ & 0 \le A(r) \le 1, & 0 \le r \le 1/2, \end{array}$$

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\textbf{maximize} & \displaystyle\int_0^{1/2} A(r) 2\pi r dr & \\
\textbf{subject to} & -10^{-5} E(0) \le E(\rho) \le 10^{-5} E(0), & \rho_{\text{iwa}} \le \rho \le \rho_{\text{owa}}, \\
& 0 \le A(r) \le 1, & 0 \le r \le 1/2, \\
& -50 \le A''(r) \le 50, & 0 \le r \le 1/2
\end{array}
$$

An infinite dimensional linear programming problem.

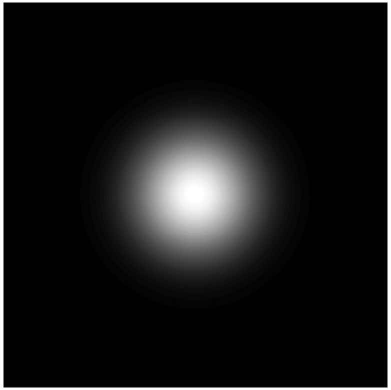
The AMPL Model for Apodization

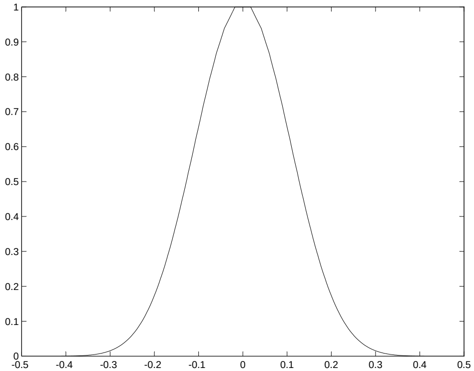
```
function J0;

param pi := 4*atan(1);
param N := 400; # discretization parameter
param rho0 := 4;
param rho1 := 60;

param dr := (1/2)/N;
set Rs ordered := setof {j in 0.5..N-0.5 by 1} (1/2)*j/N;

var A {Rs} >= 0, <= 1, := 1/2;

set Rhos ordered := setof {j in 0..N} j*rho1/N;
set PlanetBand := setof {rho in Rhos: rho>=rho0 && rho<=rho1} rho;

var E0 {rho in Rhos} =
    2*pi*sum {r in Rs} A[r]*J0(2*pi*r*rho)*r*dr;

maximize area: sum {r in Rs} 2*pi*A[r]*r*dr;
subject to sidelobe_pos {rho in PlanetBand}: E0[rho] <= 10^(-5)*E0[0];
subject to sidelobe_neg {rho in PlanetBand}: -10^(-5)*E0[0] <= E0[rho];

subject to smooth {r in Rs: r != first(Rs) && r != last(Rs)}:
    -50*dr^2 <= A[next(r)] - 2*A[r] + A[prev(r)] <= 50*dr^2;

solve;
```

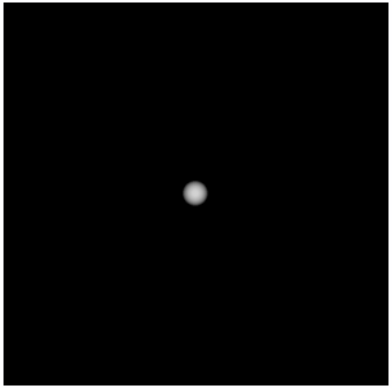
The Optimal Apodization

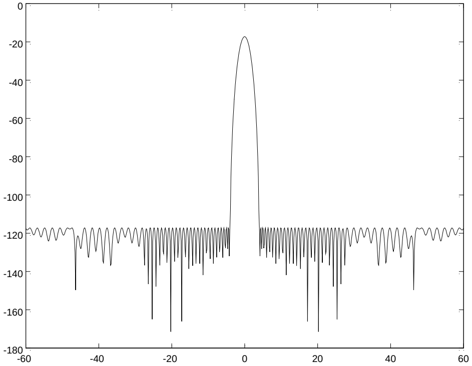
$$\rho_{\text{iwa}} = 4 \quad \mathcal{T}_{\text{Useful}} = 9\%$$

Excellent dark zone. Unmanufacturable.









Concentric Ring Masks

Recall that for circularly symmetric apodizations

$$E(\rho) = 2\pi \int_0^{1/2} J_0(r\rho) A(r) r dr,$$

where J_0 denotes the 0-th order Bessel function of the first kind.

Let

$$A(r) = \begin{cases} 1 & r_{2j} \le r \le r_{2j+1}, \qquad j = 0, 1, \ldots, m-1 \\ 0 & \text{otherwise,} \end{cases}$$

where

$$0 \le r_0 \le r_1 \le \cdots \le r_{2m-1} \le 1/2.$$

The integral can now be written as a sum of integrals and each of these integrals can be explicitly integrated to get:

$$E(\rho) = \sum_{j=0}^{m-1} \frac{1}{\rho} \left(r_{2j+1} J_1\left(\rho r_{2j+1}\right) - r_{2j} J_1\left(\rho r_{2j}\right) \right).$$

Concentric Ring Optimization Problem

$$\textbf{maximize} \quad \sum_{j=0}^{m-1} \pi (r_{2j+1}^2 - r_{2j}^2)$$

$$\textbf{subject to:} \quad -10^{-5}E(0) \le E(\rho) \le 10^{-5}E(0), \quad \textbf{for } \rho_0 \le \rho \le \rho_1$$

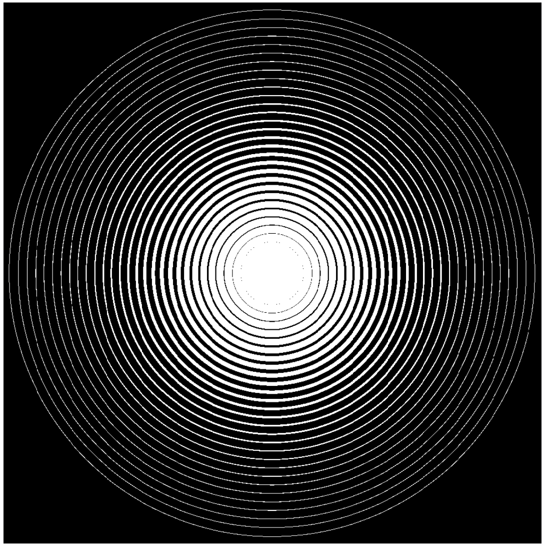
where $E(\rho)$ is the function of the r_j's given on the previous slide.

This problem is a semiinfinite nonconvex optimization problem.

The AMPL Model for Concentric Rings

```
function intrJ0;

param pi := 4*atan(1);
param N := 400; # discretization parameter
param rho0 := 4;
param rho1 := 60;

var r {j in 0..M} >= 0, <= 1/2, := r0[j];

set Rhos2 ordered := setof {j in 0..N} (j+0.5)*rho1/N;
set PlanetBand2 := setof {rho in Rhos2: rho>=rho0 && rho<=rho1} rho;

var E {rho in Rhos2} =
    (1/(2*pi*rho)^2)*sum {j in 0..M by 2}
(intrJ0(2*pi*rho*r[j+1]) - intrJ0(2*pi*rho*r[j]));

maximize area2: sum {j in 0..M by 2} (pi*r[j+1]^2 - pi*r[j]^2);
subject to sidelobe_pos2 {rho in PlanetBand2}: E[rho] <= 10^(-5)*E[first(rhos2)];
subject to sidelobe_neg2 {rho in PlanetBand2}: -10^(-5)*E[first(rhos2)] <= E[rho];

subject to order {j in 0..M-1}: r[j+1] >= r[j];

solve mask;
```

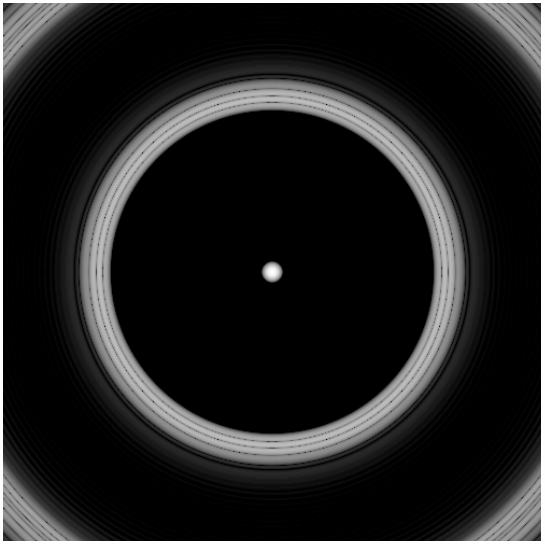
The Best Concentric Ring Mask

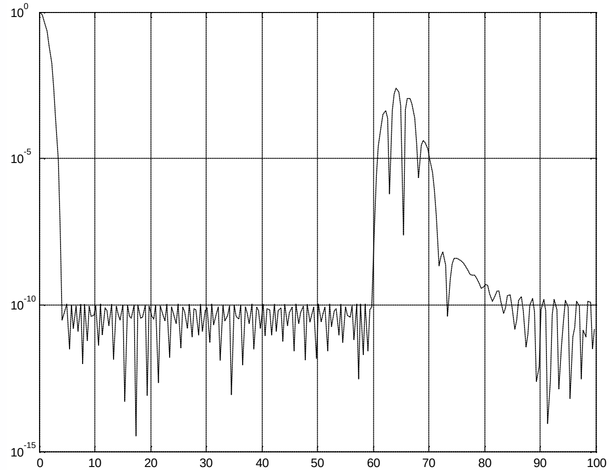


$\rho_{\text{iwa}} = 4 \quad \rho_{\text{owa}} = 60$

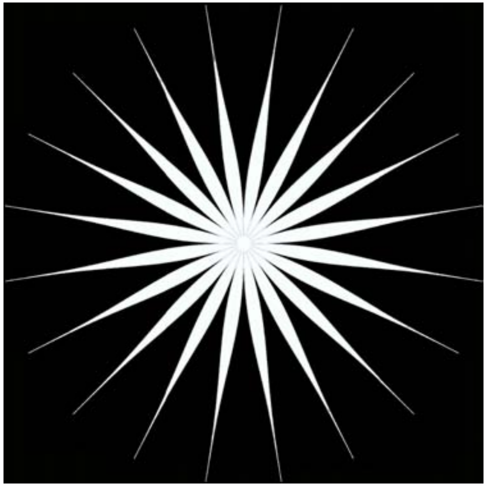
$\mathcal{T}_{\text{Useful}} = 9\%$

Lay it on glass?

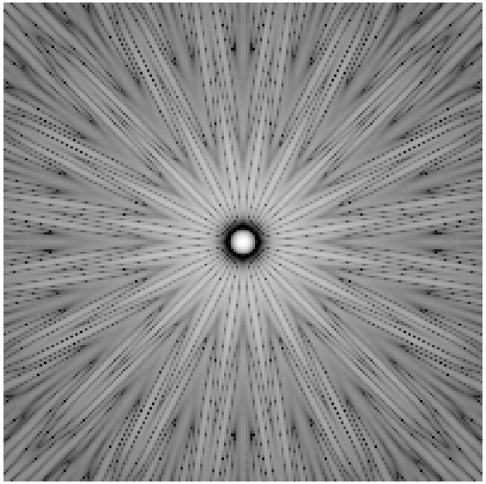




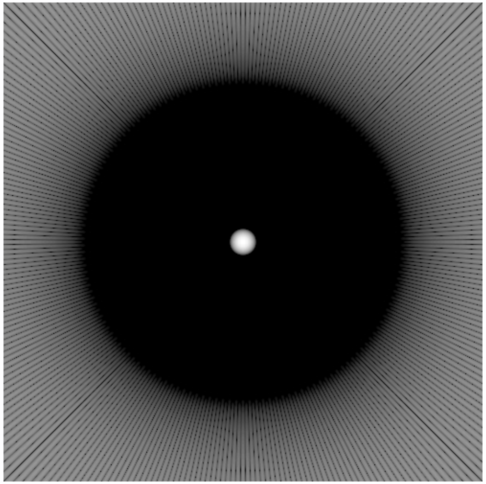
Petal-Shaped Mask



20 petals



150 petals



Ripple Masks

Consider a mask consisting of an opening given by

$$A(x, y) = \begin{cases} 1 & |y| \leq a(x) \\ 0 & \text{else} \end{cases}$$

Only consider masks that are symmetric with respect to x and y axes. Hence, $a()$ is nonnegative and even.

The electric field $E(\xi, \zeta)$ is given by

$$\begin{aligned} E(\xi, \zeta) &= \int_{-\frac{1}{2}}^{\frac{1}{2}} \int_{-a(x)}^{a(x)} e^{i(x\xi + y\zeta)} dy dx \\ &= 4 \int_{0}^{\frac{1}{2}} \cos(x\xi) \frac{\sin(a(x)\zeta)}{\zeta} dx \end{aligned}$$

Maximizing Throughput

Because of the symmetry, we only need to optimize in the first quadrant:

$$\text{maximize } 4\int_0^{\frac{1}{2}} a(x)dx$$

$$\begin{aligned} \text{subject to } & -10^{-5}E(0,0) \leq E(\xi,\zeta) \leq 10^{-5}E(0,0), && \text{for } (\xi,\zeta) \in \mathcal{O} \\ & 0 \leq a(x) \leq 1/2, && \text{for } 0 \leq x \leq 1/2 \end{aligned}$$

The objective function is the total open area of the mask. The first constraint guarantees 10^{-10} light intensity throughout a desired region of the focal plane, and the remaining constraint ensures that the mask is really a mask.

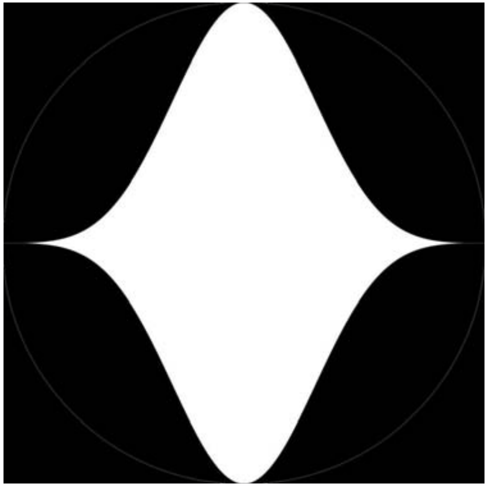
If the set $\mathcal{O}$ is a subset of the x-axis, then the problem is an infinite dimensional linear programming problem.

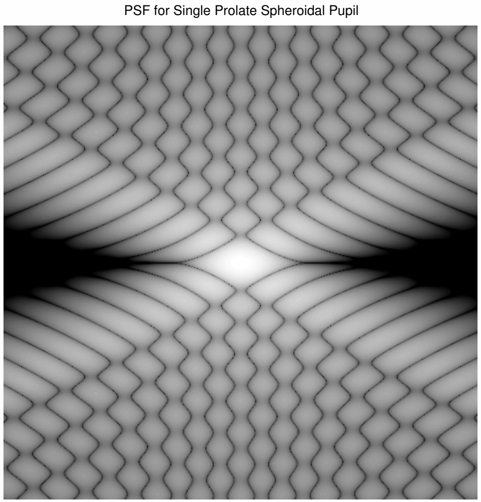
One Pupil w/ On-Axis Constraints

Spergel-Kasdin-Vanderbei Pupil

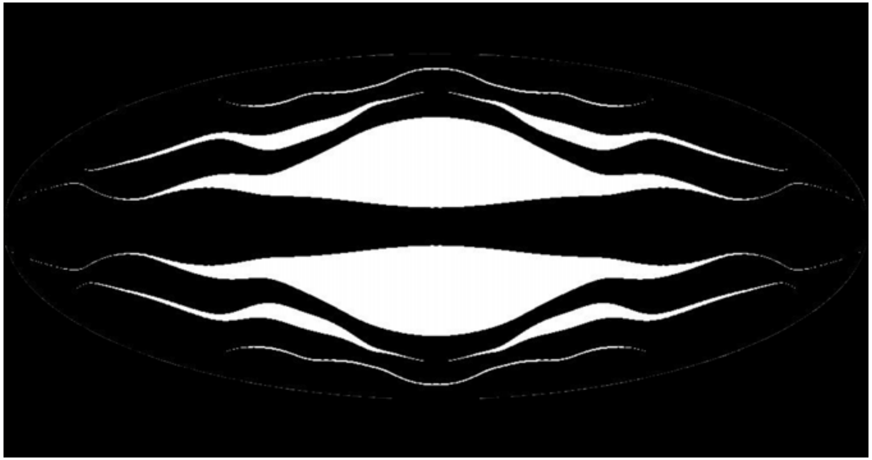
$$\rho_{\mathsf{iwa}} = 4 \quad \mathcal{T}_{\mathsf{Useful}} = 43\%$$

Small dark zone...Not feasible





Ripple3 Mask



$$\rho_{\text{iwa}} = 4$$

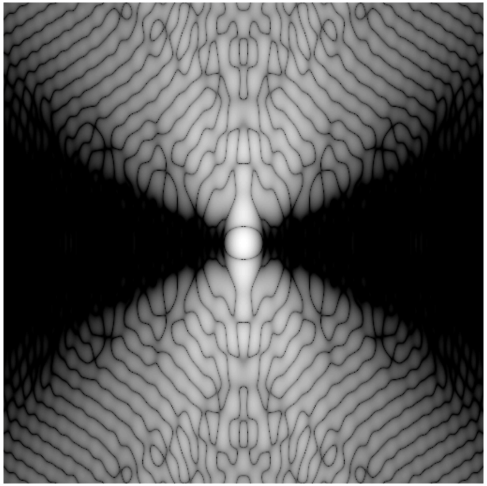
$$\mathcal{T}_{\text{Useful}} = 30\%$$

Throughput relative to ellipse

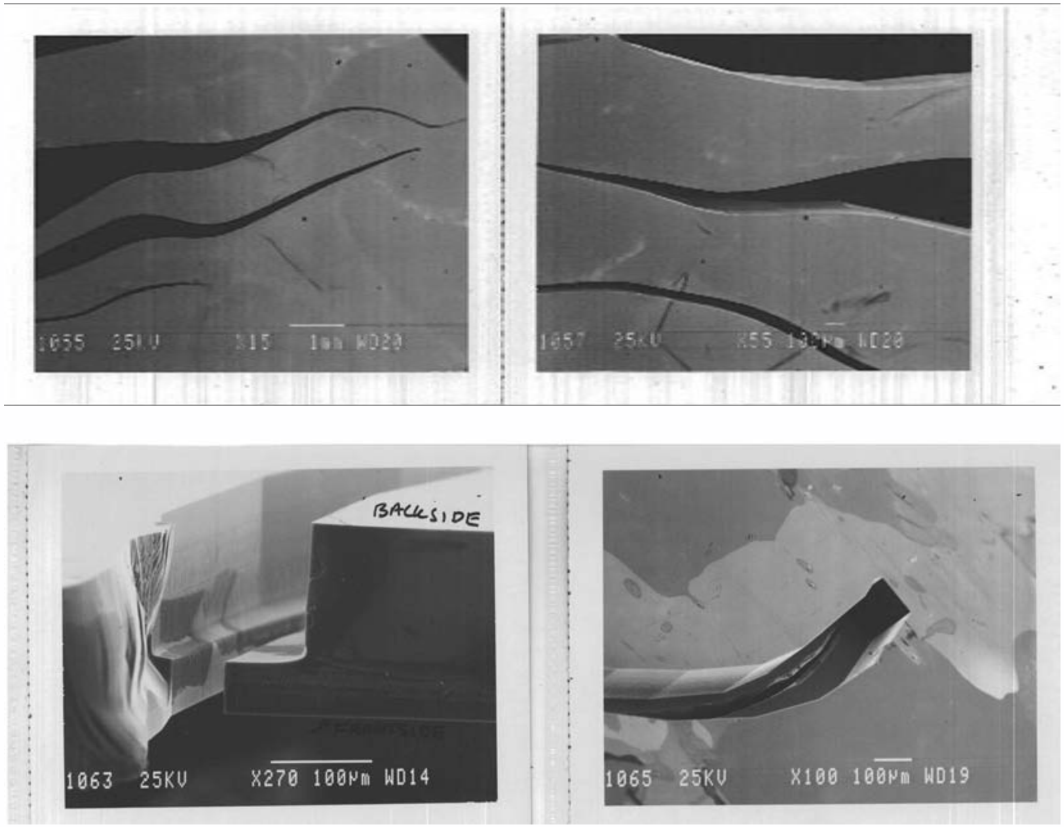
11% central obstr.

Easy to make

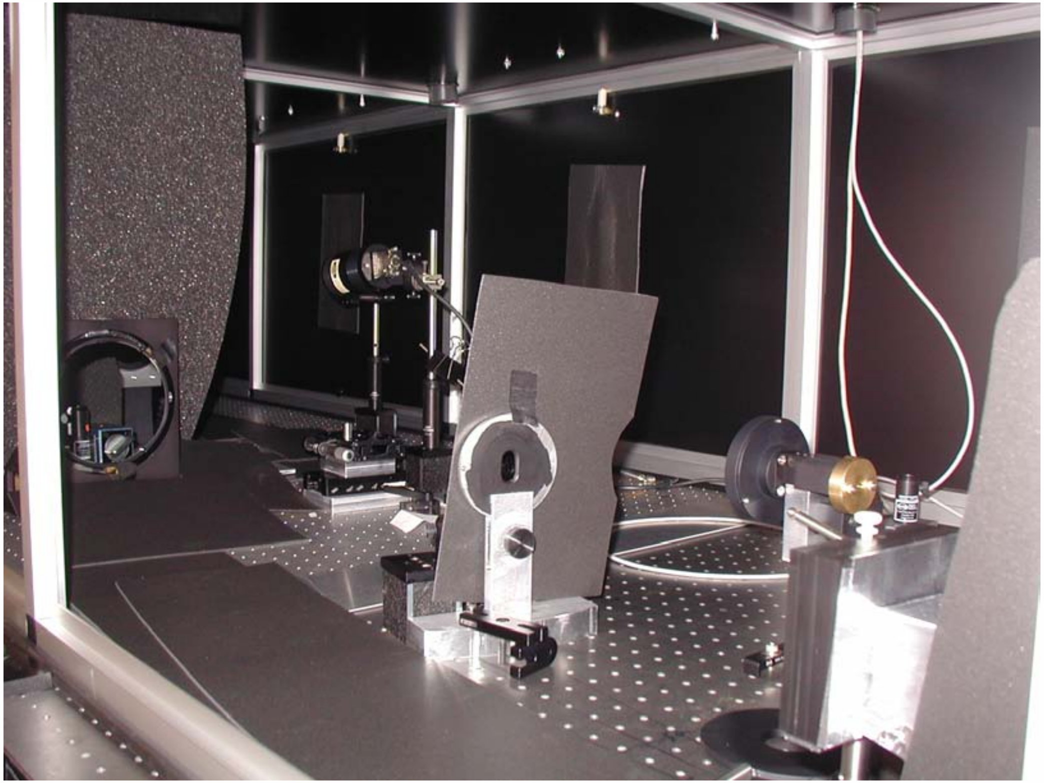
Only a few rotations



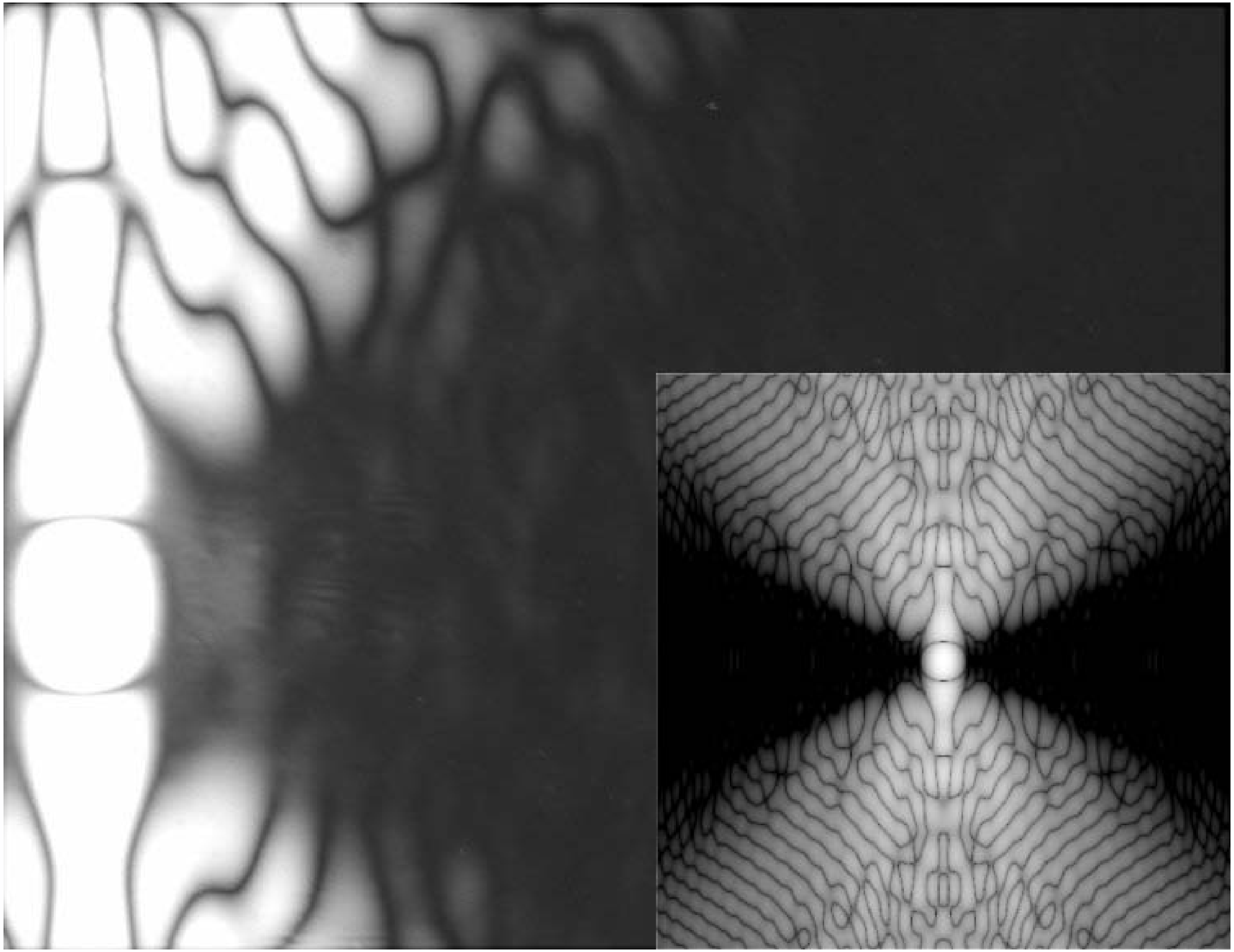
Masks from NIST



Our TPF Optics Lab



Lab Results: Theory vs. Practice



Brightest pixel $\approx 1,642,000,000$. **Sum of** 21 **one-hour exposures.**

Optimization Success Story

From an April 12, 2004, letter from Charles Beichman:

> Dear TPF-SWG,
>
> I am writing to inform you of exciting new developments for TPF. As part of the Presidents new vision for NASA, the agency has been directed by the President to conduct advanced telescope searches for Earth-like planets and habitable environments around other stars. Dan Coulter, Mike Devirian, and I have been working with NASA Headquarters (Lia LaPiana, our program executive; Zlatan Tsvetanov, our program scientist; and Anne Kinney) to incorporate TPF into the new NASA vision. The result of these deliberations has resulted in the following plan for TPF:
>
> 1. Reduce the number of architectures under study from four to two: (a) the moderate sized coronagraph, nominally the 4x6 m version now under study; and (b) the formation flying interferometer presently being investigated with ESA. Studies of the other two options, the large, 10-12 m, coronagraph and the structurally connected interferometer, would be documented and brought to a rapid close.
>
> 2. Pursue an approach that would result in the launch of BOTH systems within the next 10-15 years. The primary reason for carrying out two missions is the power of observations at IR and visible wavelength regions to determine the properties of detected planets and to make a reliable and robust determination of habitability and the presence of life.
>
> 3. Carry out a modest-sized coronagraphic mission, TPF-C, to be launched around 2014, to be followed by a formation-flying interferometer, TPF-I, to be conducted jointly with ESA and launched by the end of decade (2020). This ordering of missions is, of course, subject to the readiness of critical technologies and availability of funding. But in the estimation of NASA HQ and the project, the science, the technology, the political will, and the budgetary resources are in place to support this plan.
>
> ...

Other Issues

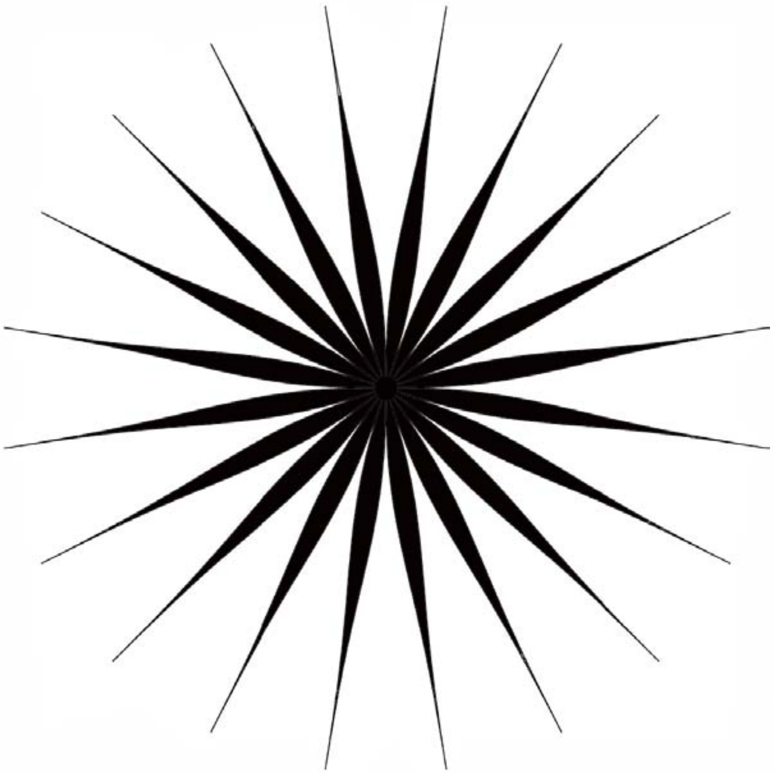
- **Wavefront errors due to imperfect mirrors.**
- **Polarization effects.**
- **An 8-meter mirror is huge (Hubble is 2.4).**
- **The glow of zodiacal dust might hide a planet.**

Other Approaches

- **Pupil Mapping**
- **Space-Based Petal-Shaped Occulter**

A Space-Based Occulter

The Complement of a Petal Mask



Vol 442|6 July 2006|doi:10.1038/nature04930 nature

LETTERS

Detection of Earth-like planets around nearby stars using a petal-shaped occulter

Webster Cash[1]

Direct observation of Earth-like planets is extremely challenging, because their parent stars are about 10^{10} times brighter but lie just a fraction of an arcsecond away[1]. In space, the twinkle of the atmosphere that would smear out the light is gone, but the problems of light scatter and diffraction in telescopes remain. The two proposed solutions—a coronagraph internal to a telescope and nulling interferometry from formation-flying telescopes—both require exceedingly clean wavefront control in the optics[2]. An attractive variation to the coronagraph is to place an occulting shield outside the telescope, blocking the starlight before it even enters the optical path[3]. Diffraction and scatter around or through the occulter, however, have limited effective suppression in practically sized missions[4–6]. Here I report an occulter design that would achieve the required suppression and can be built with existing technology. The compact mission architecture of a coronagraph is traded for the inconvenience of two spacecraft, but the daunting optics challenges are replaced with a simple deployable sheet 30 to 50 m in diameter. When such an occulter is flown in formation with a telescope of at least one metre aperture, terrestrial planets could be seen and studied around stars to a distance of ten parsecs.

A starshade suitable for planet hunting must be designed such that the diffracted light is minimized. This means the sum of the phases of the light from all the paths through and around the shade must be extremely close to zero, implying a wide range of phases in the focal plane. The number of extra wavelengths ($m\lambda$) a ray must travel to reach the centre of the shadow around an occulter of radius R at a distance F is given by $R^2 = 2m\lambda F$. Noting that R/F is the angular diameter of the shade, the relation becomes $R = 2m\lambda/\theta$. For planet-finding θ is 5×10^{-7}, so a planet just 0.1 arcsec from its parent star may be detected, and in the visible band $\lambda = 6 \times 10^{-7}$ m. If m is at least 10, then to create a large range of incident phases, R must be at least 20 m. So, based only on wavelength and planet–star angle, one finds that the starshade must be a large distance ($R/\theta \approx 40{,}000$ km) from the telescope. Conveniently, occulters with diameters of tens of metres can also fully shade the large (up to 10 m in diameter) telescopes suitable for studying the planets.

A recent study of transmitting apertures showed it was possible to efficiently suppress diffraction over a broad spectral band to the 10^{-10} level very close to a stellar image[7]. The results had the further key feature of being 'binary' (either fully transmitting or fully opaque at each and every point), thus avoiding the problem of a partially transmitting sheet that would be difficult to manufacture to the needed tolerances and might reintroduce scatter. Then, a class of circularly symmetric apertures was shown to enable suppression in all directions simultaneously between some inner and outer working angles[8]. These apertures could be made binary and still function well by approximating the circularly symmetric fall-off with an array of petal-shaped apertures.

By numerical integration of the Fresnel diffraction equations and by subsequent mathematical derivation (provided in the Supplementary Information) I have shown there exists a class of shaping