On Designing NASA's Terrestrial Planet Finder Space Telescope

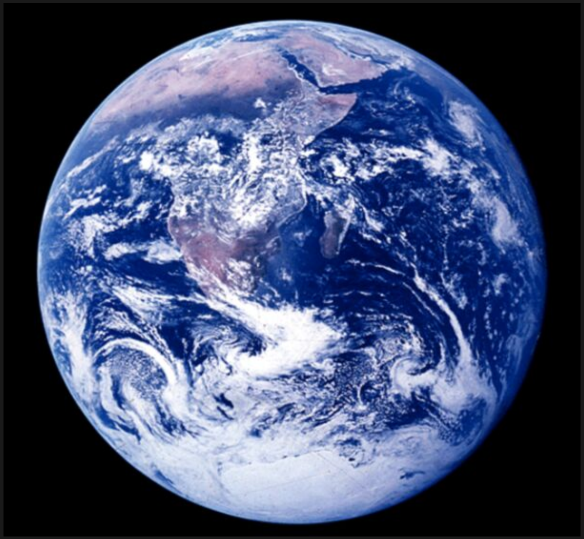
Robert J. Vanderbei
N. Jeremy Kasdin
David N. Spergel

October 20, 2003
INFORMS, Atlanta

Princeton University
http://www.princeton.edu/∼rvdb

The Big Question: Are We Alone?

- Are there Earth-like planets?
- Are they common?
- Is there life on some of them?



Home Page

Title Page

Contents

Go Back

Full Screen

Close

Quit

Exosolar Planets—Where We Are Now

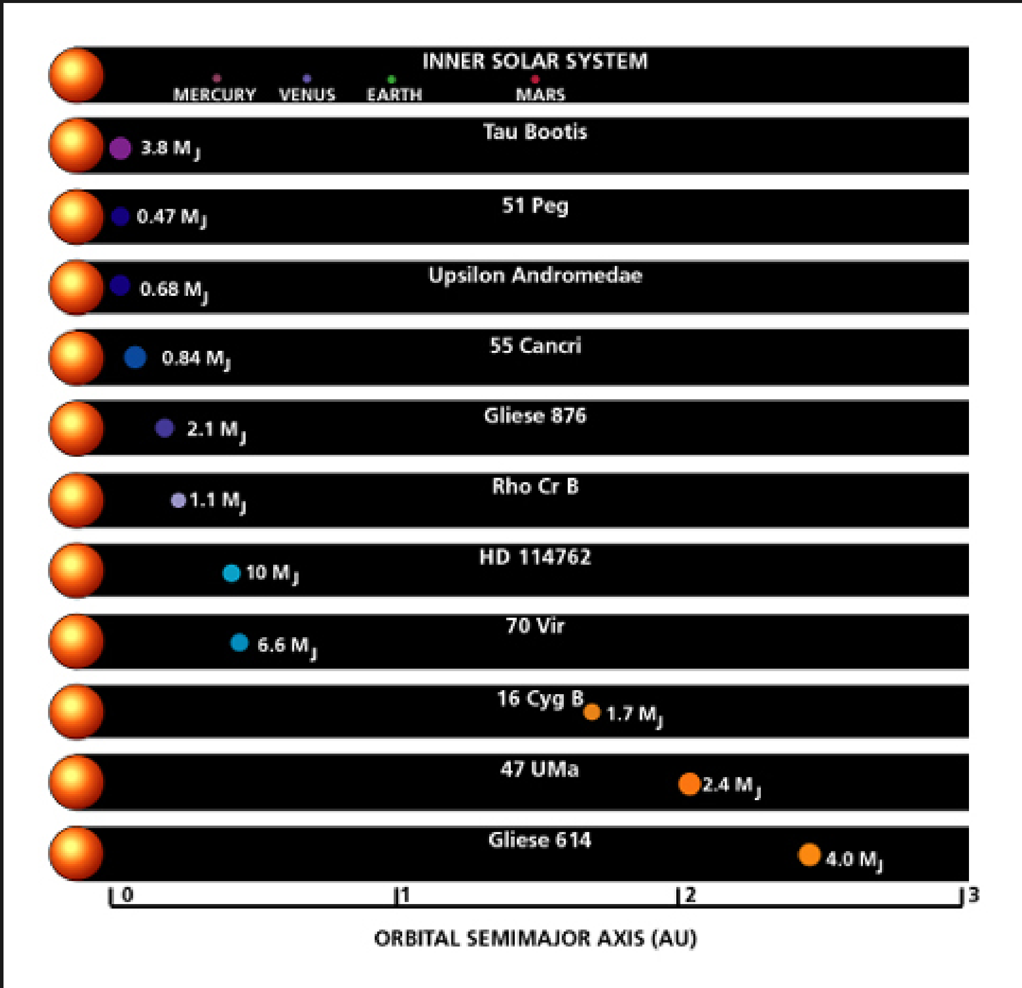
There are more than 100 Exosolar planets known today.

Most of them have been discovered by detecting a sinusoidal doppler shift in the parent star's spectrum due to gravitationally induced wobble.

This method works best for large Jupiter-sized planets with close-in orbits.

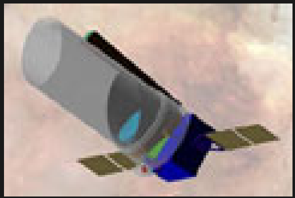
One of these planets, HD209458b, also transits its parent star once every 3.52 days. These transits have been detected photometrically as the star's light flux decreases by about 1.5% during a transit.

Some of the ExoPlanets



Future Exosolar Planet Missions

- 2006, Kepler a space-based telescope to monitor 100,000 stars simultaneously looking for “transits”.



- 2007, Eclipse a space-based telescope to directly image Jupiter-like planets.

- 2009, Space Interferometry Mission (SIM) will look for astrometric wobble.
- 2014, Darwin is a space-based cluster of 6 telescopes used as an interferometer.
- 2014, Terrestrial Planet Finder (TPF) space-based telescope to directly image Earth-like planets.

Terrestrial Planet Finder Telescope

- DETECT: Search 150-500 nearby (5-15 pc distant) Sun-like stars for Earth-like planets.

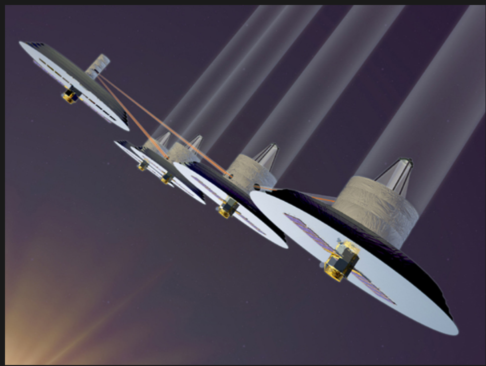
- CHARACTERIZE: Determine basic physical properties and measure "biomarkers", indicators of life or conditions suitable to support it.

Why Is It Hard?

- If the star is Sun-like and the planet is Earth-like, then the reflected visible light from the planet is 10^{-10} times as bright as the star. This is a difference of 25 magnitudes!

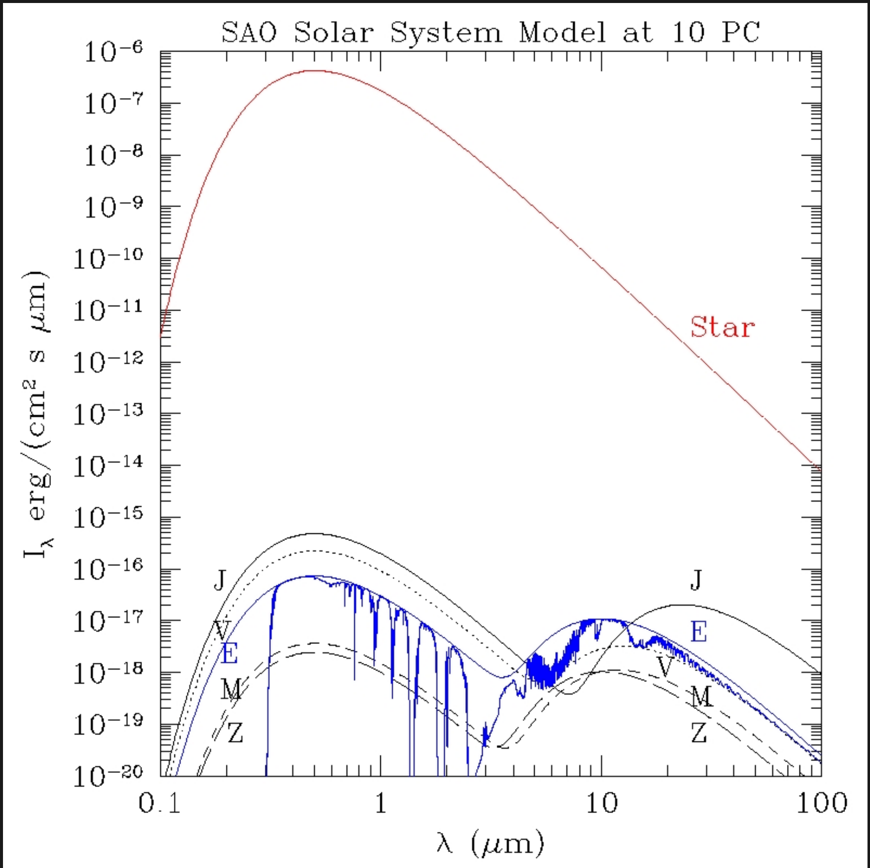
- If the star is 10 pc (33 ly) away and the planet is 1 AU from the star, the angular separation is 0.1 arcseconds!

Originally, it was thought that this would require a space-based multiple mirror nulling interferometer.



However, a more recent idea is to use a single large telescope with an elliptical mirror (4 m x 10 m) and a *shaped pupil* for diffraction control.

Visible vs. Infrared



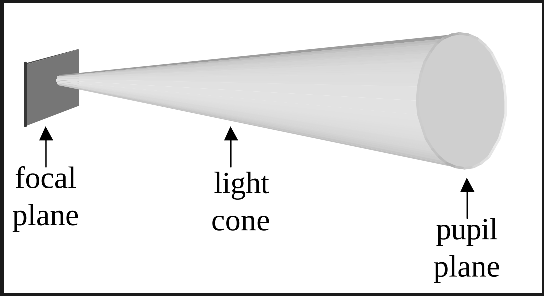
HD209458 is the bright (mag. 7.6) star in the center of this image.
The dimmest stars visible in this image are magnitude 16.
An Earth-like planet 1 AU from HD209458 would be magnitude 33, and would be located 0.2 pixels from the center of HD209458.

Shape Optimization (Telescope Design)

The problem is to design and build a space telescope that will be able to "see" planets around nearby stars (other than the Sun).

Consider a telescope. Light enters the front of the telescope. This is called the *pupil plane*.

The telescope focuses all the light passing through the pupil plane from a given direction at a certain point on the *focal plane*, say $(0, 0)$.



However, the wave nature of light makes it impossible to concentrate all of the light at a point. Instead, a small disk, called the *Airy disk*, with diffraction rings around it appears.

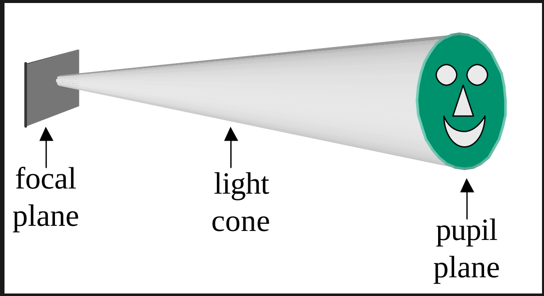
These diffraction rings are bright relative to any planet that might be orbiting a nearby star and so would completely hide the planet. The Sun, for example, would appear 10^{10} times brighter than the Earth to a distant observer.

By placing a mask over the pupil, one can design the shape and strength of the diffraction rings. The problem is to find an optimal shape so as to put a very deep *null* very close to the Airy disk.

Shape Optimization (Telescope Design)

The problem is to design and build a space telescope that will be able to "see" planets around nearby stars (other than the Sun).

Consider a telescope. Light enters the front of the telescope. This is called the *pupil plane*.



The telescope focuses all the light passing through the pupil plane from a given direction at a certain point on the *focal plane*, say $(0, 0)$.

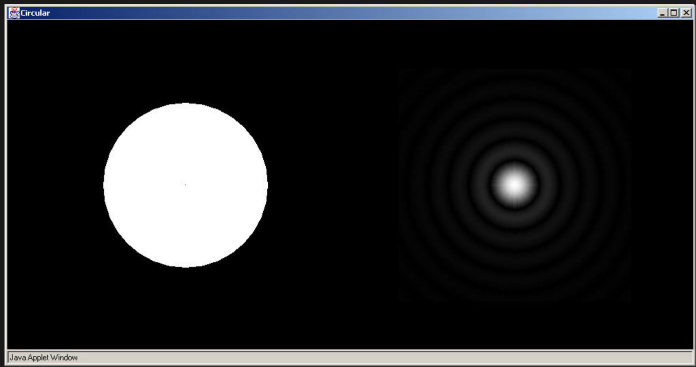
However, the wave nature of light makes it impossible to concentrate all of the light at a point. Instead, a small disk, called the *Airy disk*, with diffraction rings around it appears.

These diffraction rings are bright relative to any planet that might be orbiting a nearby star and so would completely hide the planet. The Sun, for example, would appear 10^{10} times brighter than the Earth to a distant observer.

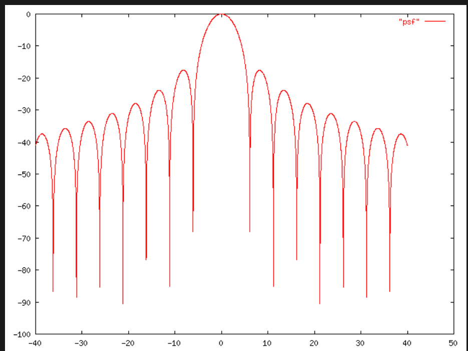
By placing a mask over the pupil, one can design the shape and strength of the diffraction rings. The problem is to find an optimal shape so as to put a very deep *null* very close to the Airy disk.

Airy Disk and Diffraction Rings

A conventional telescope has a circular openning as depicted by the left side of the figure. Visually, a star then looks like a small disk with rings around it, as depicted on the right.

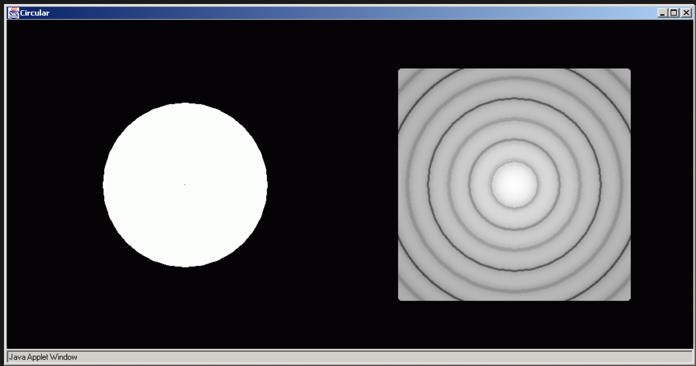


The rings grow progressively dimmer as this log-plot shows:



Airy Disk and Diffraction Rings—Log Scaling

Here's the same Airy disk from the previous slide plotted using a logarithmic brightness scale with $10^{-11} = -110$dB set to black:



The problem is to find an aperture mask, i.e. a pupil plane mask, that yields a -100 dB *null* somewhere near the first diffraction ring. *A hard problem!* Such a null would appear almost black in this log-scaled image.

Electric Field

Consider an aperture mask consisting of an openning given by

$$\left\{(x, y) : -\frac{1}{2} \le x \le \frac{1}{2}, -A(x) \le y \le A(x)\right\}.$$

We only consider masks that are symmetric with respect to both the x and y axes. Hence, the function $A()$ is a nonnegative even function.

In such a situation, the electric field $E(\xi, \zeta)$ is real and also symmetric about both the x and y axes. It is given by

$$\begin{aligned} E(\xi, \zeta) &= \int_{-\frac{1}{2}}^{\frac{1}{2}} \int_{-A(x)}^{A(x)} e^{i(x\xi + y\zeta)} dy dx \\ &= 4 \sum_j \int_0^{\frac{1}{2}} \cos(x\xi) \frac{\sin(A(x)\zeta)}{\zeta} dx \end{aligned}$$

The intensity of the light at (ξ, ζ) is given by the square of the electric field.

Maximizing Throughput

Because of the symmetry, we only need to optimize in the first quadrant:

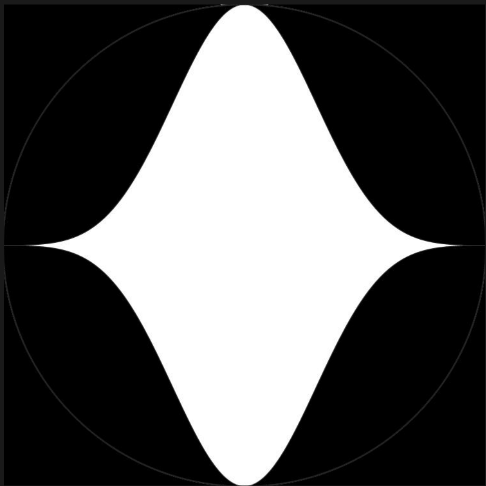
$$\text{maximize } 4\int_0^{\frac{1}{2}} A(x)dx$$

$$\begin{aligned} \text{subject to } & -10^{-5}E(0,0) \le E(\xi,\zeta) \le 10^{-5}E(0,0), && \text{for } (\xi,\zeta) \in \mathcal{O} \\ & 0 \le A(x) \le 1/2, && \text{for } 0 \le x \le 1/2 \end{aligned}$$

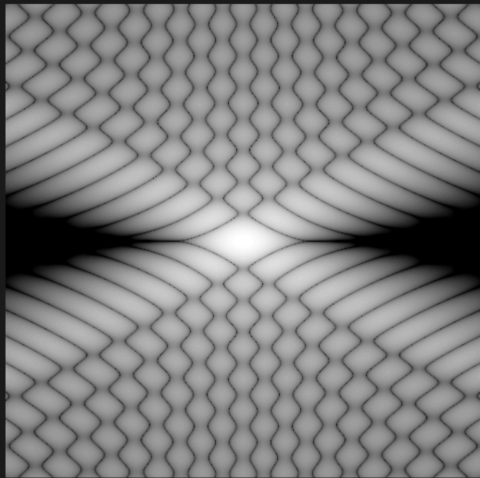
The objective function is the total open area of the mask. The first constraint guarantees 10^{-10} light intensity throughout a desired region of the focal plane, and the remaining constraint ensures that the mask is really a mask.

If the set $\mathcal{O}$ is a subset of the x-axis, then the problem is entirely linear (a linear programming problem).

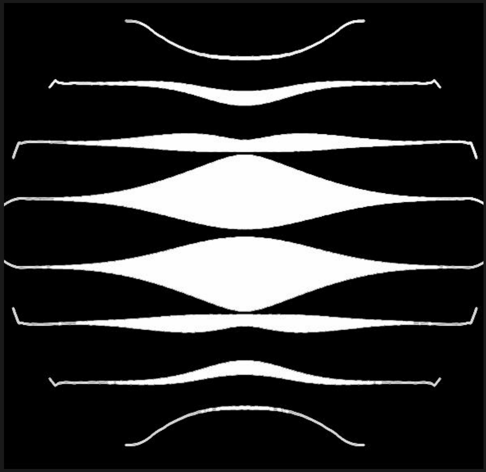
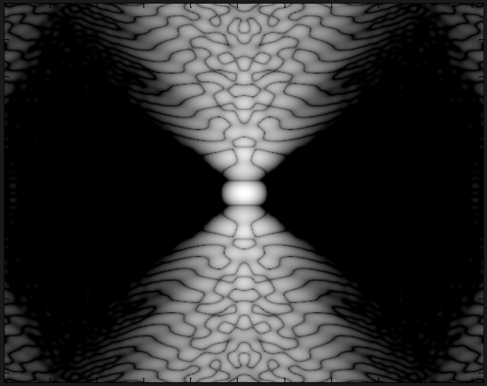
One Pupil w/ On-Axis Constraints



PSF for Single Prolate Spheroidal Pupil



Best Mask: 8-Pupil Mask

Circularly Symmetric Masks

- My original question was "Why not work with circularly symmetric optics?" In this case, one could think of making a variable filter. That is, at point (x, y) have the filter transmit a fraction $A(x, y)$ of the light.
- Such a filter is called an *apodization*.
- The answer is that apodizations are hard to make *accurately*.
- For small working bands, the square-aperture masks are essentially bang-bang all-or-nothing masks.
- It suggests looking for similar circularly symmetric masks.
- They can be thought of as apodizations in which the apodizing function $A(r)$ is zero-one valued.
- On the next few slides we derive the formulas for circularly symmetric apodization and then restrict attention to the zero-one valued case.

Circularly Symmetric Apodization

Instead of a square mask, we consider now a circularly symmetric apodized aperture:

$$E(\xi,\zeta) = \int_0^{1/2}\int_{-\pi}^{\pi} A(r) e^{-2\pi i(x\xi+y\zeta)} r\,d\theta\,dr$$

where, of course, $x = r\cos\theta$ and $y = r\sin\theta$.

WLOGWMAT, $\zeta = 0$ and hence we look at

$$\begin{aligned} E(\xi) &= \int_0^{1/2} rA(r)\left(\int_{-\pi}^{\pi} e^{-2\pi i\xi r\cos\theta} d\theta\right) dr \\ &= \int_0^{1/2} 2\pi r A(r) J_0\left(2\pi r\xi\right) dr \end{aligned}$$

Circularly Symmetric Masks

Let

$$A(r) = \begin{cases} 1 & r_{2j} \leq r \leq r_{2j+1}, \qquad j = 0, 1, \ldots, m-1 \\ 0 & \text{otherwise,} \end{cases}$$

where

$$0 \leq r_0 \leq r_1 \leq \cdots \leq r_{2m-1} \leq 1/2.$$

The integral on the previous slide can now be written as a sum of integrals and each of these integrals can be explicitly integrated to get:

$$E(\xi) = \sum_{j=0}^{m-1} \frac{1}{\xi} \Big(r_{2j+1} J_1 \left(2\pi\xi r_{2j+1}\right) - r_{2j} J_1 \left(2\pi\xi r_{2j}\right) \Big).$$

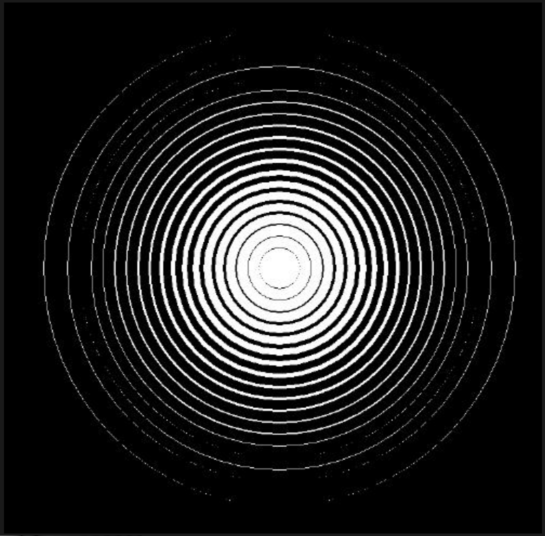
Circularly Symmetric Masks Optimization Problem

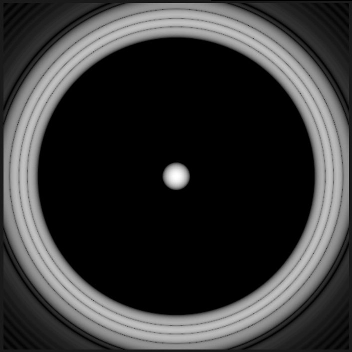
$$\text{maximize} \sum_{j=0}^{m-1} \pi(r_{2j+1}^2 - r_{2j}^2)$$

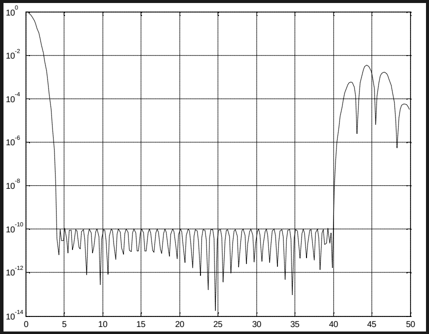
$$\text{subject to:} \quad -10^{-5}E(0) \leq E(\xi) \leq 10^{-5}E(0), \quad \text{for } \xi_0 \leq \xi \leq \xi_1$$

where $E(\xi)$ is the function of the r_j's given on the previous slide.

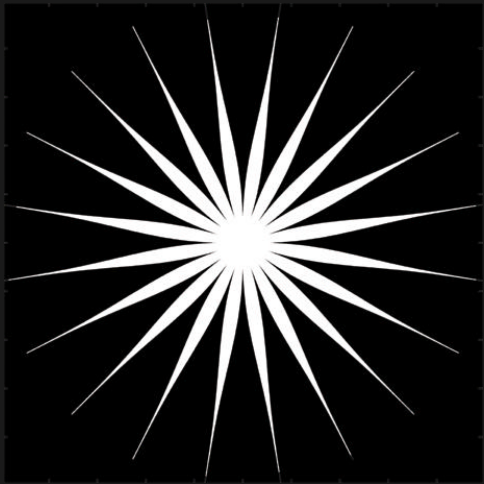
$\xi_0 = 4$ and $\xi_1 = 40$ and $m = 18$

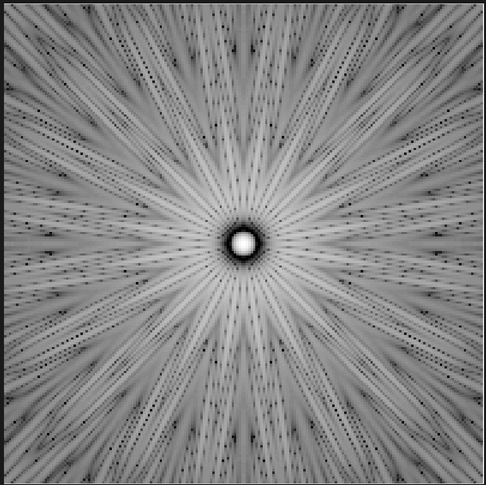


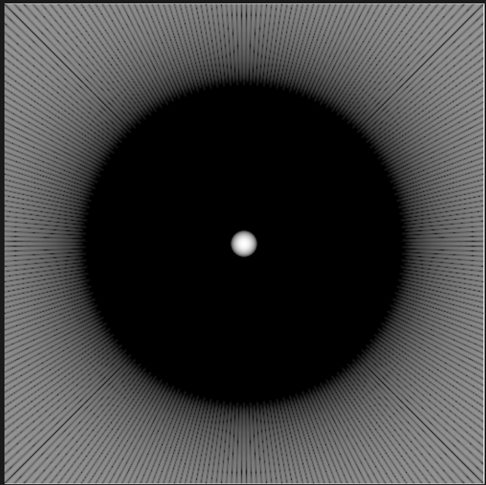




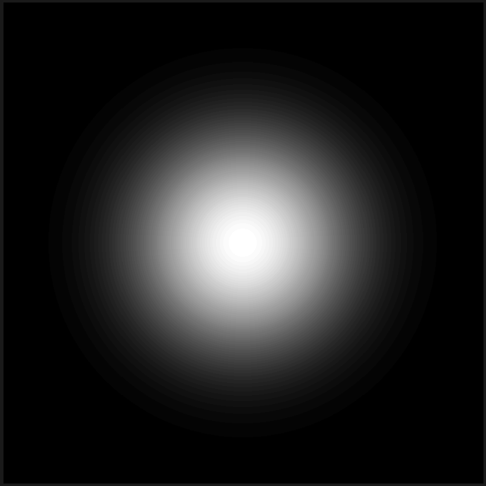
Starshaped Masks

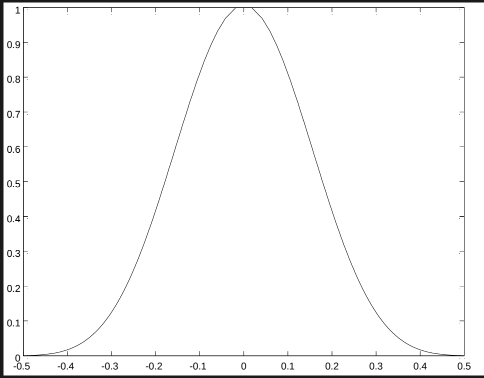


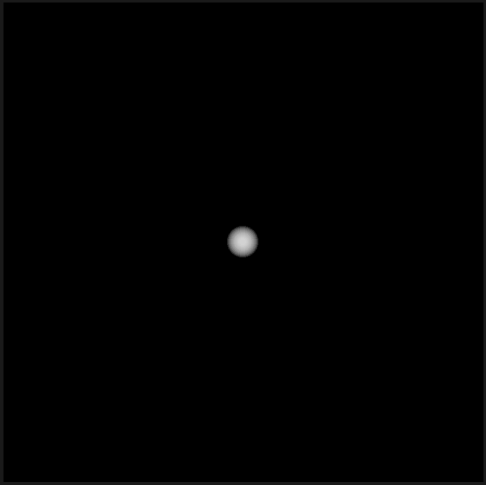


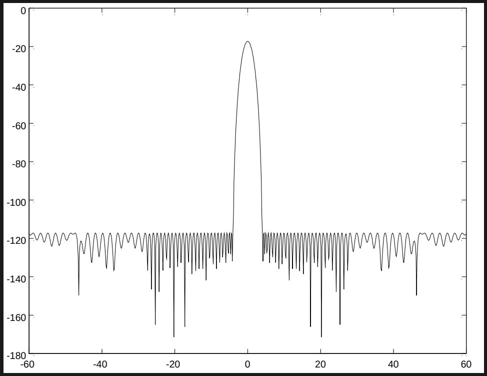


Apodization—Tinting Glass









Contents