



Extinguishing Poisson's Spot with Linear Programming

Robert J. Vanderbei

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<http://www.princeton.edu/~rvdb>

Are We Alone?



Indirect Detection Methods

Over 300 planets found—more all the time

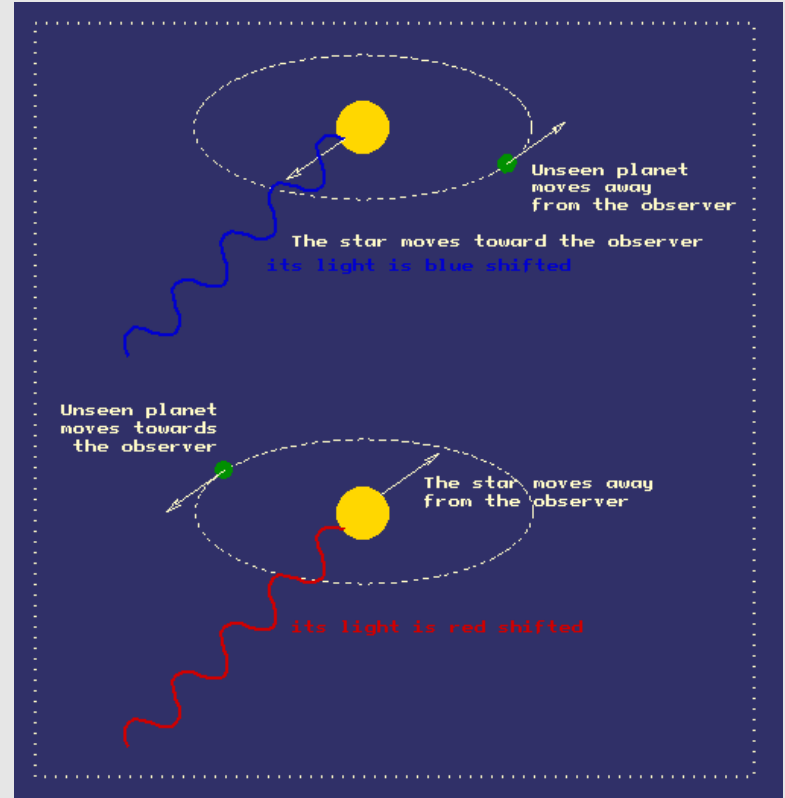
Wobble Methods

Radial Velocity (RV).

For approximately edge-on systems.
Measure periodic doppler shift.

Astrometry.

Best for face-on systems.
Measure circular wobble against back-
ground stars.

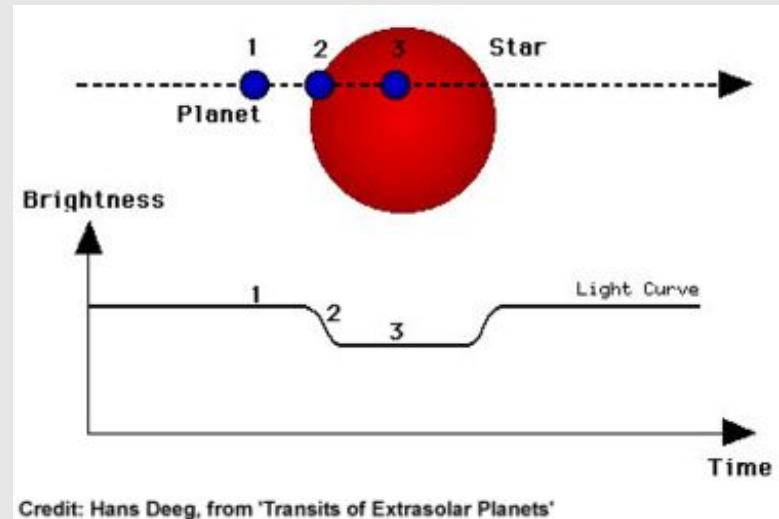


Transit Method

- HD209458b confirmed both via RV and transit.
- Period: 3.5 days
- Separation: 0.045 AU (0.001 arcsecs)
- Radius: $1.3R_J$
- Intensity Dip: $\sim 1.7\%$
- Venus Dip = 0.01%, Jupiter Dip: 1%



Venus Transit (R.J. Vanderbei)



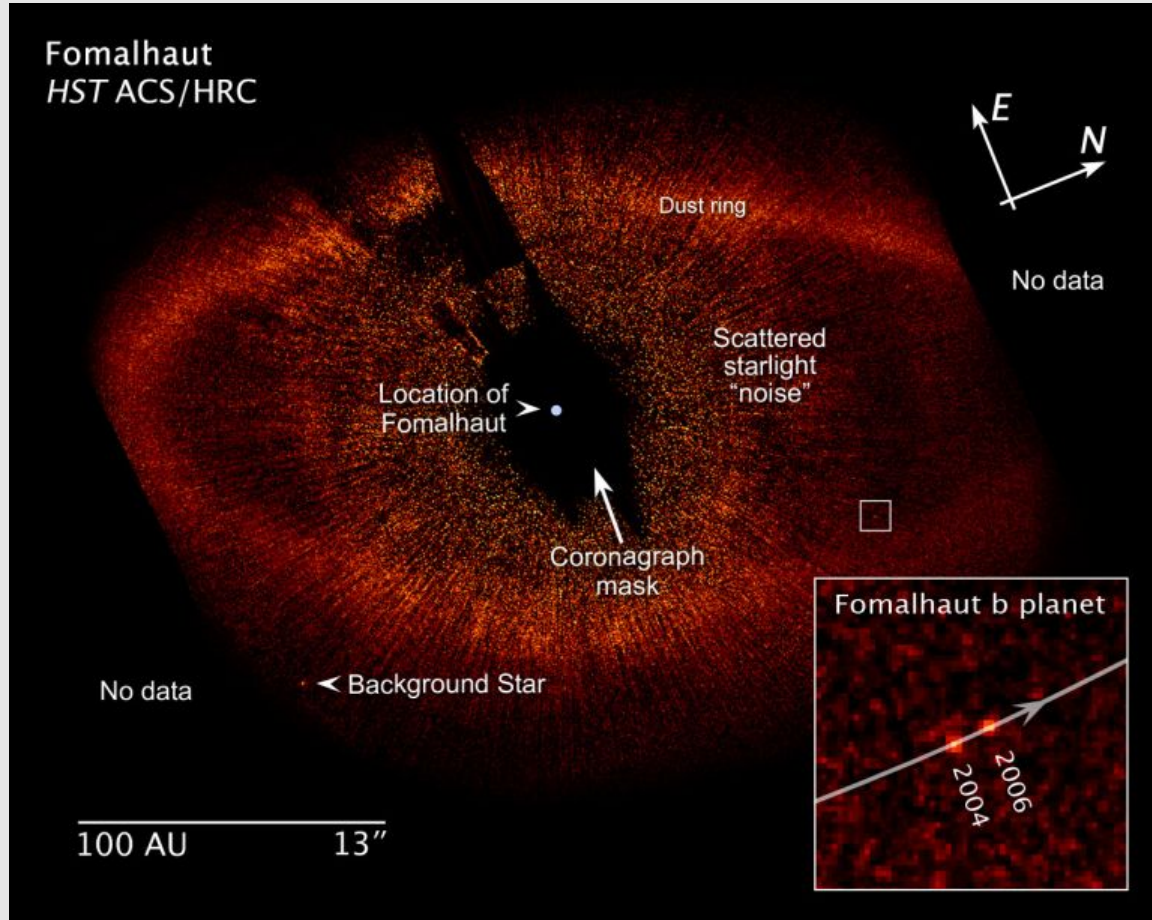
Credit: Hans Deeg, from 'Transits of Extrasolar Planets'

Direct Imaging

Fomalhaut.

Distance: 25 light-years

Star-Planet Separation: 100 AU



Terrestrial Planet Finder Telescope (TPF)

- DETECT: Search 150-500 nearby (5-15 pc distant) Sun-like stars for Earth-like planets.
- CHARACTERIZE: Determine basic physical properties and measure “biomarkers”, indicators of life or conditions suitable to support it.

Why Is It Hard? Can't Hubble do it?

- If the star is Sun-like and the planet is Earth-like, then the reflected visible light from the planet is 10^{-10} times as bright as the star.
- If the star is 10 pc (33 ly) away and the planet is 1 AU from the star, the angular separation is at most 0.1 arcseconds!
- A point source (i.e. star) produces not a point image but an *Airy pattern* consisting of an *Airy disk* surrounded by a system of *diffraction rings* completely swamping light from the nearby planet.
- By *apodizing* the entrance pupil, one can control the shape and strength of the diffraction rings.

Space Occulter Design for Planet-Finding

Space-based Occulter (TPF-O)

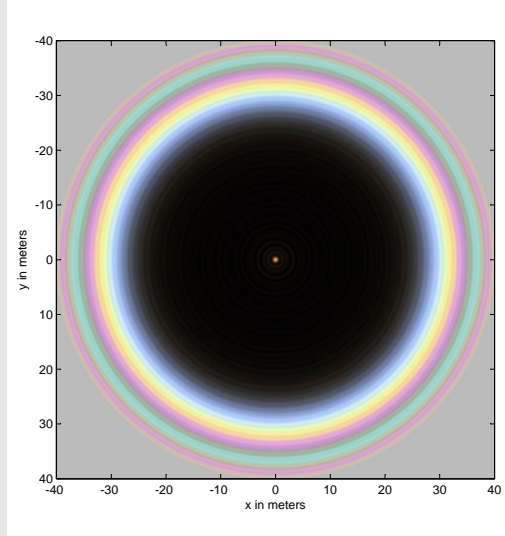
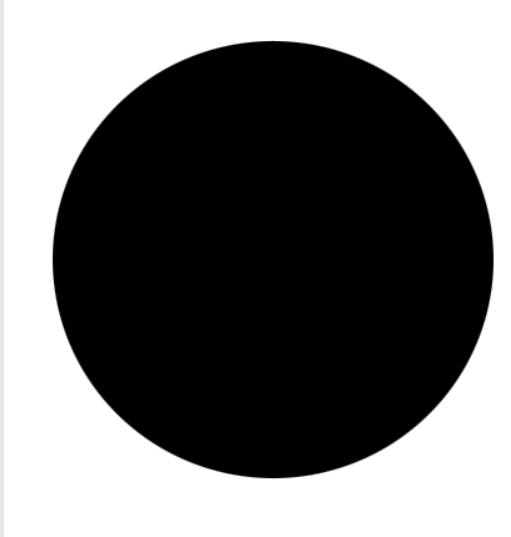


Telescope Aperture: 4m, Occulter Diameter: 50m, Occulter Distance: 72,000km

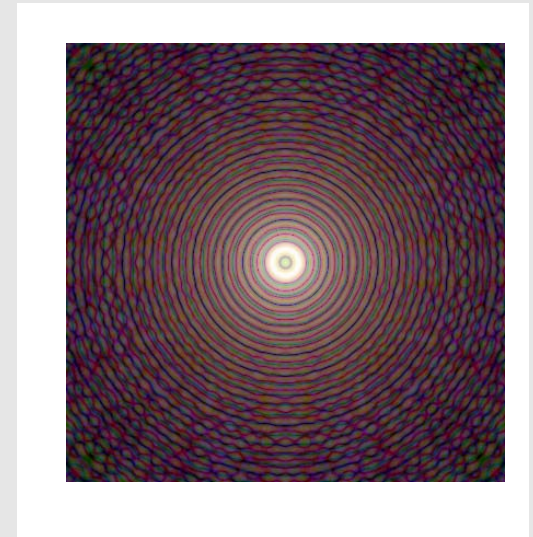
Plain External Occulter (Doesn't Work!)

Shadow \Rightarrow

Circular Occulter

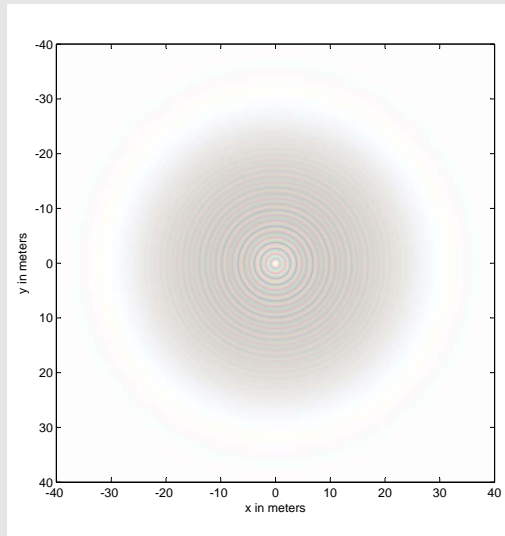


\Leftarrow Note bright spot at center
(Poisson's spot)



Telescope Image

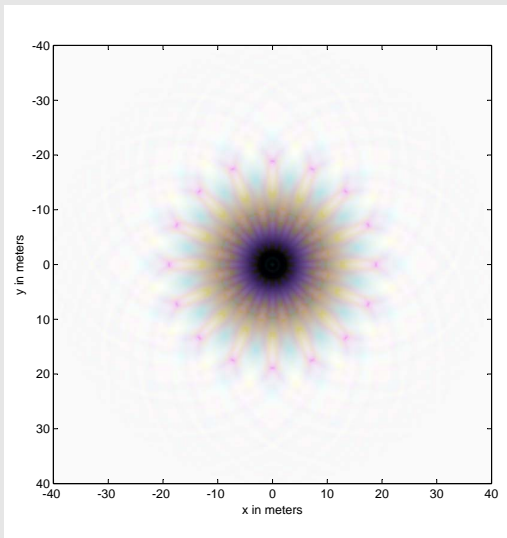
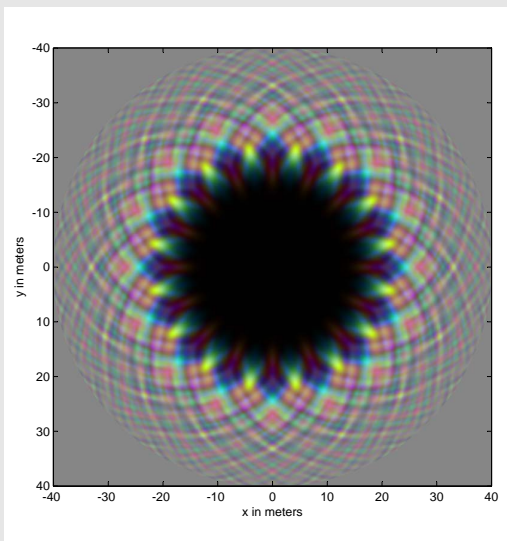
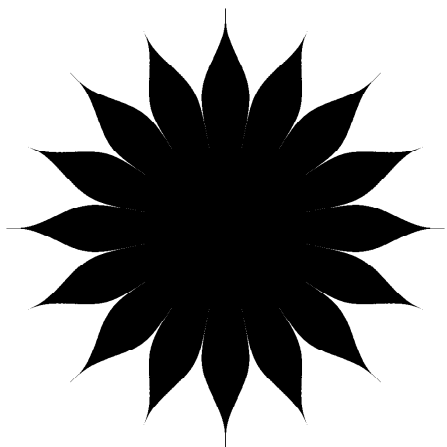
\Leftarrow Shadow (Log Stretch)



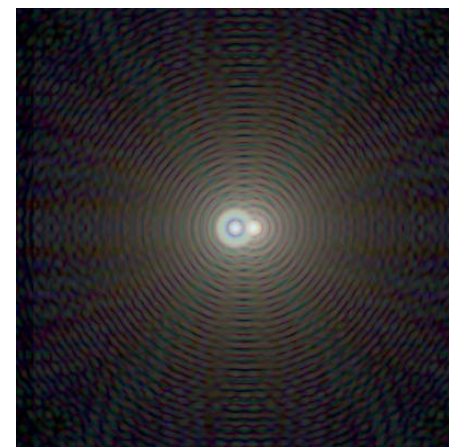
Shaped Occulter—Eliminates Poisson's Spot

Shadow \Rightarrow

Shaped Occulter



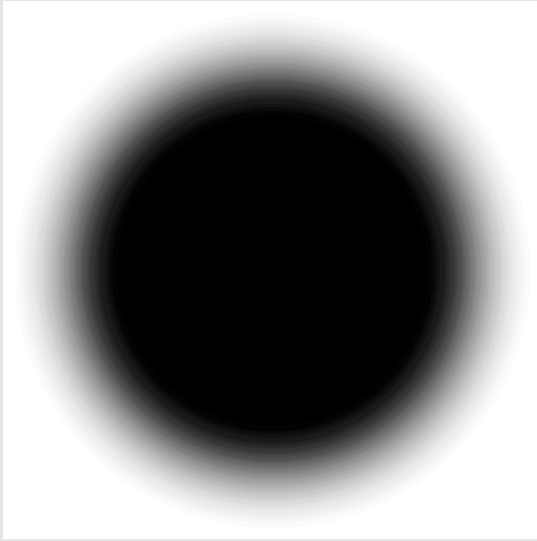
\Leftarrow Bright spot is gone



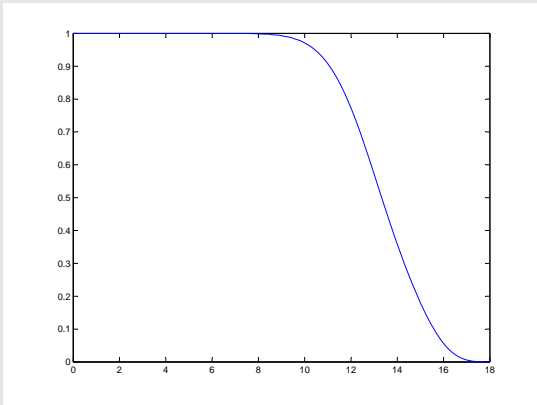
\Uparrow
Telescope image shows planet

\Leftarrow Shadow is dark
(Log Stretch)

Apodized Occulters



Apodized Occulter



Radial Attenuation $A(r)$

- The problem (as mentioned before) is *diffraction*.
- Abrupt edges create unwanted diffraction.
- *Solution*: Soften the edges with a partially transmitting material—an *apodizer*.
- Let $A(r, \theta)$ denote *attenuation* at location (r, θ) on the occulter.
- The *intensity* of the downstream light is given by the *square of the magnitude of the electric field* $E(\rho, \phi)$.
- *Babinet's principle* plus *Fresnel propagation* gives a formula for the downstream electric field:

$$E(\rho, \phi) = 1 - \frac{1}{i\lambda z} \int_0^\infty \int_0^{2\pi} e^{\frac{i\pi}{\lambda z}(r^2 + \rho^2 - 2r\rho \cos(\theta - \phi))} A(r, \theta) r d\theta dr.$$

where

- z is distance “downstream” and
- λ is wavelength of light.

Attenuation Profile Optimization

$$\begin{array}{ll} \text{minimize} & \gamma \\ \text{subject to} & -\gamma \leq \Re(E(\rho)) \leq \gamma \quad \text{for } \rho \in \mathcal{R}, \quad \lambda \in \Lambda \\ & -\gamma \leq \Im(E(\rho)) \leq \gamma \quad \text{for } \rho \in \mathcal{R}, \quad \lambda \in \Lambda \\ & A'(r) \leq 0 \quad \text{for } 0 \leq r \leq R \\ & -d \leq A''(r) \leq d \quad \text{for } 0 \leq r \leq R \end{array}$$

Specific choice:

$$R = 25, \quad d = 0.04, \quad \mathcal{R} = [0, 3], \quad \Lambda = [0.4, 1.1] \times 10^{-6}$$

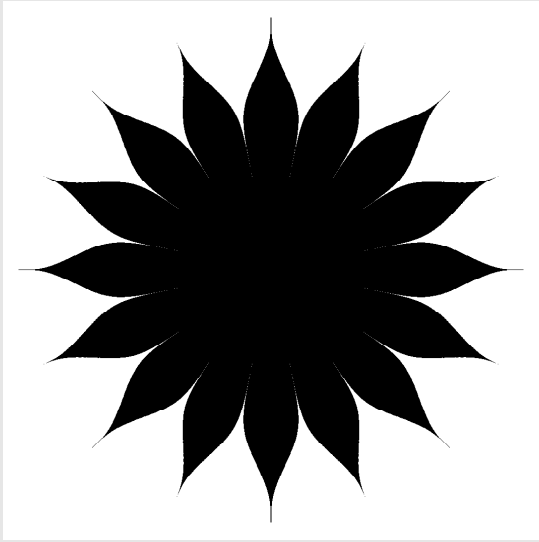
where all metric quantities are in meters.

An infinite dimensional linear programming problem.

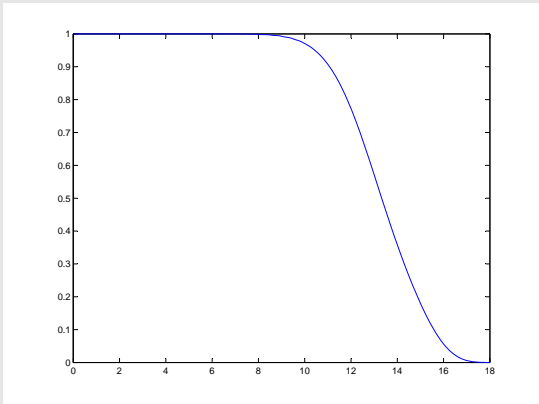
Discretize:

- $[0, R]$ into 5000 evenly space points.
- \mathcal{R} into 150 evenly spaced points.
- Λ into increments of 0.1×10^{-6} .

Petal-Shaped Occulters



16-Petal Occulter $A(r, \theta)$



Radial Attenuation $A(r)$

- From Jacobi-Anger expansion we get:

$$E(\rho, \phi) = 1 - \frac{2\pi}{i\lambda z} \int_0^R e^{\frac{i\pi}{\lambda z}(r^2 + \rho^2)} J_0\left(\frac{2\pi r \rho}{\lambda z}\right) A(r) r dr \\ - \sum_{k=1}^{\infty} \frac{2\pi(-1)^k}{i\lambda z} \left(\int_0^R e^{\frac{i\pi}{\lambda z}(r^2 + \rho^2)} J_{kN}\left(\frac{2\pi r \rho}{\lambda z}\right) \frac{\sin(\pi k A(r))}{\pi k} r dr \right) \\ \times \left(2 \cos(kN(\phi - \frac{\pi}{2})) \right)$$

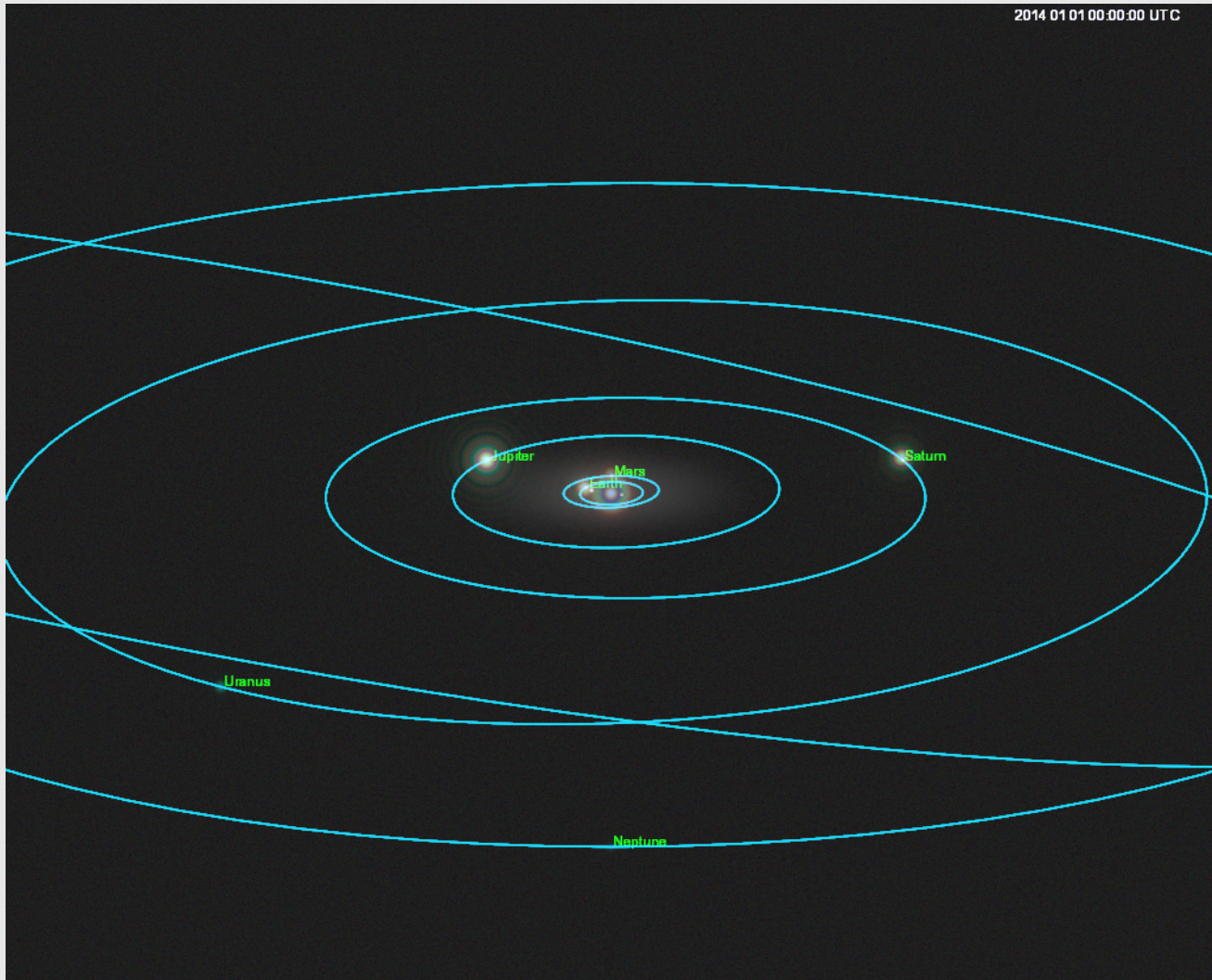
where N is the number of petals.

- For small ρ , truncated summation well-approximates full sum.
- Truncated after 10 terms.
- $\lambda \in [0.4, 1.1]$ microns.
- $z = 72,000$ km, $R = 25$ m.
- In angular terms, $R/z = 0.073$ arcseconds.

Our Solar System As Seen From Fomalhaut

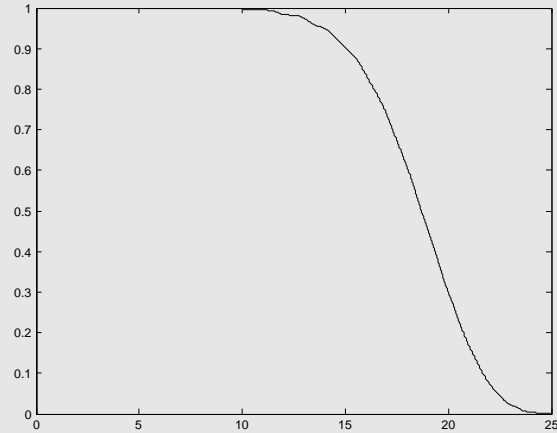


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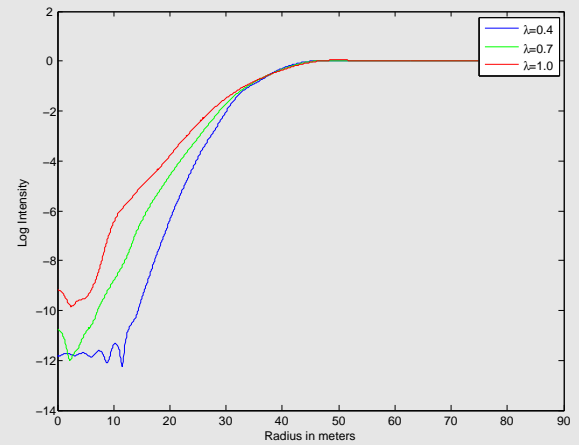
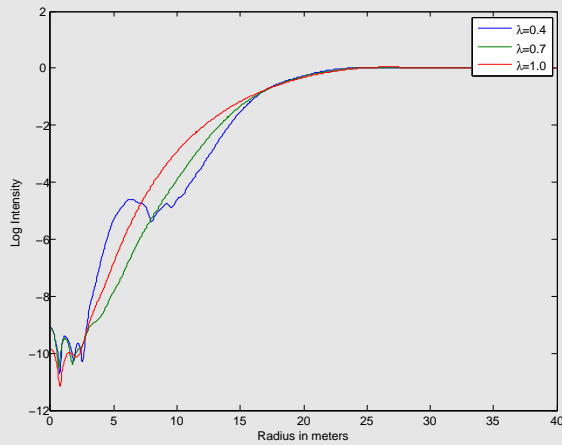
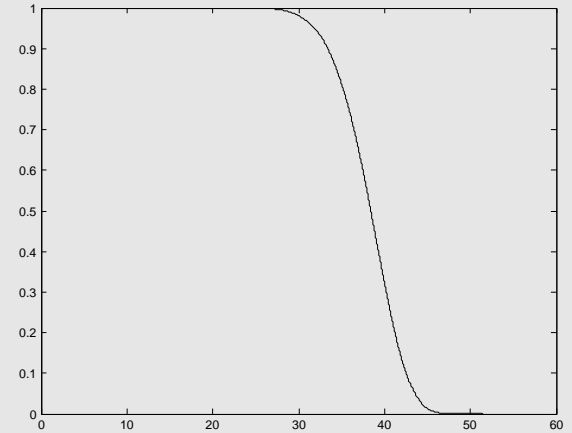


Optimal vs. Sub-Optimal But Explicit Design

Optimal



Hypergaussian



Size (tip-to-tip):

50 meters

Distance:

72,000 km

104 meters

150,000 km