The Search for Earth-like Planets

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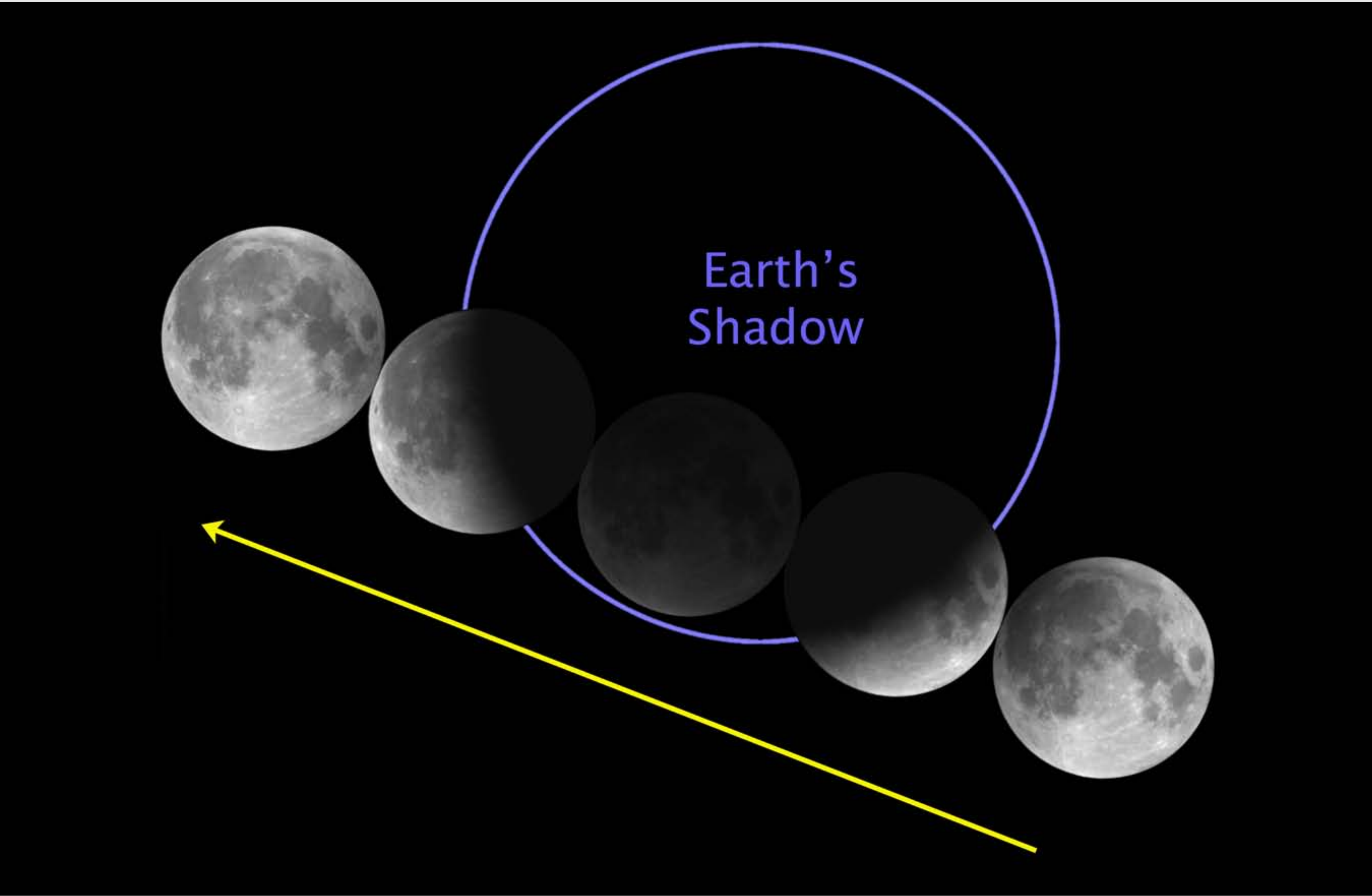
REU Seminar
DIMACS
Rutgers University

http://www.princeton.edu/~rvdb

Are We Alone?

Some History—The Earth is a Sphere



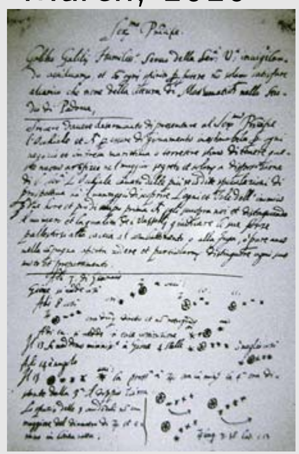
1609: Telescope is Invented (Hans Lippershey)

1610: Galileo Looks at Jupiter

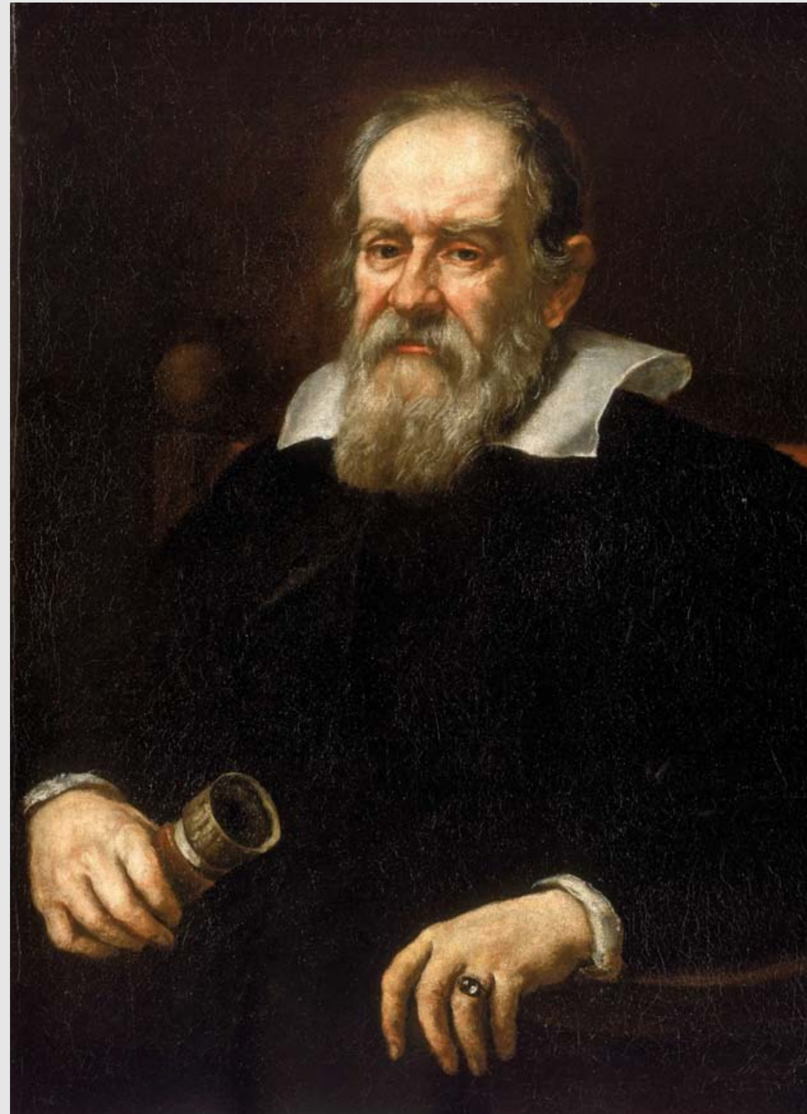
Galileo Galilei

"I should disclose and publish to the world the occasion of discovering and observing four Planets, never seen from the beginning of the world up to our own times, their positions, and the observations... about their movements and their changes of magnitude; and I summon all astronomers to apply themselves to examine and determine their periodic times...."

March, 1610

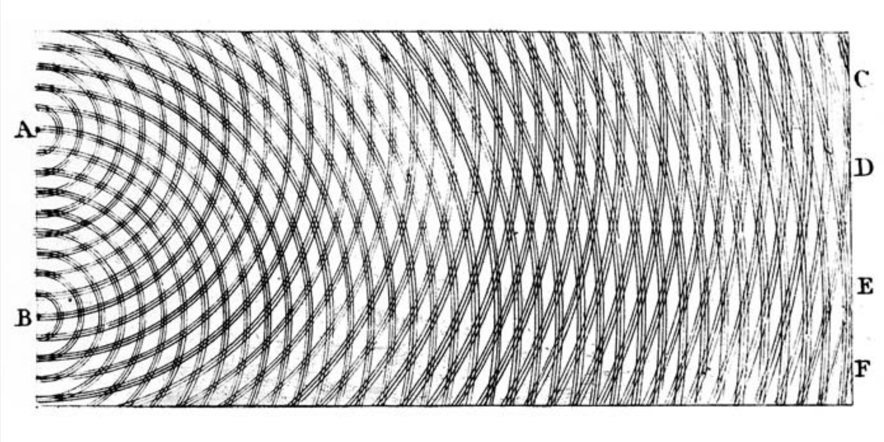


(Convicted of heresy, 1633.
House arrest until his death.
Sentence rescinded October, 1992)

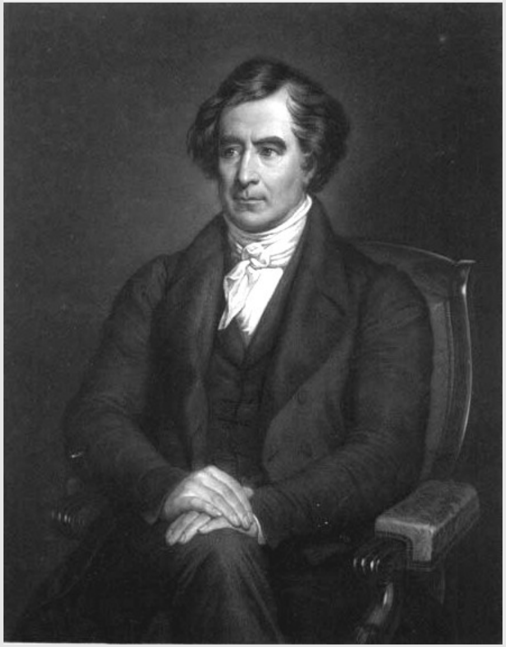


Christiaan Huygens (1678): Light is a Wave

Young's two-slit diffraction experiment (1801):



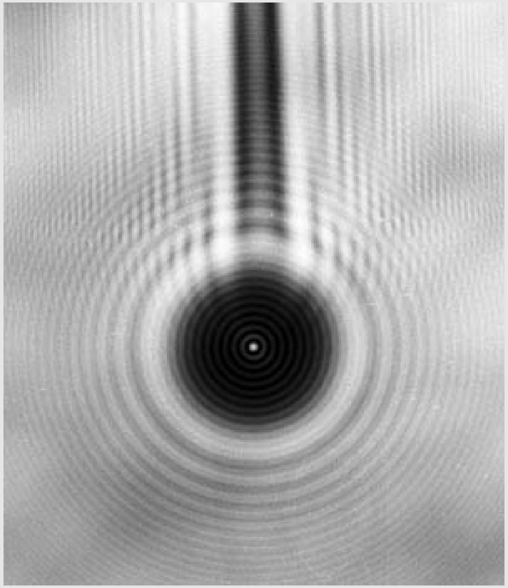
Siméon Poisson/Francois Arago (1818)



Poisson didn't believe the wave theory of light.
He pointed out that light falling on a circular object would have a bright spot at the center of its shadow.

Arago did the experiment.

Poisson was wrong.



Poisson's spot

A Modern Analysis of Poisson's Spot

Made elegant with Complex Variables

Here's what you need to know...

$$i = \sqrt{-1}$$

$$x + iy = re^{i\theta}$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

And, finally, all the rules of algebra and calculus are the same for complex variables as they are for real variables.

A Modern Analysis Continued

Here we use Huygen's Wavelets and Babinet's Principle

$$E = \iint \frac{1}{i\lambda d} e^{2\pi i d/\lambda} dx dy = e^{2\pi i z/\lambda} - \iint \frac{1}{i\lambda d} e^{2\pi i d/\lambda} dx dy,$$

where

$$\begin{aligned} d &= \sqrt{x^2+y^2+z^2} \\ &= \sqrt{x^2+y^2+z^2} - z + z \\ &= \frac{(\sqrt{x^2+y^2+z^2} - z)(\sqrt{\cdots} + z)}{\sqrt{\cdots}+z} + z \\ &= \frac{x^2+y^2}{\sqrt{\cdots}+z} + z \\ &\approx \frac{1}{2z}\left(x^2+y^2\right) + z. \end{aligned}$$

So, we get

$$E \approx e^{2\pi i z/\lambda} - \iint \frac{1}{i\lambda z} e^{\frac{\pi i}{\lambda z}(x^2+y^2) + 2\pi i z/\lambda} dx dy.$$

A Modern Analysis Continued

And the rest is just Calculus

First switch to polar coordinates:

$$E \approx e^{2\pi iz/\lambda} - \frac{e^{2\pi iz/\lambda}}{i\lambda z} \int_0^R \int_0^{2\pi} e^{\frac{\pi i}{\lambda z} r^2} r d\theta dr.$$

The inner θ-integral is trivial and the outer r-integral is straightforward:

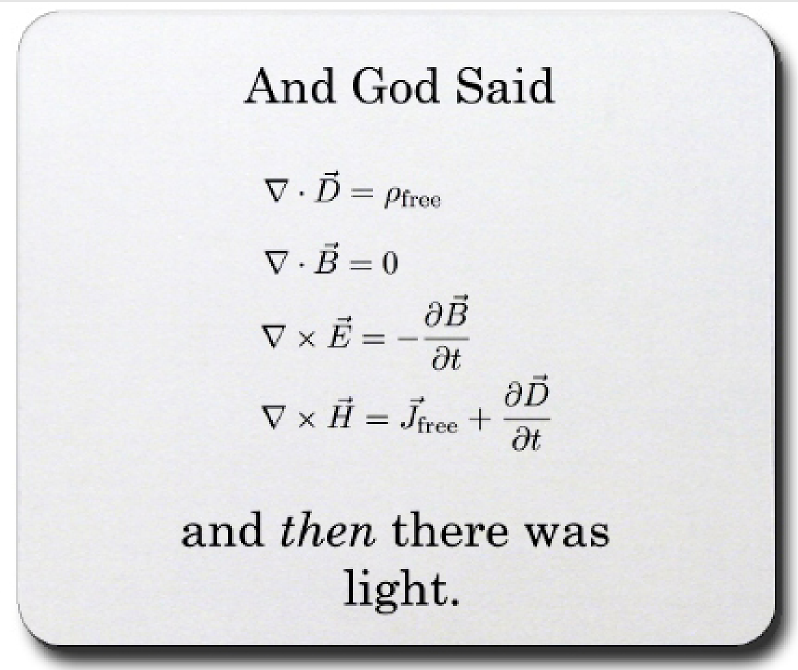
$$\begin{aligned} E &\approx e^{2\pi iz/\lambda} - \frac{2\pi e^{2\pi iz/\lambda}}{i\lambda z} \int_0^R e^{\frac{\pi i}{\lambda z} r^2} r dr \\ &= e^{2\pi iz/\lambda} - e^{2\pi iz/\lambda} \int_0^{\frac{\pi i R^2}{\lambda z}} e^u du \\ &= e^{2\pi iz/\lambda} e^{\frac{\pi i R^2}{\lambda z}}. \end{aligned}$$

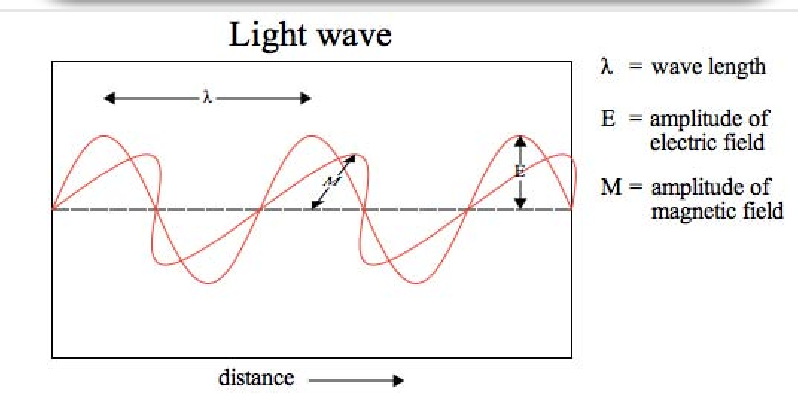
Finally, our eyes see *intensity* which is $|E|^2$:

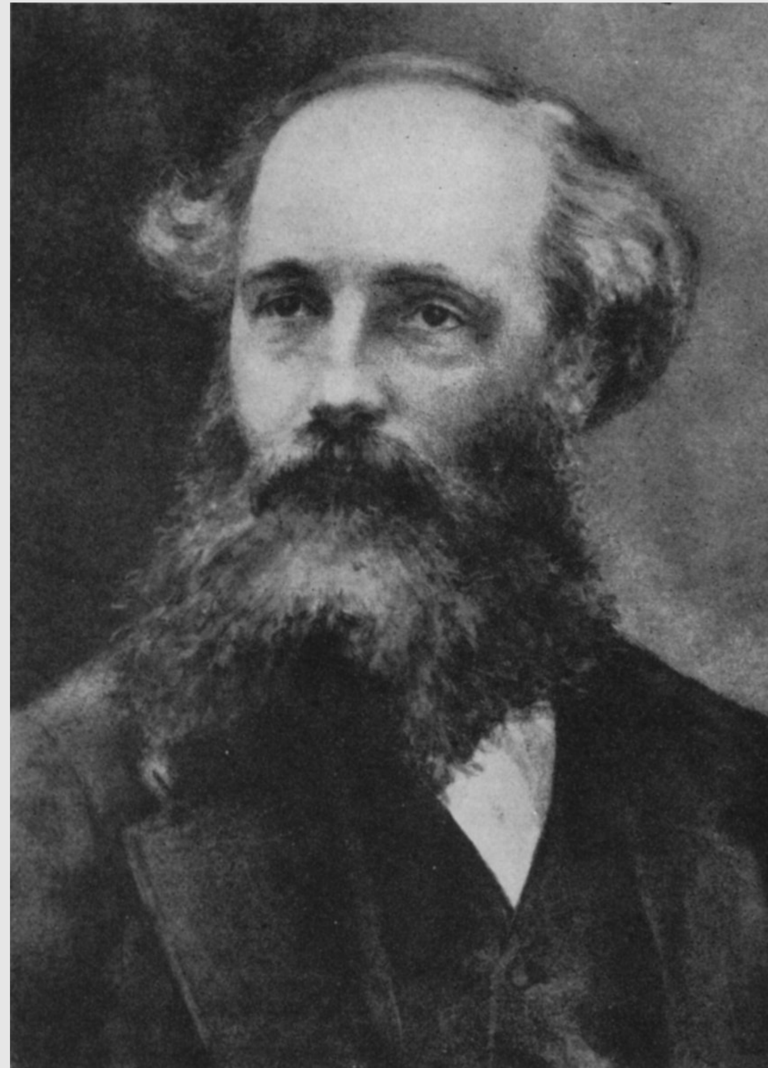
$$|E|^2 = 1.$$

Conclusion: The intensity at the center of the shadow is the same as the intensity outside the shadow.

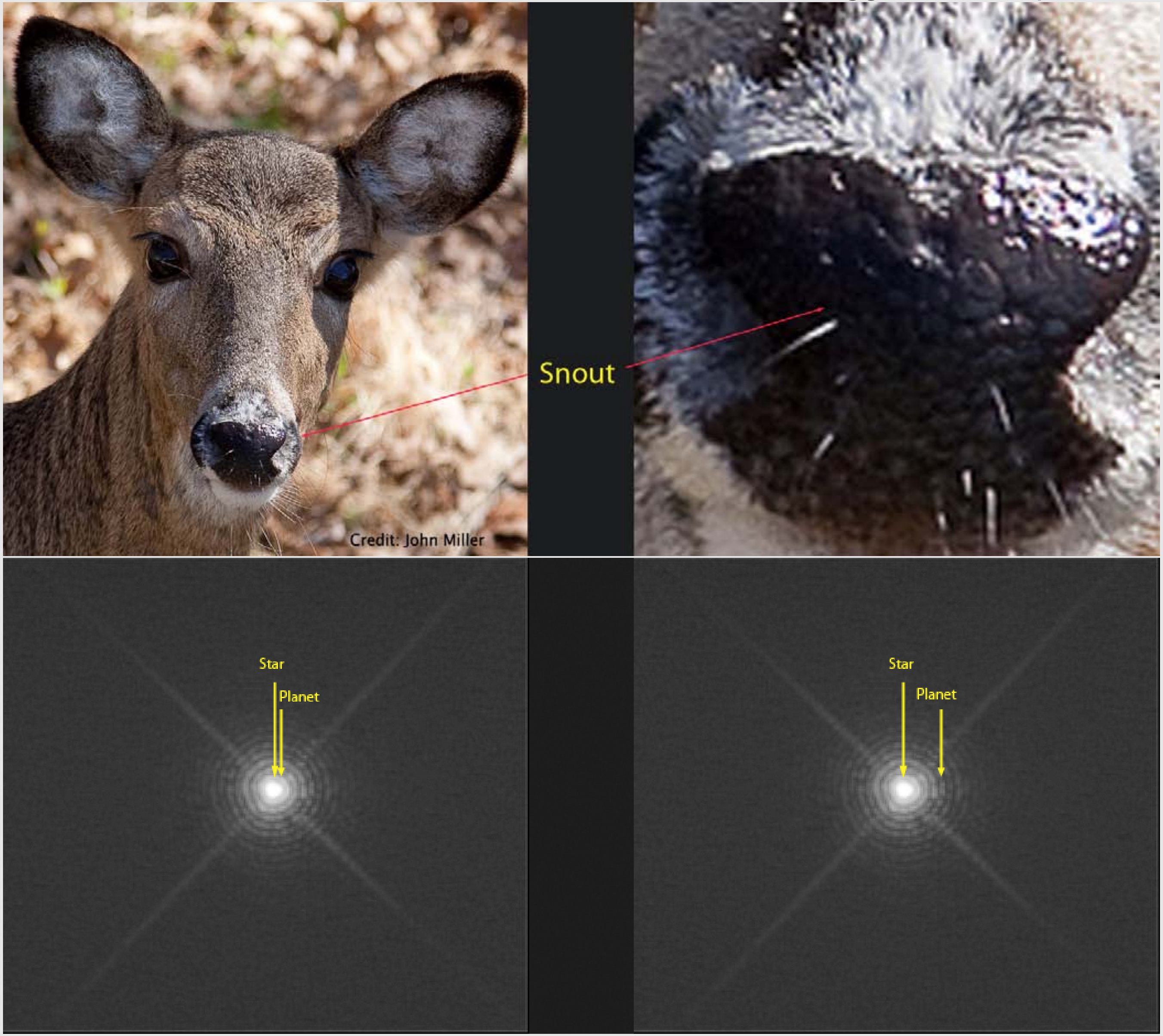
James Clerk Maxwell (1862): Light is an Electro-Magnetic Wave







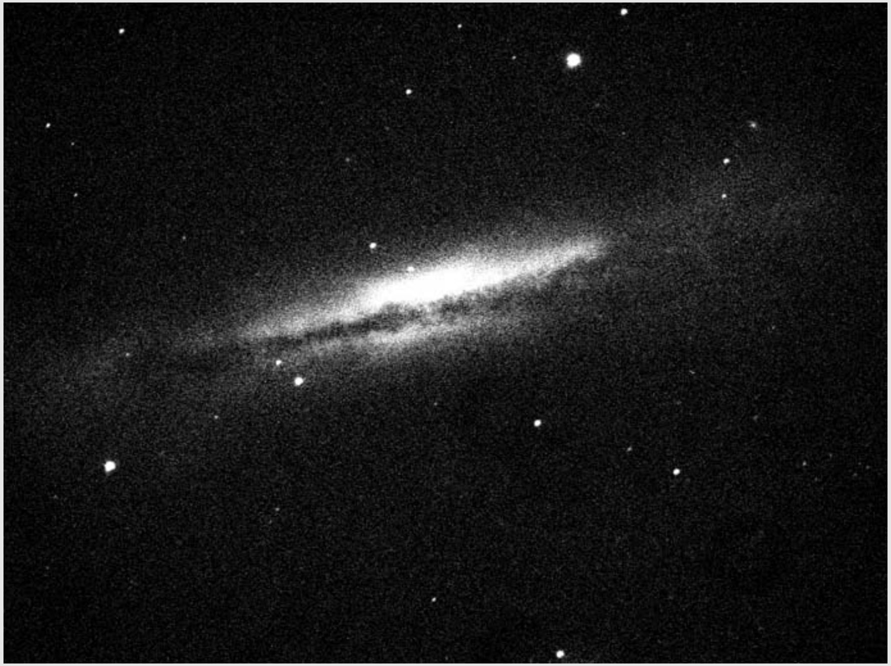
Telescope | $6\times$ Bigger Telescope

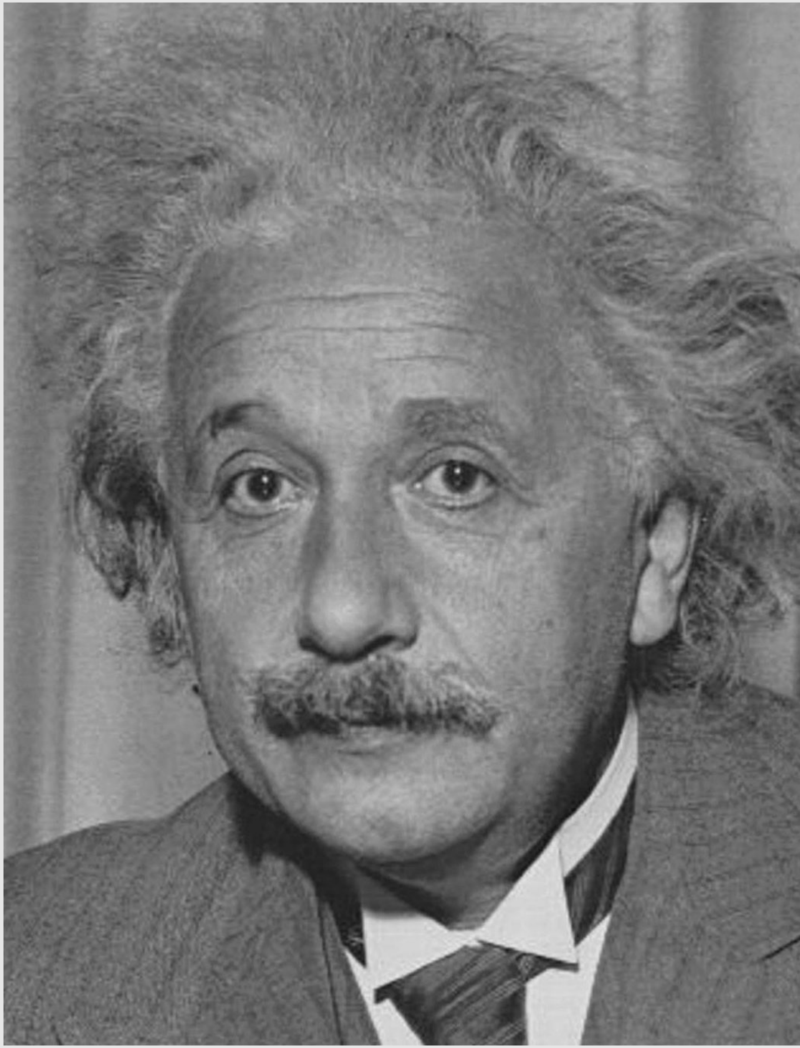


Albert Einstein (1905): Light is a Particle

Explained the photoelectric effect,
which led to the new field of *quantum mechanics*.
Einstein himself never accepted it.

Modern CCD cameras count *photons*.





21st Century Question: Are We Alone?

Indirect Detection Methods

More than 400 planets found so far

(since 1995)

Wobble Methods

Radial Velocity.

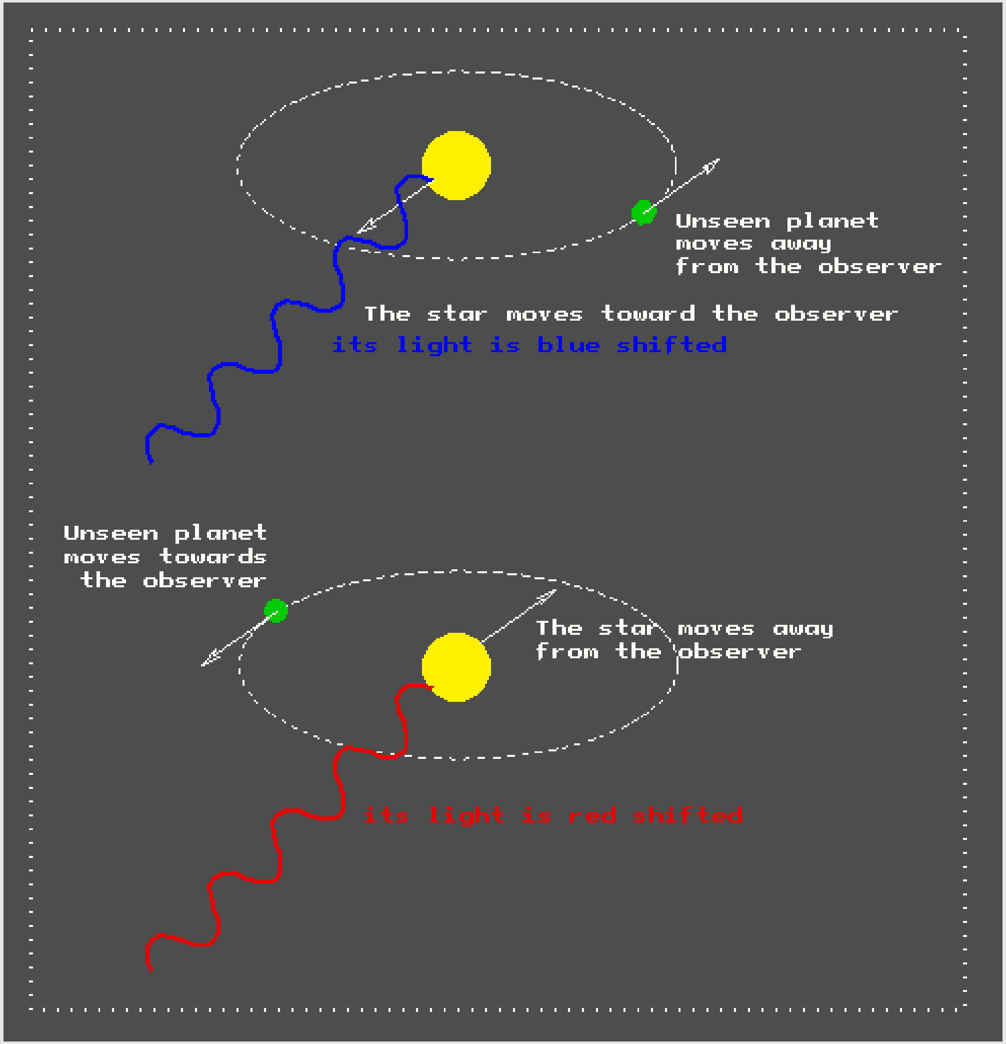
For edge-on systems.

Measure periodic doppler shift.

Astrometry.

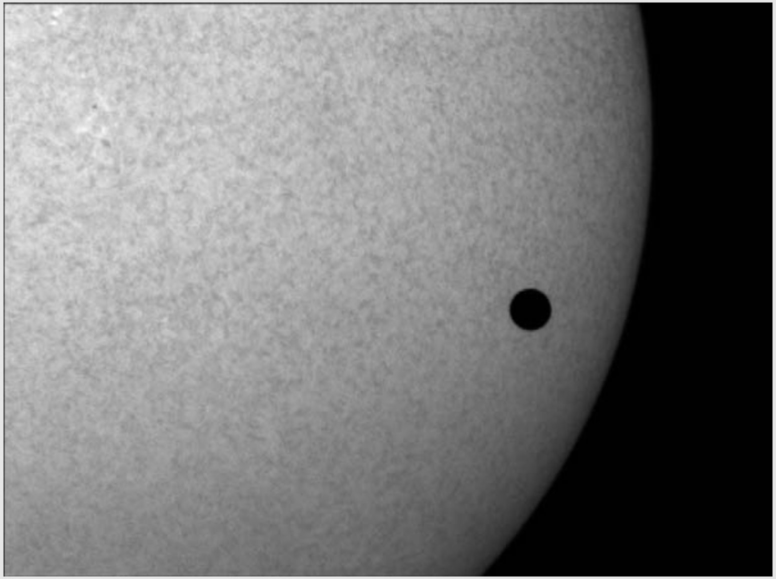
Best for face-on systems.

Measure circular wobble against background stars.

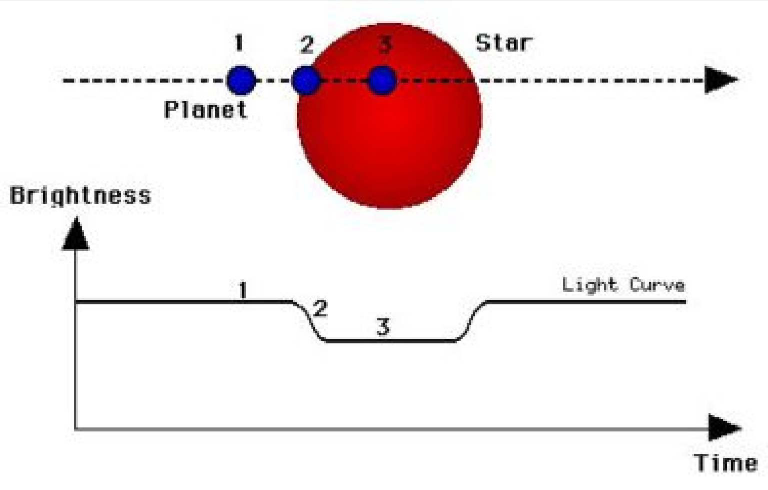


Transit Method

- HD209458b confirmed both via RV and transit.
- Period: 3.5 days
- Separation: 0.045 AU (0.001 arcsecs)
- Intensity Dip: $\sim 1.7\%$
- Radius: $1.3R_{\mathrm{J}}$
- (Venus Dip $= 0.01\%$, Jupiter Dip: 1%)



Venus Transit (R.J. Vanderbei)



We Want "Earths"

Venus

Hot and Humid

Earth

Just Right

Mars

A Bit Cold

Jupiter

Too Gassy

Saturn

Brrr

$\Longleftarrow$ This way to Mercury and the Sun

Where Should We Look?


Not in a star-birth region—stars are too young

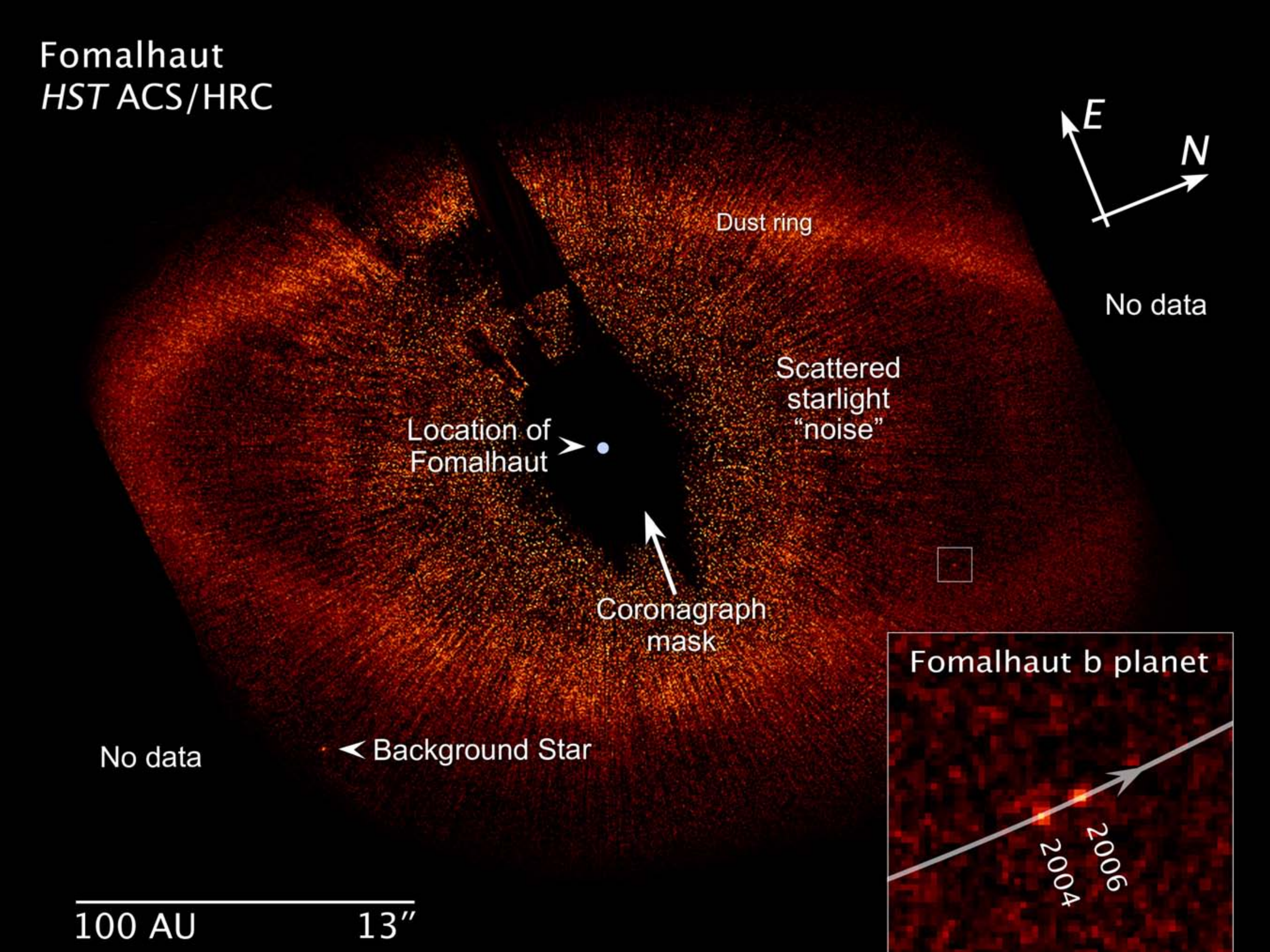
Not in a young open cluster—stars are too young

Not near old dying stars

Not in a globular cluster—too much dynamical instability


HERE!
Around run-of-the-mill middle-aged stars

Direct Detection


Fomalhaut
HST ACS/HRC
E
N
Dust ring
No data
Scattered starlight "noise"
Location of Fomalhaut
Coronagraph mask
Fomalhaut b planet
2004
2006
No data
Background Star
100 AU
13"

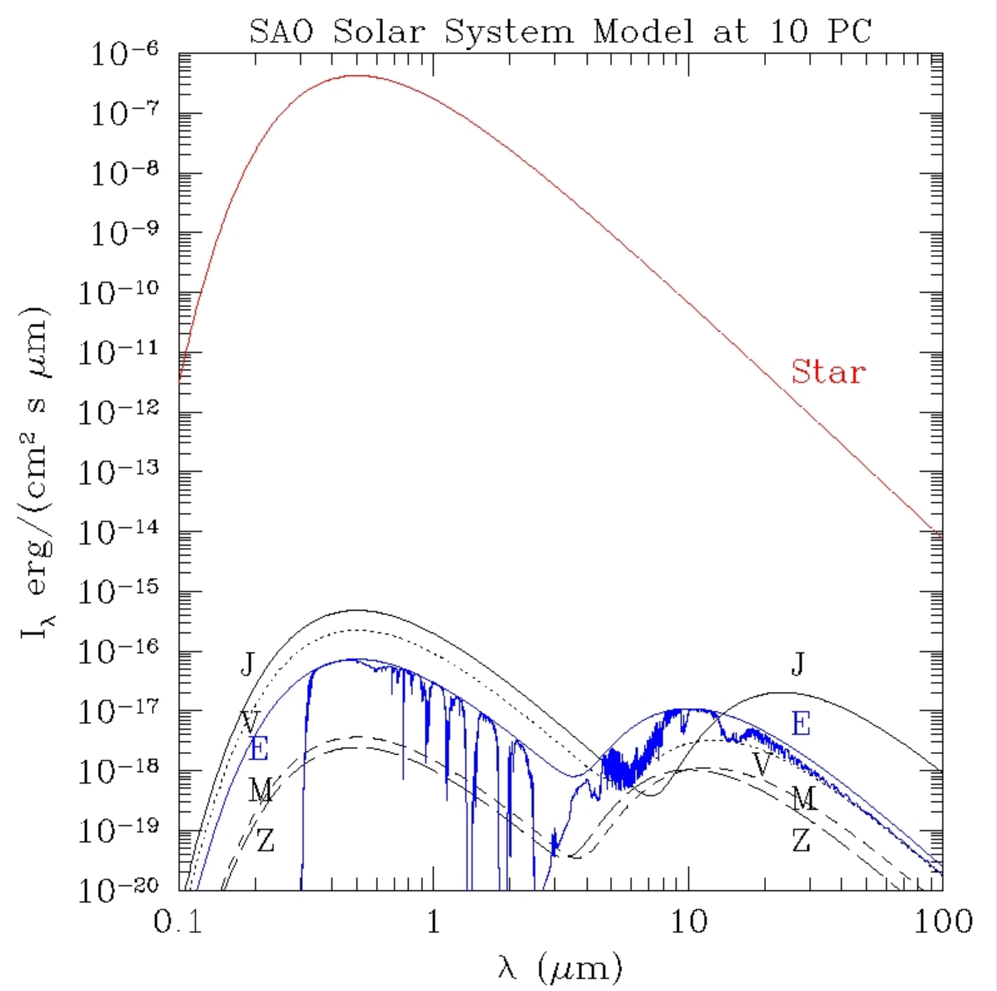
Why It's Hard

Premise: If there is intelligent life "out there", it probably is similar to life as we know it on Earth.

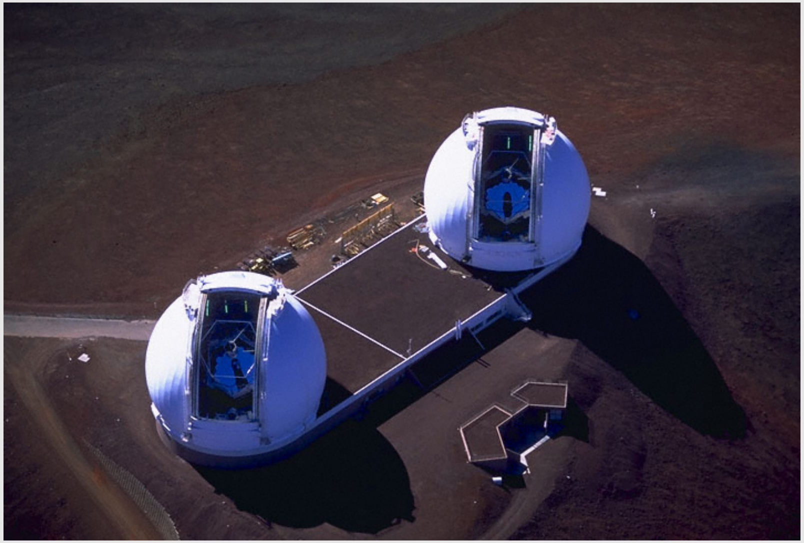
- *Bright Star/Faint Planet:* In visible light, our Sun is ten billion times brighter than Earth.

- *Close to Each Other:* A planet at 1 AU from a star at 33 light-years can appear at most 0.1 arcseconds in separation. (The full moon is 1800 arcseconds in diameter.)

- *Far from Us:* There are less than 100 Sun-like stars within 10 parsecs.



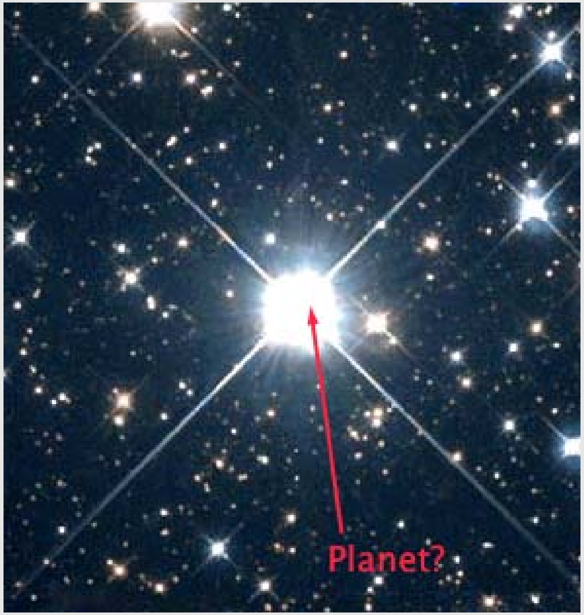
Can Ground-Based Telescopes Do It?



- Atmospheric distortion limits *resolution* to about 1 arcsec.
  Note: Resolution refers to equally bright objects.
  If one is much brighter than the other, then it is more difficult.
- Large aperture with adaptive optics.
- Interferometry.

No they can't!

Can Hubble Do It?



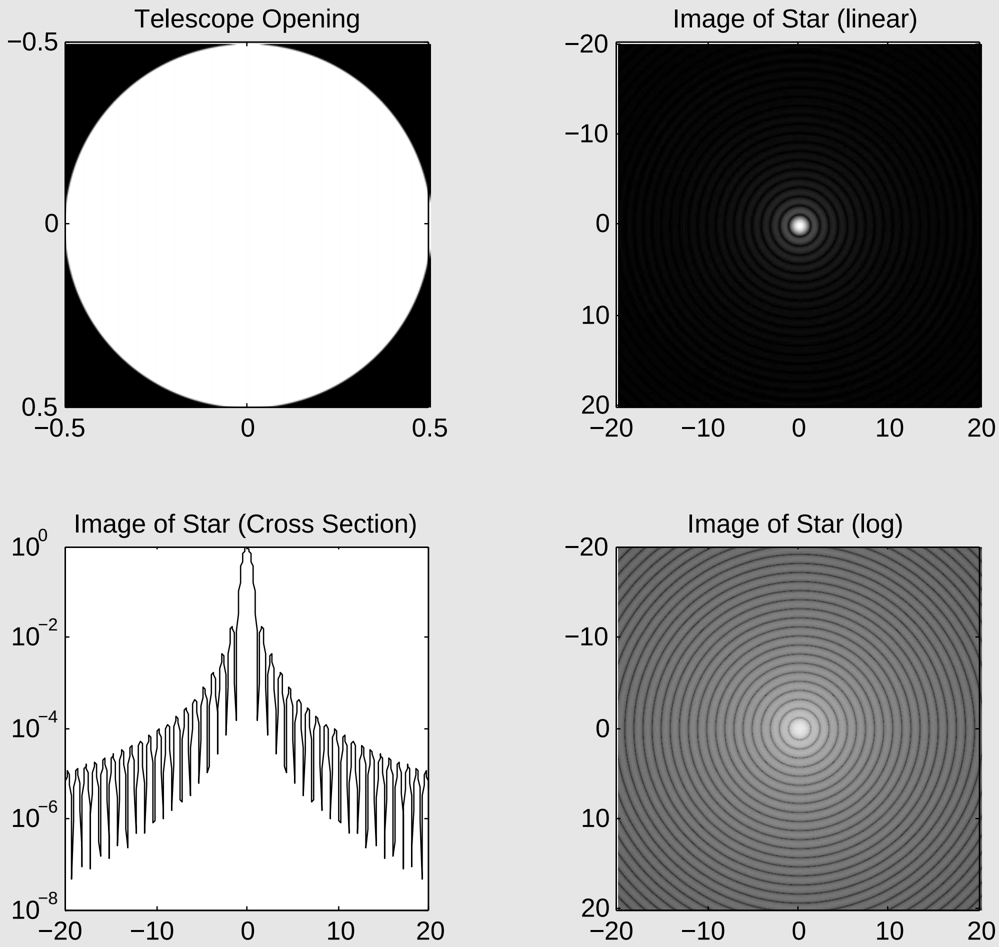
No it can't!

The problem is diffraction

Would have to be $1000\times$ bigger (in each dimension!)

The Problem is Diffraction

Requires a telescope with a mirror measured in *kilometers* to mitigate diffraction effects.



Diffraction Control via Tinting the Pupil

Physicists have understood for a long time that the abrupt edge of the telescope's "mirror" causes the bright diffraction rings.

Solution: Use tinted glass to ease the transition from transparent to opaque.

Some of the Math

The image-plane *electric field* $E()$ produced by an on-axis plane wave (i.e., starlight) and an apodized (i.e., tinted) aperture defined by an *apodization function* $A()$ is given by the *Fourier transform*:

$$\begin{aligned} E(\xi, \zeta) &= \iint_{\bigcirc} e^{i(x\xi + y\zeta)} A(x, y) dy dx \\ &\vdots \\ E(\rho) &= 2\pi \int_0^{1/2} J_0(r\rho) A(r) r dr, \end{aligned}$$

where J_0 denotes the 0-th order Bessel function of the first kind.

NOTE: The *electric field* depends *linearly* on the *apodization function*.

The *intensity* is the square of the electric field.

The unitless pupil-plane "length" r is given as a multiple of the aperture D.

The unitless image-plane "length" ρ is given as a multiple of focal-length times wavelength over aperture $(f\lambda/D)$ or, equivalently, as an angular measure on the sky, in which case it is a multiple of just λ/D. (Example: $\lambda = 0.5\mu$m and $D = 10$m implies $\lambda/D = 10$mas.)

Optimization

Find *apodization* function $A()$ that solves:

$$\begin{aligned} \text{maximize} \quad & \int_0^{1/2} A(r) 2\pi r dr \\ \text{subject to} \quad & -10^{-5} E(0) \leq E(\rho) \leq 10^{-5} E(0), \qquad \rho_{\text{iwa}} \leq \rho \leq \rho_{\text{owa}}, \\ & 0 \leq A(r) \leq 1, \qquad 0 \leq r \leq 1/2, \\ & -50 \leq A''(r) \leq 50, \qquad 0 \leq r \leq 1/2 \end{aligned}$$

An infinite dimensional *linear programming* problem.

Mirror with "Optimal" Tinting

Mirror with Softened Edge

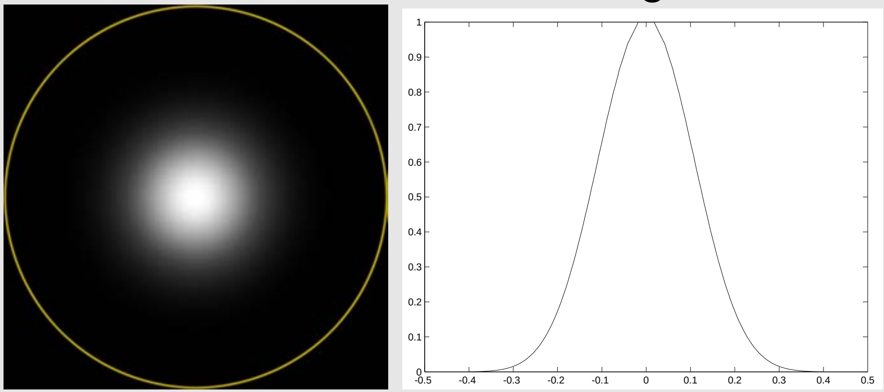
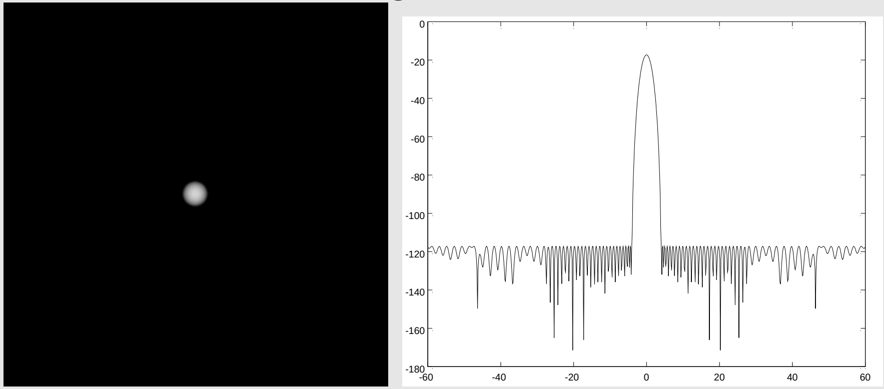
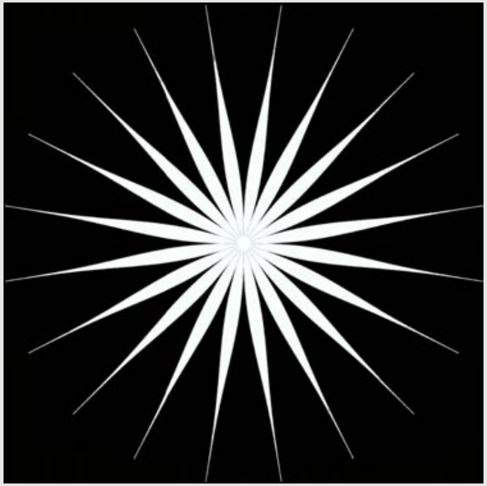


Image of Star

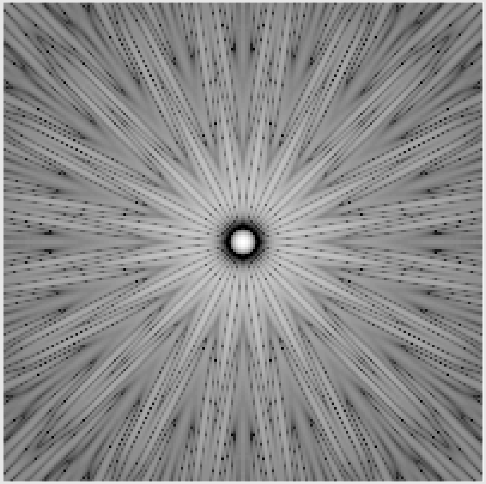


Mathematically Perfect... But Unmanufacturable!

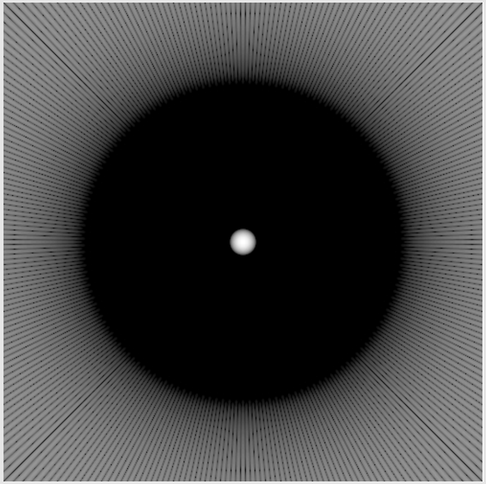
Shaped Pupil Coronagraph (TPF-C)



20 petals



150 petals



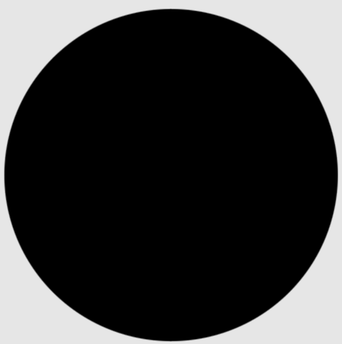
Maybe We Can!

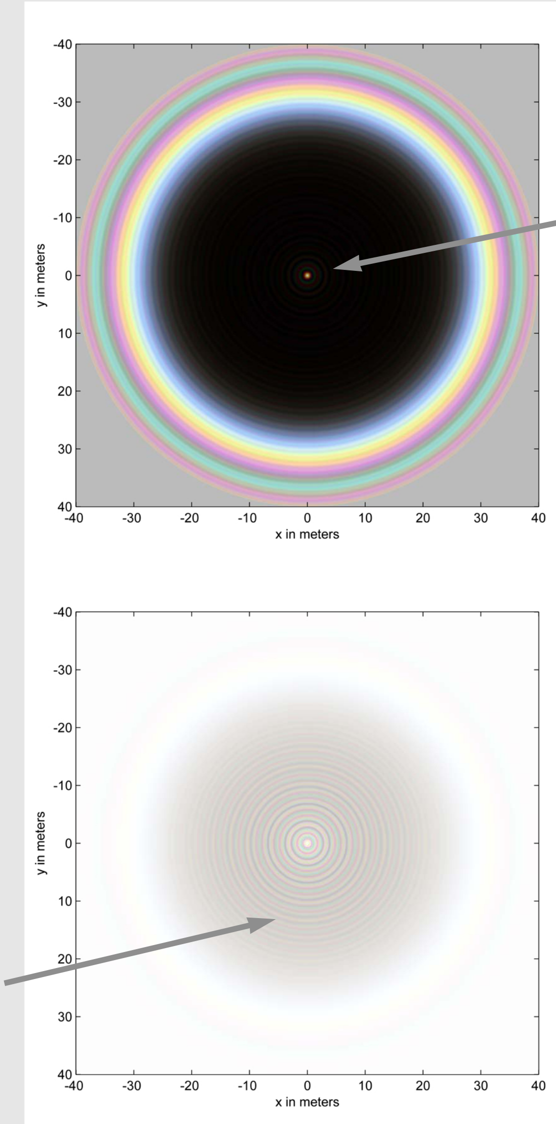
Space-based Occulter (TPF-O)

Telescope Aperture: 4m, Occulter Diameter: 50m, Occulter Distance: $72,000$km

Plain External Occulter (Doesn't Work!)

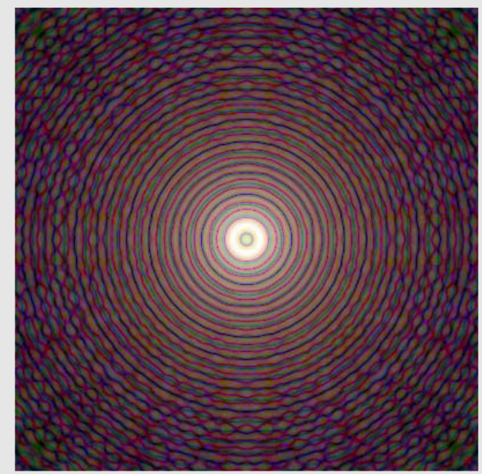
Circular Occulter





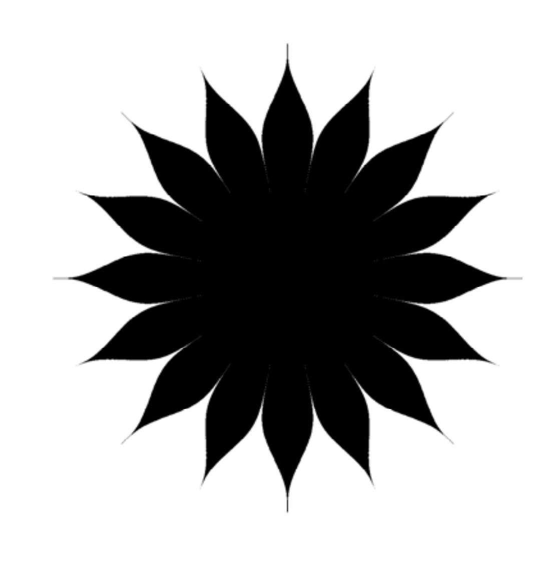
Poisson's Spot!

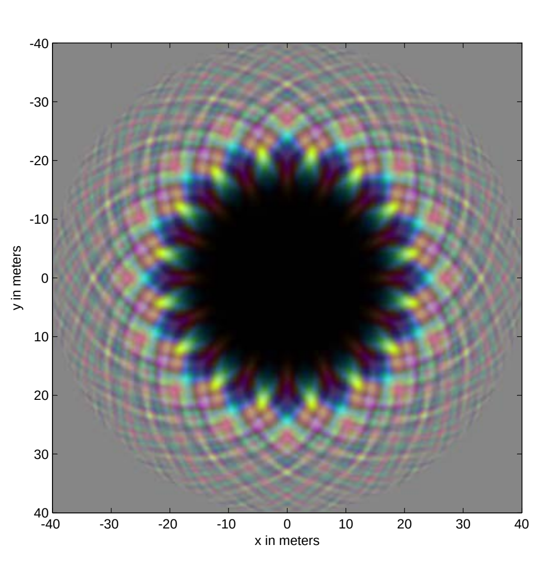
Shadow isn't dark enough

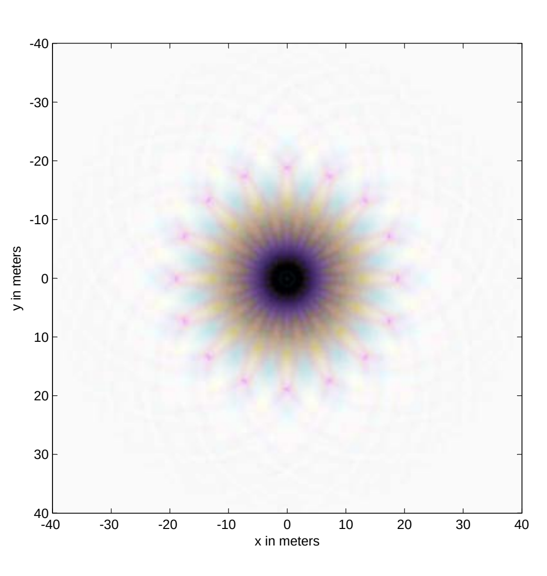


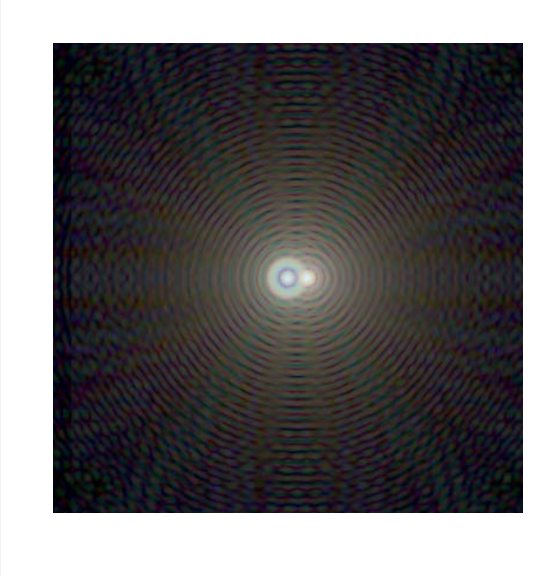
Simulated star/planet image

Shaped Occulter









Summary/Conclusion

- In the last 15 years, we have found hundreds of planets around other stars.
- These planets are mostly Jupiter-sized and close to their star—not *habitable*.
- It may be possible, in the near future, to take pictures of Earth-like planets around Sun-like stars.
- But, it will be hard.
- Such pictures would be the first step in answering the question: “Are we alone?”