## Chapter 12

# THREE BEWITCHING PARADOXES

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## ABSTRACT

For hundreds of years paradoxes dealing with questions of probability have ranked among some of the most perplexing of all mathematical paradoxes. In this chapter, we will discuss in detail three probabilistic paradoxes, each of which involves a decision to "switch" or "not to switch".

## 1. THE "LET'S MAKE A DEAL" PARADOX

This paradox has been around for a long time, but recently it has generated a great deal of interest following its appearance in syndicated puzzler Marilyn vos Savant's weekly brain teaser article that appears in many newspapers throughout the United States (see Marilyn vos Savant [8]).

Here's how the paradox goes. In the game show *Let's Make a Deal*, the following scenario arises frequently. On the stage there are three large doors. The host, Monty Hall, tells the contestant that behind one of the doors is a nice prize, but the other two doors have nothing of value behind them. The contestant is offered the chance to select one of the doors. Let's call this door A. Before showing the contestant what is behind door A, Monty Hall, who knows which door actually conceals the nice prize, shows the contestant what is behind one of the two doors that the contestant didn't choose. Let's call this door B and the door that he doesn't show you we will call door C. Monty Hall always picks for door B a door that does not have the prize. At this point, Monty asks the contestant whether he or she would like to switch to door C. Should the contestant switch? In other words, what is the probability that the prize is behind door A given that there is nothing behind door B?

When first presented with this question, most students (and professors) of probability say that it does not matter; either way, the probability of success

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is 1/2. After all, there are two choices that remain and knowing that door B is empty doesn't say anything about doors A and C. Right?

Actually, it's wrong. But the conviction that 1/2 is the correct answer can be very strong. In fact, on a recent visit to an Ivy League school one of us got into a rather intense discussion about this problem with three distinguished mathematicians (one of whom is even a probabilist). All three of them were convinced that 1/2 was the success probability whether they stick with their original door or switch.

All sorts of reasoning was applied to convince them of their error, but it was to no avail. Experimental evidence was offered first. They were told that a computer program had been written that simulates this game. The program was written so that the contestant always switches and was set to simulate 10 million plays of the game. Of the 10 million simulated contestants, only 3,332,420 of them lost. This gives an empirical success rate of 0.6667580. It was suggested that this number looks suspiciously close to 2/3. Of course, this piece of evidence did not alter their convictions. They preferred to believe that the program was flawed rather than their logic.

After experimental evidence failed, raw logic was employed. It was argued that the probability of success for door A was 1/3 before door B was revealed and, since Monty Hall is always able to find some door to be door B, how can the opening of door B say anything about door A. Hence, its probability must still be 1/3. Since door C is the only remaining door, its success probability must then be 2/3. Amazingly, this still did not convince our three distinguished friends.

Finally, they were offered the best explanation. They were shown an actual definition of a probability space that models this game and were then shown the computations that lead to 1/3 and 2/3. Here is the model. First we need a random variable $P$ that describes which door the prize is behind (i.e., $P = 1, 2$, or 3, each with probability 1/3). Without loss of generality, we may assume that the contestant always picks door 1 (so that door 1 is door A). Now, if the prize is behind door 3, Monty Hall will open door 2 (so that door 2 becomes door B and door 3 becomes door C). Similarly, if the prize is behind door 2, Monty Hall will open door 3. For these two cases Monty Hall had no choice.

However, what if the prize is actually behind door 1. Now what is Monty going to do? Maybe he always shows door 2 in this case. Or maybe he always shows door 3. Or maybe he uses some complicated secret algorithm for deciding which door to open. In any case the contestant has no knowledge as to how Monty will behave in this situation and regards Monty's two possibilities as equally likely. Hence, we may as well assume that Monty tosses a fair coin: if it comes up heads he opens door 2 while if it comes up tails he opens door 3. This random coin toss is independent of $P$. Therefore, our sample space consists of six points as shown in the table below. If the contestant switches, then success will correspond to the four sample points (H,2), (H,3), (T,2) and (T,3). Hence the probability of success is $4/6 = 2/3$. On the other hand, if the

| | 1 | 2 | 3 |
|---|---|---|---|
| H | 2 | 3 | 2 |
| T | 3 | 3 | 2 |

The columns represent the values of $P$. The two rows represent the coin that Monty flips to decide what to say in the case that $P = 1$. Each of the six sample points has probability 1/6. The numbers given in the table indicate which door Monty will show.

contestant holds onto door A, then success will correspond to sample points (H,1) and (T,1) and the success probability will be only 1/3.

Though they were unable to find any flaw in this line of argument, they were left quite puzzled since their intuition had failed them so miserably.

## 2. THE "OTHER PERSON'S ENVELOPE IS GREENER" PARADOX

Here is another paradox having to do with switching from one choice to another.

Two envelopes each contain an IOU for a specified amount of gold. One envelope is given to Ali and the other to Baba and they are told that the IOU in one envelope is worth twice as much as the other. However, neither knows who has the larger prize. Before anyone has opened their envelope, Ali is asked if she would like to trade her envelope with Baba. She reasons as follows. With 50 percent probability Baba's envelope contains half as much as mine and with 50 percent probability it contains twice as much. Hence, its expected value is

$$1/2(1/2) + 1/2(2) = 1.25,$$

which is 25 percent greater than what I already have and so yes, it would be good to switch. Of course, Baba is presented with the same opportunity and reasons in the same way to conclude that he too would like to switch. So they switch and each thinks that his/her net worth just went up by 25 percent. Of course, since neither has yet opened any envelope, this process can be repeated and so again they switch. Now they are back with their original envelopes and yet they think that their fortune has increased 25 percent twice. They could continue this process ad infinitum and watch their expected worth zoom off to infinity.

Clearly, something is wrong with the above reasoning, but where is the mistake? This paradox is quite puzzling until one carefully writes down a probabilistic model that describes the situation. Here is one possible model. Let $X_0$ denote the smaller amount of money between the two envelopes. This is a random variable taking values in the positive reals, but we (and more importantly, Ali and Baba) know nothing about its distribution. Let $X_1$ denote the larger amount of money so that $X_1 = 2X_0$. To select one of the two envelopes at random and give it to Ali means that we toss a fair coin and deliver to Ali either the envelope containing $X_0$ or the one containing $X_1$ depending on whether heads or tails appears. Mathematically, this means that we have another random variable $N$ independent of $X_0$ (and hence of $X_1$) and taking values 0 and 1, each with probability 1/2. The envelope that Ali receives contains $Y = X_N$ and the envelope that Baba receives contains $Z = X_{1-N}$.

Ali's reasoning about Baba's envelope starts by conditioning on the two possible values of $N$ and bearing in mind that on the event $\{N = 0\}$, $Z = 2Y$ and on the event $\{N = 1\}$, $Z = Y/2$. Hence,

$$E[Z] = \frac{1}{2}E[2Y|N = 0] + \frac{1}{2}E[\frac{1}{2}Y|N = 1]. \tag{1}$$

At this point she mistakenly assumes that $Y$ and $N$ are independent and continues her argument as follows:

$$E[Z] = \frac{1}{2}E[2Y] + \frac{1}{2}E[\frac{1}{2}Y] = \frac{5}{4}E[Y].$$

Of course, the correct way to complete the analysis is to first note that

$$E[2Y|N = 0] = E[2X_0|N = 0] = 2E[X_0]$$

and

$$E[\frac{1}{2}Y|N = 1] = E[\frac{1}{2}X_1|N = 1] = \frac{1}{2}E[X_1] = E[X_0].$$

Then, substituting these into (1) we see that

$$E[Z] = \frac{1}{2}2E[X_0] + \frac{1}{2}E[X_0] = \frac{3}{2}E[X_0] = E[Y].$$

## 3. "CHOOSING THE BIGGER NUMBER" PARADOX

Here is another paradox closely related to the previous one. Ali and Baba are again given two envelopes with an IOU for a specified amount of gold in each envelope. This time they know nothing about the amounts other than that they are non-negative numbers. After opening her envelope, Ali is offered the chance to switch her envelope with that of Baba. Can Ali find a strategy for deciding whether to switch which will make her chance of getting the envelope with the larger of the two numbers greater than one half? At first blush, this would appear to be impossible.

But consider the following strategy: Ali does an auxiliary experiment of choosing a number $U$ by some chance device that makes all non-negative numbers possible. For example, choose $U$ according to a exponential distribution with mean 10. If the IOU given Ali is greater than $U$ she keeps this envelope, if it is less than $U$ she switches to the other envelope.

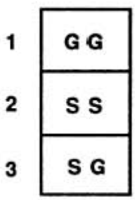
Let's see why this auxiliary experiment helps. As before, let $X_0$ be the smaller and $X_1$ the larger of the two numbers in the envelopes. Assume first that $U$ is less than both numbers, that is $U < X_0 < X_1$. Then Ali will not switch and, since she chose an envelope at random, she has a fifty percent chance of getting the larger number $X_1$. Assume next that her auxiliary number is between the two numbers in the envelopes, that is, $X_0 < U < X_1$. In this case, she is certain to get the larger number, since if her envelope has $X_0$ she will switch, and if it has $X_1$ she will keep it giving her in both cases the bigger number. Thus, in either case she ends up with the larger number. Finally, consider the case $X_0 < X_1 < U$. Then she will switch envelopes and, since her original choice of envelopes was random she again has a fifty percent chance of having the bigger number. Thus, in two of the cases Ali has a fifty percent chance and in the third case, which can happen, she is certain to get the largest number. Thus, her overall probability is greater than 1/2. Note that, to have this probability well defined, we would have to assume that there is some probability distribution that describes the probability that any two particular numbers are put in the envelopes.

This problem arose in work of David Blackwell on estimating translation parameters. His example 1 on page 397 of [4] was the following: $W$ is an unknown integer and $X$ is a random variable with values 1 or $-1$ with probability 1/2 each. You observe $Y = W + X$ and want to estimate $W$. Using method we just described, Blackwell showed that you can guess the value of $W$ with a probability greater than 1/2 of being correct.

## 4. COMMENTS ON THE MONTY HALL PROBLEM

The Monty Hall problem is a conditional probability problem that is very similar to other such problems that have puzzled students of probability throughout its history. Before commenting on the Monty Hall problem itself, we consider some other variants of this conditional probability problem. A more complete discussion of these many variants can be found in Barbeau [2] and Bar-Hillel and Falk [1].

One of the first conditional probability paradoxes is the *Box Paradox* formulated by Bertrand [3].



> A cabinet has three drawers. In the first drawer there are two gold balls, in the second drawer there are two silver balls, and in the third drawer one silver and one gold ball. A drawer is picked at random and a ball chosen at random from the two balls in the drawer. Given that a gold ball was drawn, what is the probability that the drawer with the two gold balls was chosen?

The intuitive answer might be $1/2$ but, of course, that is wrong. The possible outcomes for this experiment are displayed in the tree diagram of Figure 1. We assign the appropriate branch probabilities and path probabilities as products of these branch probabilities along the path.

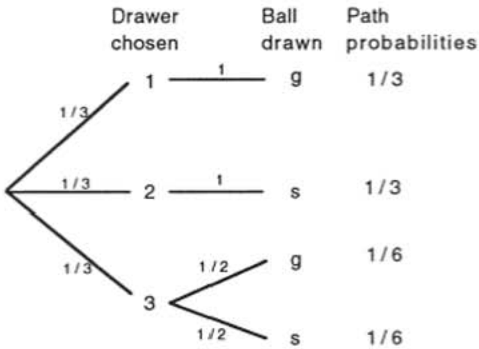



**FIGURE 1.** Unconditional probabilities for the box paradox.

This tree diagram provides the probabilities for the unconditional problem. To find the conditional probabilities, given that a gold ball was drawn, we need only delete the paths that do not result in a gold ball being drawn and renormalize the probabilities for the remaining paths of the tree to add to one. We can then compute the desired conditional probability by adding the conditional probabilities for the paths that give the desired outcome. We do this in Figure 2 and we see that the conditional probability that the drawer with two gold balls was drawn is $2/3$ and not $1/2$.

This is a particularly simple version of the conditional probability paradox because there is not a lot of argument about how to set up the model. It is pretty clear what "picking a box at random" and then "picking a ball at random" means. The next version of the problem called *the sibling problem* gets more complicated. In its simplest form this problem may be stated:

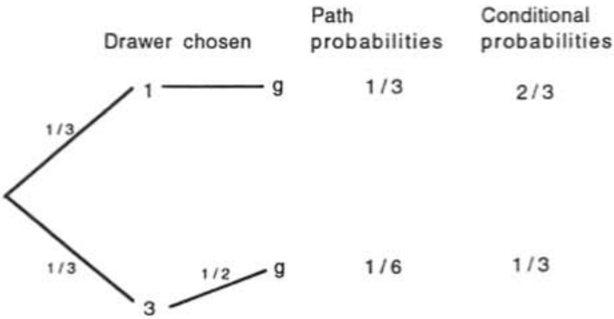



**FIGURE 2.** Conditional probabilities for the box paradox.

> Consider a family of two children. Given that there is a boy in the family what is the probability that there are two boys in the family?

Again we are supposed to say 1/2 and find that we are wrong. The "text book" solution would be to draw the tree diagram and then form the conditional tree by deleting paths to leave only those paths that are consistent with the given information. The result is shown in Figure 3. We see that the probability of two boys given a boy in the family is not 1/2 but rather 1/3.

One often says that the more intuitive answer 1/2 is the correct answer if the given information is that the youngest child is a boy.

This problem and others like it are discussed in [1]. These authors stress that the answer to conditional probabilities of this kind can change depending upon how the information given was actually obtained. For example, they show that 1/2 is the correct answer for the following scenario presented in [5].

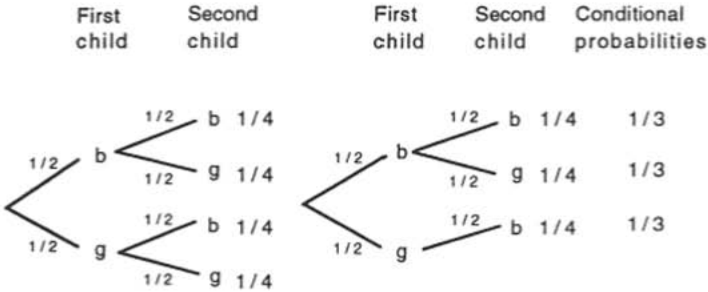



**FIGURE 3.** A family with two children has a boy. What is the probability that the family has two boys?

> Mr. Smith is the father of two. We meet him walking along the street with a young boy whom he proudly introduces as his son. What is the probability that Mr. Smith's other child is also a boy?

As usual we have to make some additional assumptions. For example, we will assume that, if Mr. Smith has a boy and a girl, he is equally likely to choose either one to accompany him on his walk. In Figure 4 we show the tree analysis of this problem and we see that 1/2 is, indeed, the correct answer.

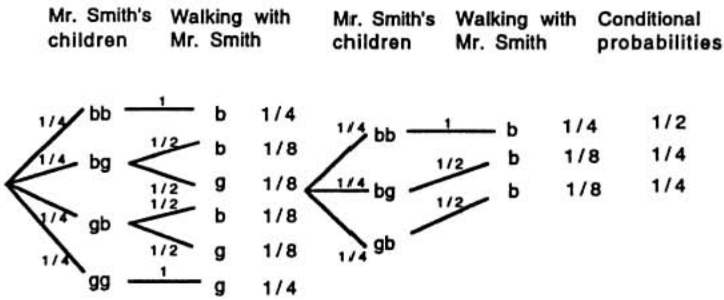



**FIGURE 4.** You meet Mr. Smith out walking with a son, one of his two children. What is the probability that his other child is a boy?

In his popular book *Innumeracy* John (see John Paulos [10]) decided to jazz the problem up a bit by asking:

> Consider some randomly selected family of four (i.e., two children) which is known to have at least one daughter. Say Myrtle is her name. Given this, what is the conditional probability that Myrtle's sibling is a brother?

He gave the answer 2/3 but, as pointed out to us by Bill Vinton and George Wolford, this is no longer a reasonable answer. There is a new consideration in the process, namely, that the family has a girl named Myrtle. Assume that a family names a daughter Myrtle with probability $p$. Then a tree diagram for the unconditional problem is shown in Figure 5 with resulting conditional probability tree in Figure 6.

Adding the two conditional probabilities for the sequences that result in Myrtle having a brother, we see that the conditional probability that she has a sister is $\frac{(2-p)}{(4-p)}$. Thus this probability depends upon the probability $p$ that a family will name a girl Myrtle. If $p$ is 1 we get a probability 1/3 as in the standard version. We have obtained no new information in this case. As $p$ decreases to 0 this conditional probability increases to 1/2.

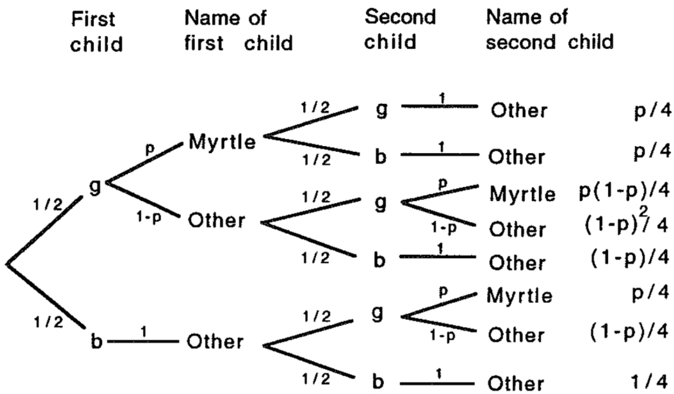



**FIGURE 5.** A family with two children has a daughter named Myrtle. What is the probability that Myrtle has a brother?

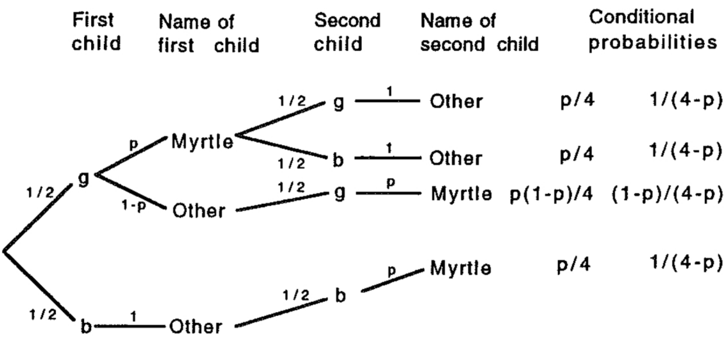



**FIGURE 6.** Conditional probabilities given a child named Myrtle.

It is not so easy to think of reasonable scenarios that would lead to the classical 1/3 answer. An attempt was made by Stephen Geller in proposing this problem to Marilyn vos Savant [9]. He writes:

> A shopkeeper says she has two new baby beagles to show you, but she doesn't know whether they're male, female or a pair. You tell her that you want only a male, and she telephones the fellow who's giving them a bath. "Is at least one a male?" she asks. "Yes," she

> informs you with a smile. What is the probability that the *other* one is male?

The next version historically is the *two aces problem*. This problem, dating back to 1936, has been attributed to the English mathematician J. H. C. Whitehead. (see Gridgeman [7]). This problem was also submitted to Marilyn vos Savant by the master of mathematical puzzles Martin Gardner, who remarks that it is one of his favorites.

> A bridge hand has been dealt. Are the following two conditional probabilities equal? Given that the hand has an ace what is the probability that it has two aces? Given that the hand has the ace of hearts, what is the probability that the hand has two aces?

It is customary to choose two cards from a smaller four card deck that contains say: the ace of hearts, the ace of spades, the king of hearts and the king of spades. The textbook solution to the problem is shown in Figure 7.

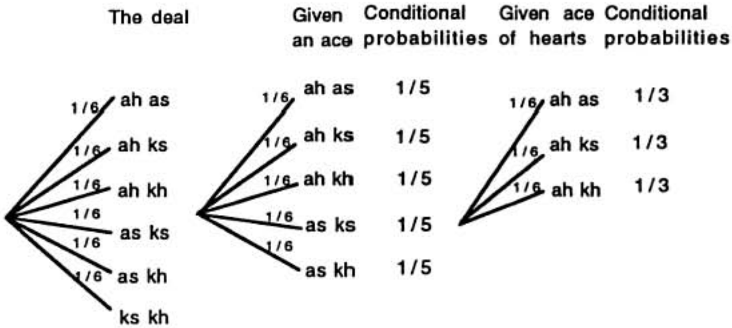



**FIGURE 7.** Probability of two aces given an ace, as compared to given the ace of hearts.

We see the somewhat surprising result that the conditional probability, given an ace, is 1/5 and, given the ace of hearts, is 1/3. It is natural to ask "how do we get the information that you have an ace?" Gridgeman [7] considers several different ways that we might get this information. For example, assume that the person holding the hand is asked to "name an ace in your hand" and answers "the ace of hearts." Then what is the probability that he has two aces? The tree analysis is shown in Figure 8.

We see that the answer is 1/5 which agrees with our solution to the probability of two aces given an ace. Now suppose you ask the more direct question "do you have the ace of hearts" and the answer is "yes". Then we have the tree analysis in Figure 9. We see that in this case the answer is 1/3 in agreement with the probability of two aces given the ace of hearts.

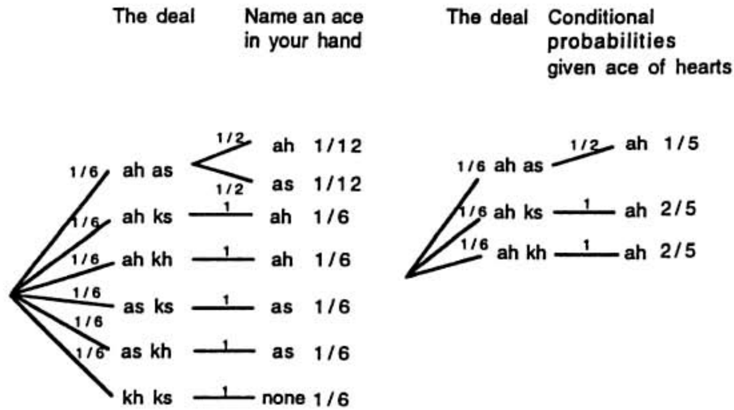



**FIGURE 8.** Probability of two aces given you answer "ace of hearts" to the request "name an ace in your hand".

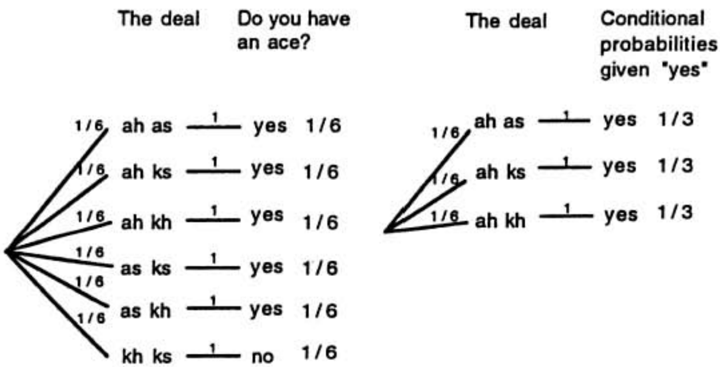



**FIGURE 9.** Probability of two aces given that you answer "yes" to the question "do you have the ace of hearts?".

Thus we can get either of the standard solutions by asking the appropriate question. Just how to figure out the correct question seems to be a bit of an art. Also, we have certainly not explained the paradoxical fact that very slight changes in the way you ask the question can give a completely different answer.

We consider next a problem that is often called the *prisoner's dilemma.* (Not to be confused with the more famous *prisoner's dilemma of game theory.*) It seems to have first appeared in Martin Gardner's book (see Gardner [6]). He writes:

> A wonderful confusing little problem involving three prisoners and a warden, even more difficult to state unambiguously, is now making the rounds. Three men — A, B, and C — were in separate cells under sentence of death when the governor decided to pardon one of them. He wrote their names on three slips of paper, shook the slips in a hat, drew out one of them and telephoned the warden, requesting that the name of the lucky man be kept secret for several days. Rumor of this reached prisoner A. When the warden made his morning rounds, A tried to persuade the warden to tell him who had been pardoned. The warden refused.
>
> "Then tell me," said A, "the name of one of the others who will be executed. If B is to be pardoned, give me C's name. If C is to be pardoned, give me B's name. And if I'm to be pardoned, flip a coin to decide whether to name B or C."

The warden tells A that B is to be executed and A assumes now that his probability of being executed has decreased from 2/3 to 1/2 by virtue of this information. Is he correct? The solution is given in Figure 10.

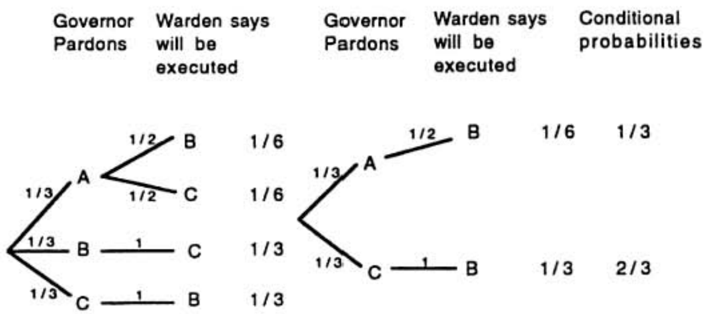



**FIGURE 10.** Solution to the prisoner's dilemma.

We see that the probability that A was pardoned is still 1/3 and so he has not improved his chances of being executed by learning that it is between him and B. Had we not assumed that the warden tosses a coin when he has a choice, the probability would have changed and, as usual, would depend upon the probability that the guard chooses B when he has a choice between B and C.

We return now to the *Monty Hall* problem. That we have already solved. We want to now solve it by our standard tree diagram method. This problem has also been discussed at length in letters in *The American Statistician* (see Selvin [11]). The problem was revived by a letter from Craig Whitaker to Marilyn vos Savant for consideration in her column in *Parade Magazine* (see Marilyn vos Savant [8]). Craig wrote:

> Suppose you're on Monty Hall's *Let's Make a Deal !* You are given the choice of three doors, behind one door is a car, the others, goats.
>
> You pick a door, say 1, Monty opens another door, say 3 which has a goat. Monty says to you "Do you want to pick door 2?" Is it to your advantage to switch your choice of doors?

After posing the puzzle Craig goes on to say:

> I've worked out two different situations (based upon Monty's prior behavior, i.e., whether or not he knows what's behind the doors). In one situation it is to your advantage to switch, in the other there is no advantage to switch. What do you think?

Here we have a problem that is purported to be a real life problem and so we have to decide on the appropriate scenario. Craig in his letter already suggests that a basic question is whether Monty knows where the car is.

In her discussion of the problem, Marilyn vos Savant assumed that Monty did know where the car was and that he would open a door that did not have the car, but not the one tentatively chosen by the contestant. Thus, if the car is behind door 2 he must open door 3 and if it is behind door 3 he must open door 2. If the car is behind door 1, Monty has a choice and, as usual, we can either assume that he tosses a coin (as Marilyn vos Savant did) or more generally that he chooses door 3 with probability $p$ and 2 with probability $1 - p$. Let's make the more general assumption. Now our unconditional tree is a bit larger than usual. The first step would show where the car is put. (We assume the choice is random.) The second step would show which door the contestant tentatively chooses. (Again we assume a random choice.) The third step would show Monty's choice. Rather than draw the somewhat large tree, we realize by now that to solve the conditional probability questions we need only draw the branches that are possible under the information given. Thus the answer is provided by the tree diagram in Figure 11.

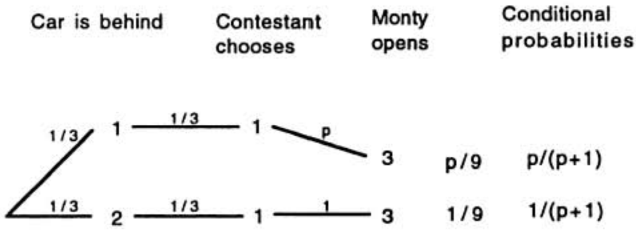



**FIGURE 11.** Solution to the Monty Hall problem.

Under our assumptions, given that the contestant chose door 1 and Monty chose door 3, the probability that the car is behind door 1 is $\frac{p}{1+p}$. From this we see that if $p = 1/2$ the probability the car is behind door 1 is 1/3 and that it is behind door 2 is 2/3, so the contestant should certainly switch. This was the solution of Marylin vos Savant, and was perfectly correct for her assumptions, but many readers found this solution difficult to believe. Since $\frac{p}{1+p} > 1/2$ for all $p$ except $p = 1$, in all but this extreme case it is to your advantage to switch and even in this case you do not lose by switching. Thus you might as well switch.

If you assume that Monty does not know where the car is and just opens a door at random then, as Craig Whitaker remarked, there is no advantage to switching. Of course, this being a real life situation you can make lots of other assumptions. For example, it may well be that the host is out to trick you and sometimes offers you the choice (for example, if he sees that you have chosen the door with the car) and other times doesn't. When Monty Hall was interviewed by the *New York Times* (see Tierney [12]) he stated that

> "If the host is required to open a door all the time and offer you a switch, then you should take the switch. But if he has the choice whether to allow a switch or not, beware. *Caveat emptor*. It all depends on his mood."

## 5. COMMENTS ON THE TWO ENVELOPE PROBLEM

One of the tricks of making paradoxes is to make them slightly more difficult than is necessary to further befuddle us. As John Finn suggested to us, in this paradox we could just have well started with a simpler problem. Suppose Ali and Baba know that I am going to give then either an envelope with \$5 or one with \$10 and I am going to toss a coin to decide which to give to Ali, and then give the other to Baba. Then Ali can argue that Baba has $2x$ with probability 1/2 and $x/2$ with probability 1/2 and then the expected value would be $1.25x$. But now it is clear that this is nonsense, since, if Ali has the \$5, Bala cannot possibly have 1/2 of this, namely, \$2.50, since that was not even one of the choices. Similarly, if Ali has \$10, Baba cannot have twice as much, \$20. In fact, in this simpler problem the possibly outcomes are given by the tree diagram

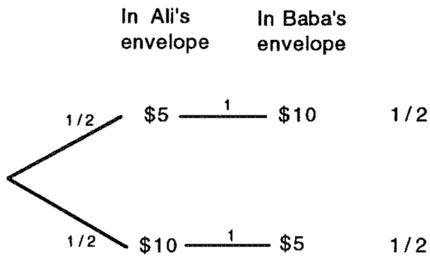



**FIGURE 12.** Tree diagram to compute Ali's expected winning.

From this we see that the expected amount received by either Ali or Baba is $1/2 \cdot 5 + 1/2 \cdot 10 = 7.5$.

## 6. COMMENTS ON THE CHOOSING THE BIGGER NUMBER PARADOX

This paradox shows that, in the first switching problem, there is an advantage to being allowed to switch. If Ali is allowed to switch or keep her number after looking at her envelope, he can improve her chances. Of course, we would have to assume that Baba must switch if Abi wants to. Let's try to see where this advantage really comes from. Assume that Ali were to know the distribution by which the two numbers were chosen. She will then know the probability $f(x)$ that the minimum of the two numbers is $x$. Then Ali can calculate the probability that she has the minimum given that she has an I.O.U worth $a$. That is,

$$\begin{aligned} \mathrm{P}(a \text{ is the minimum} \mid \text{Ali has } a) &= \frac{\mathrm{P}(\text{Ali has } a \text{ and } a \text{ is the min})}{\mathrm{P}(\text{Ali has } a)} \\ &= \frac{1/2 f(a)}{1/2 f(a) + 1/2 f(a/2)} \\ &= \frac{f(a)}{f(a) + f(a/2)}. \end{aligned}$$

Thus the probability that Ali has the minimum of the two numbers is greater than one half only when $f(a/2) < f(a)$ and in this case, she should switch. Of course, Baba could reason in the same way and would not want to switch in precisely the cases where Ali would want to switch.

Now if we were picking the numbers and wanted to assure that Ali could not take advantage of the chance to switch, we would want to make the density $f(x)$ a constant so that she would never have an advantage. Unfortunately, there is no uniform density on the positive axis and so that is not possible. Thus one way to look at this paradox is to say that it comes about because we can't pick a real number with all possibilities equally likely.

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