

# On Graphical Representations of Voting Results

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## ABSTRACT

We describe and illustrate various ways to represent election results graphically. The advantages and disadvantages of the various methods are discussed. While there is no one perfect way to fairly represent the outcomes, it is easy to come up with methods that are superior to those used in recent elections.

## 1. Introduction

While most daily newspapers were published in black and white until just a few years ago, USA Today has always been published in full color. Furthermore, it uses color to its advantage by featuring many colorful pictures, charts, and diagrams. Hence, after an election, it is natural to look to USA Today for colorful graphical depictions of election results. Indeed, after the 2000 presidential election and, again, after the 2004 presidential election, the paper featured a large map of the US showing county-by-county how the election had gone. Counties in which the democratic candidate received the most votes were painted blue and counties in which the republican candidate received most votes were painted red. These two maps from USA Today are shown in Figure 1.

But, what is the value of such a map? As political entities, counties are irrelevant in national elections. In the electoral college, states give all of their electoral votes to the candidate that received the most votes in the state. Hence, for a red-blue, winner-take-all map to make any sense, it should be drawn state-by-state, not county-by-county. Of course, USA Today was trying to show results at a finer-grained level. Michigan usually goes to the democratic candidate, but that is because of Detroit. Most of the rest of the state leans toward the republican. In fact, this is generally true—the large metropolitan areas tend to vote democratic whereas the rural areas vote republican. Since most states have both metropolitan areas and rural areas, it seems interesting to depict the election at this finer grained level. But, a quick look at the maps in Figure 1 immediately shows the flaw of this approach—these two elections were very close, yet the maps give the impression of a stunning victory for the republicans.

This paper is about better ways to depict election results. First we will discuss the use of blended colors to represent better the subtleties of the voting patterns. Then we will discuss ways to address the population density bias.

## 2. Color Perception.

Light is a wave. As such, it has a wavelength. Our eyes are sensitive to light with wavelengths between about 350 nanometers and 650 nanometers. We perceive the short wavelengths as blue and the long wavelengths as red. At intermediate wavelengths we see all of the colors of the rainbow. Scientific instruments can identify with great precision the various wavelengths contained in some light source. If our eyes were capable of such distinctions the world would be a much more colorful place than what we see it as. Our eyes actually only perceive color in a rather rudimentary fashion. The color receptors in our eyes are called *cones*. There are three types of cones. Cones of the first type are sensitive to short wavelengths, the second to medium wavelengths, and the third to long wavelengths. Figure 2 shows the sensitivity of these cones to different wavelengths of light.

If one could make a light source having an intensity vs. wavelength profile as shown by the blue curve in Figure 2 (labeled “S” for short), that light source would look blue to us. Similarly, a light source with an intensity profile shown in red in the figure (and labeled “L” for long) would appear red and a light source corresponding to the green curve (labeled “M” for medium) would look green to us. Hence, the *primary colors* of an additive color scheme are red, green, and blue (RGB)—and this system is based on the physiology of the human eye.

Modern TVs and computer displays are based on this RGB system. Each pixel has three emitters—one for each primary color. All other colors correspond to some mixture of these three primary colors. Mix red and blue and you get magenta. Mix blue and green and you get cyan. Mix green and red and you get yellow. These pairwise mixtures, cyan, magenta, and yellow (CMY), are called the *secondary additive colors*. Modern color printers are based on laying down ink which darkens a white paper. Hence, they use primary subtractive colors, which are the same thing as the secondary additive colors, i.e., the CMY system.

One could employ blended colors to represent voting outcomes using either the primary additive colors RGB or the primary subtractive colors CMY. But, since red is already identified with republicans, and blue with the democrats, it seems natural to work with the primary additive colors R, G, and B. The right-hand plot in Figure 2 shows all blends of the three primary colors. We should point out that the center of the triangle is not white but

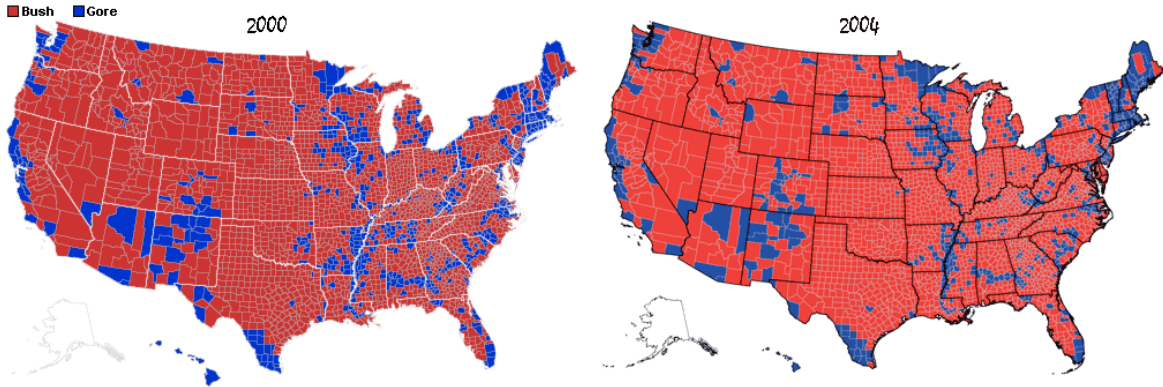


Fig. 1.— County-by-county election results as published by USA Today after the 2000 and 2004 elections. Each county is painted either pure red or blue—red if the republican candidate received more votes than the democratic candidate and painted blue otherwise.

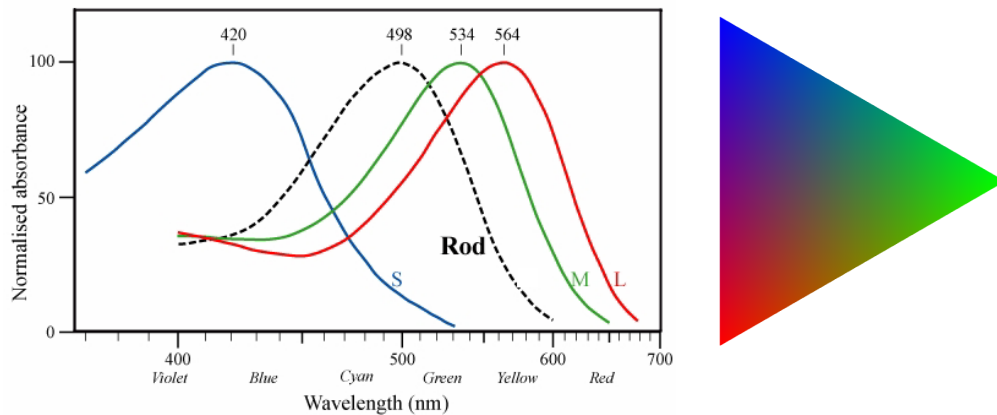


Fig. 2.— *Left.* Receptor response curves for the three types of cones and the gray-scale rods (source: J.K. Bowmaker and H.J.A. Dartnall, "Visual pigments of rods and cones in a human retina." *J. Physiol.* **298**: pp501-511 (1980)). *Right.* Colors achievable as mixtures of red, green, and blue.

gray. This is because we have given each color intensity  $1/3$ , not 1. For the same reason, yellow, which is along the edge halfway between red and green is a grayish yellow rather than a vibrant yellow. Similarly, we could have made magenta and cyan brighter too but such a brightened mixture would seem to unnaturally emphasize these colors.

### 3. Mapping Colors to Votes

Because our eyes have only three types of cones, we can only represent blends of three extremes. For example, we can choose red to represent republicans, blue to represent democrats, and green to represent all other minor parties. The color triangle on the right in Figure 2 shows all of the various mixtures of these three colors. A county that votes 100% republican appears red. A county that votes 50% republican and 50% democrat appears purple (magenta). A county that votes 33% republican, 33% democrat, and 33% other would appear as gray.

Figures 3 and 4 show all presidential elections from 1960 to 2004 using the color triangle shown in Figure 2. In most recent elections, third party candidates played such a minor role that there is no green to be seen. The exceptions are 1960, when Harry Byrd won a significant fraction of the votes in Alabama and Mississippi, 1968, when George Wallace carried much of the south winning a total of 13.5% of the votes nationwide, and 1992 when Ross Perot took 18.9% of votes nationwide preventing Bill Clinton from winning a majority.

### 4. Accounting for Population Density

Even coloring the maps with shades of purple, the vast sparsely populated expanse of the rocky mountain states gives the impression that the republicans won decisive victories in 2000 and 2004.

Perhaps the most logical solution to this problem would be to use brightness to represent population density. Clearly if each person who voted were to hold up a candle, colored red, green, or blue according to his or her vote, then viewed from space the total intensity of blue would be about the same as the total intensity of red and this would seem like a fair way to represent the data. Of course, most of the light would come from the densely populated areas. Since the population density in cities is thousands of times greater than it is in rural areas, we would have to make the bright areas really bright, shield-your-eyes bright, in order to see how both the cities and the rural areas voted all at once. Otherwise, the rural areas just look black. Figure 5 illustrates this problem. On the left, we show a picture

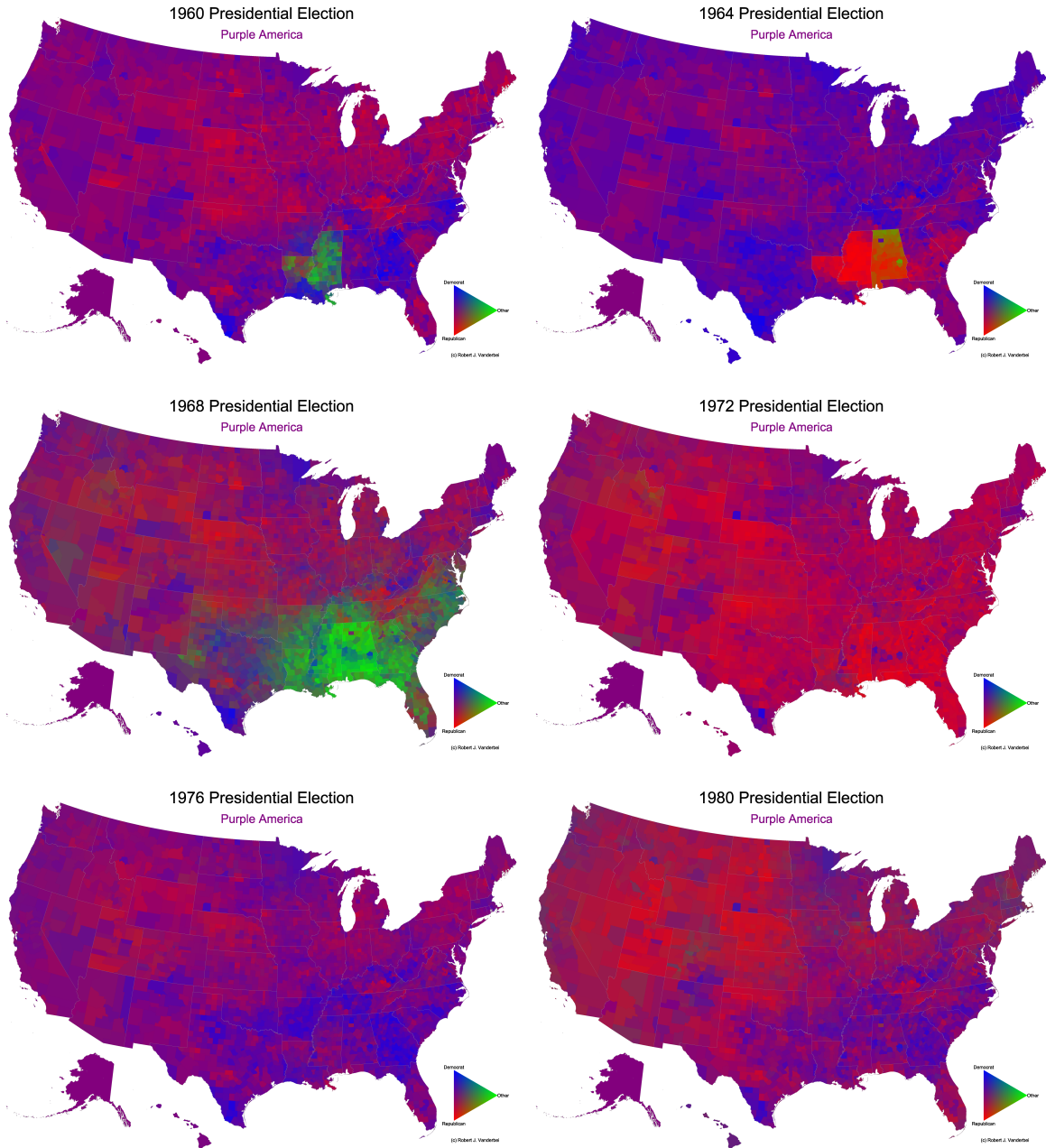


Fig. 3.— Elections 1960 to 1980. Red is republican, blue is democrat, and green is all others.

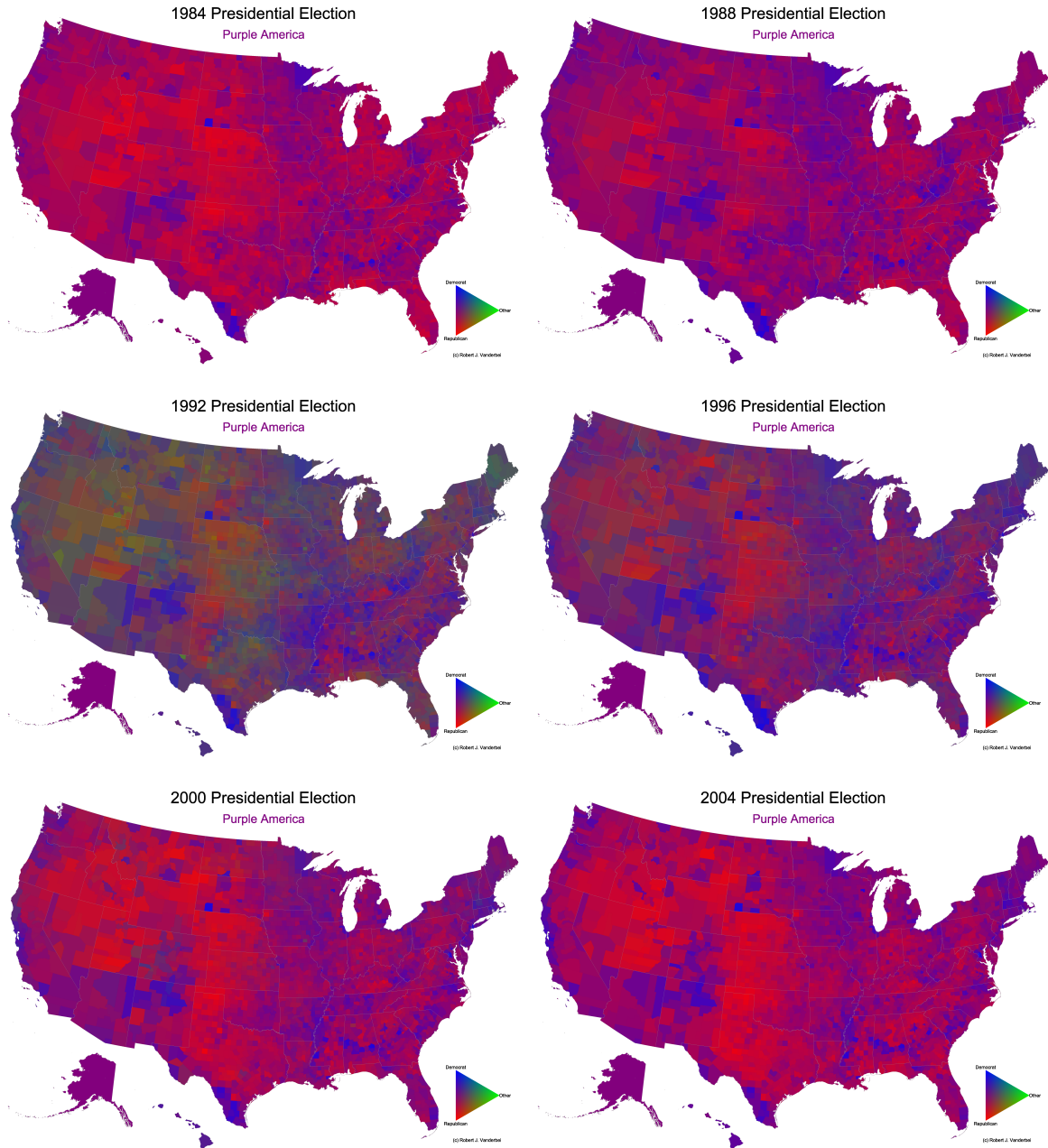


Fig. 4.— Elections 1984 to 2004. Red is republican, blue is democrat, and green is all others.

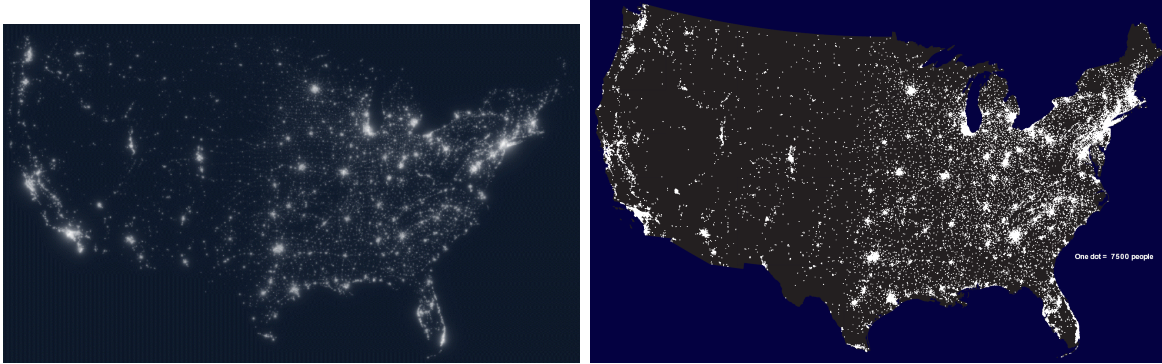


Fig. 5.— *Left.* Lights at night from the Dark Sky Association. *Right.* Population concentration according to the Census Bureau.

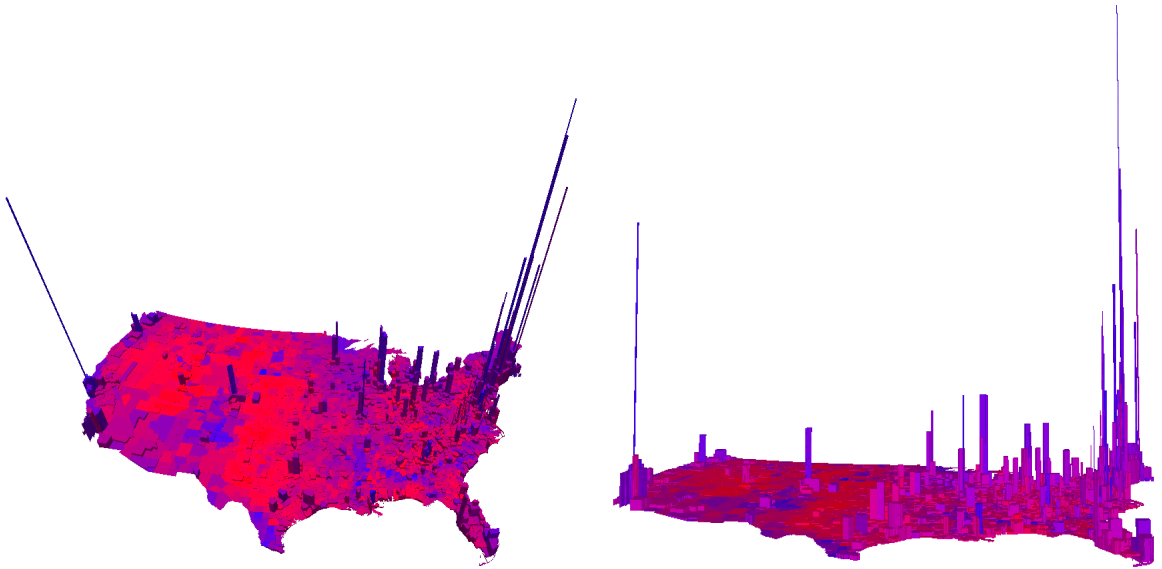


Fig. 6.— Three dimensional version. Here the height of the “tower” over each county is proportional to population density so that the volume of the solid is proportional to total population in the county.

of the United States at night (with clouds removed!). Densely populated areas show up as white. Sparsely populated areas are virtually black. But, this picture has been stretched nonlinearly. A purely linear stretch shows everything almost black except for New York City and a few other cities. The right-hand image in the figure is a diagram where a small speck of white is placed for every 7500 people according to the most recent Census statistics. It too has a problem in that overlapping dots are just white even when they should be ultra-white. Anyway, the conclusion is that an intensity-scaled representation is inadequate at conveying the differences in population density.

A second approach to solving the density problem is to place a “volume” of color over every county, making the volume proportional to total population in the county so that height is proportional to population per unit area, i.e., population density. Two views from such a representation are shown in Figure 6. While this representation is accurate, it still gives the false impression of a decisive republican victory. The reason is that we perceive size according to surface area, not according to volume. A cube that is 10 inches on a side looks 100 times bigger than a cube that is one inch on a side even though the volume is 1000 times larger.

The obvious solution, then, is to distort a map to make the area of each county be proportional to its population. Such a distorted map is called a *cartogram*. Cartograms are useful when the map distortions are not so great as to make the map completely unintelligible. Gastner et al. (2005) have made such maps for the 2004 election. With their permission, two of their maps are shown in Figure 7. Clearly, distorting a map based on population is a borderline case. The large isolated island positioned where one would expect to see New York, is actually Long Island. The population of Long Island is almost as large as the population of Michigan. The rocky mountain states have been squished to almost nothingness. In these maps, the visual impact is of an even split between red and blue. But, much of the interior of the country is distorted to the point that if you lived say in Iowa you would be hard pressed to find the part of the map that represents your home.

## 5. The 2006 Congressional Election.

We end this paper with a brief look at the 2006 Congressional election. Figure 8 shows three representations of the outcomes in this election. At the top, is the usual county-by-county “purple” depiction. The graphic on the left shows the breakdown by congressional district. Since these are the areas represented by the election winners, it make sense to draw the map this way. But, for those districts that appear purple in the map, it is hard to tell which side actually won and, in this case, there is a winner associated with each district.

Hence, the “margin of victory” plot shown on the right is more informative for this type of election.

## REFERENCES

M.T. Gastner, C.R. Shalizi, and M.E.J. Newman. Maps and cartograms of the 2004 us presidential election results. *Advances in Complex Systems*, 8:117–123, 2005.

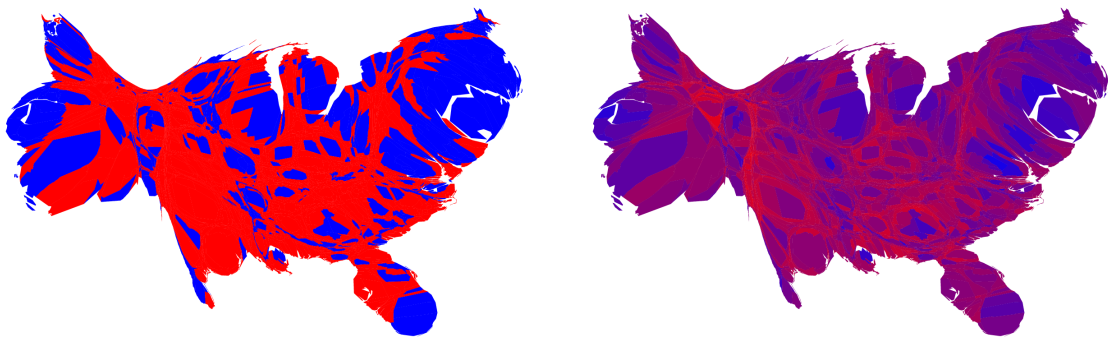


Fig. 7.— The cartograms of M. Gastner, C. Shalizi, and M. Newman.

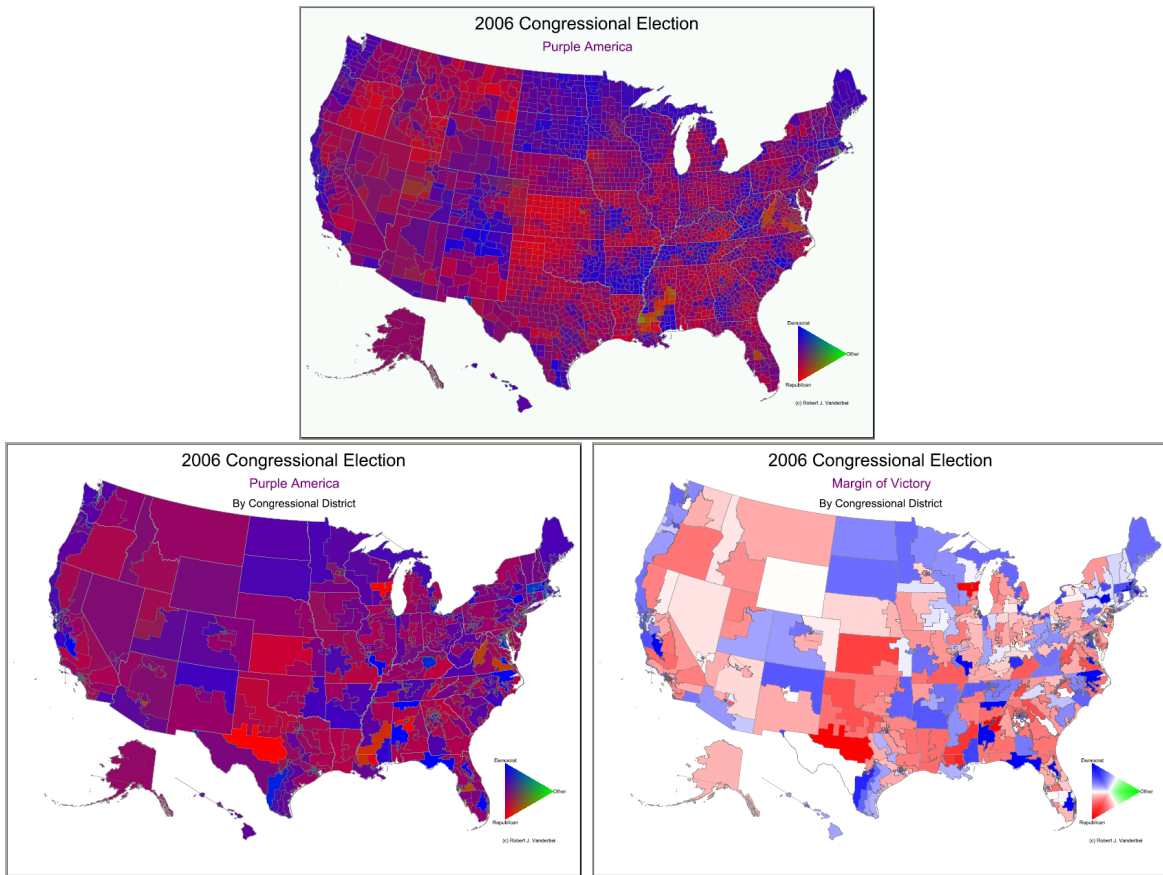


Fig. 8.— The 2006 Congressional election. *Top*. County-by-county results. *Left*. District-by-district results. *Right*. District-by-district using “margin-of-victory” color scheme.