

# Designing a Space Telescope to Image Earth-like Planets

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# The Big Question: Are We Alone?

- Are there Earth-like planets?
- Are they common?
- Is there life on some of them?



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# Exosolar Planets—Where We Are Now

There are more than 100 Exosolar planets known today.

Most of them have been discovered by detecting a sinusoidal doppler shift in the parent star's spectrum due to gravitationally induced **wobble**.



This method works best for large Jupiter-sized planets with close-in orbits.

One of these planets, HD209458b, also transits its parent star once every 3.52 days. These transits have been detected photometrically as the star's light flux decreases by about 1.5% during a transit.

Recent transit spectroscopy of HD209458b shows it is a gas giant and that its atmosphere contains sodium, as expected.



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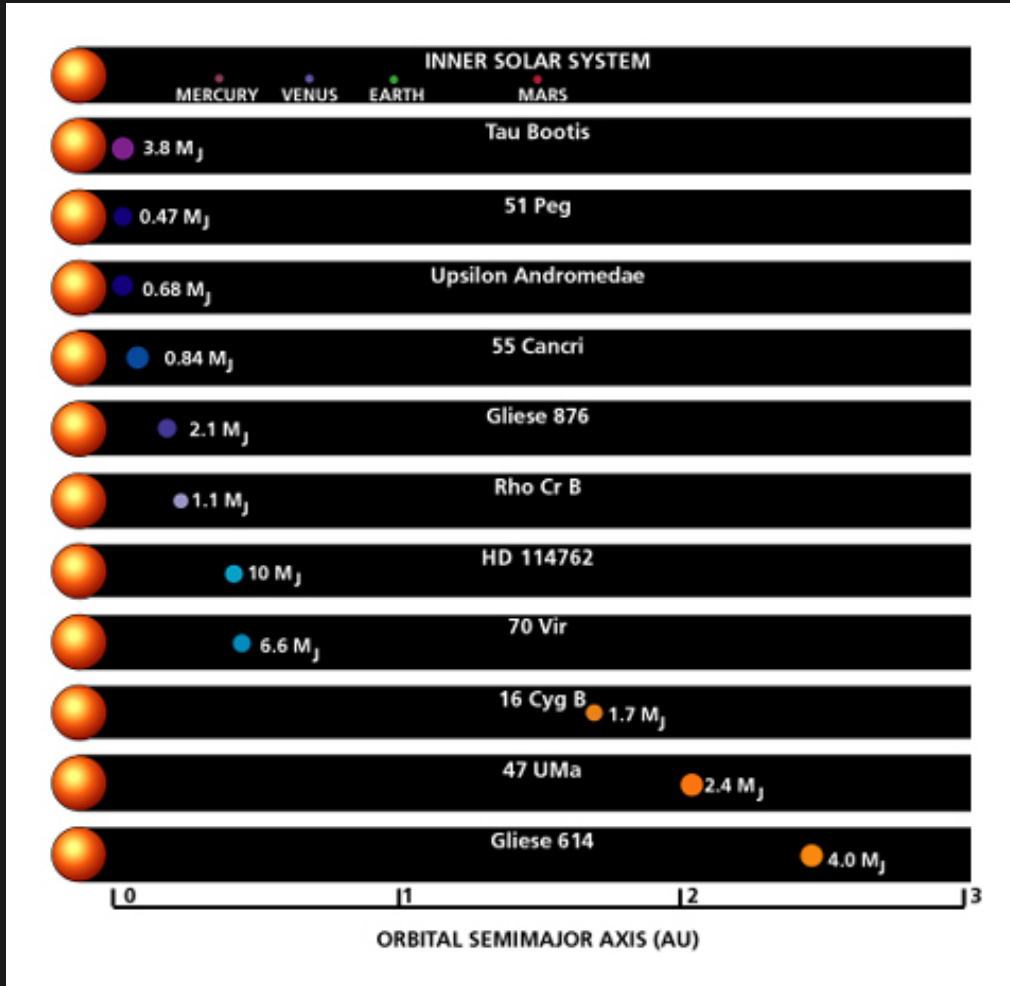
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# Some of the ExoPlanets



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# Terrestrial Planet Finder Telescope

- DETECT: Search 150-500 nearby (5-15 pc distant) Sun-like stars for Earth-like planets.
- CHARACTERIZE: Determine basic physical properties and measure “biomarkers”, indicators of life or conditions suitable to support it.



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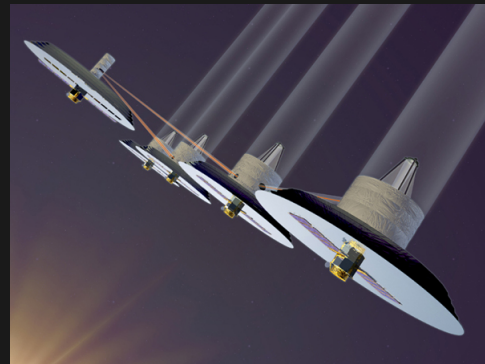
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# Why Is It Hard?

- If the star is Sun-like and the planet is Earth-like, then the reflected visible light from the planet is  $10^{-10}$  times as bright as the star. This is a difference of 25 magnitudes!
- If the star is 10 pc (33 ly) away and the planet is 1 AU from the star, the angular separation is 0.1 arcseconds!

Originally, it was thought that this would require a space-based multiple mirror nulling interferometer.

However, a more recent idea is to use a single large telescope with an elliptical mirror (4 m x 10 m) and a *shaped pupil* for diffraction control.



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HD209458 is the bright (mag. 7.6) star in the center of this image. The dimmest stars visible in this image are magnitude 16. An Earth-like planet 1 AU from HD209458 would be magnitude 33, and would be located 0.2 pixels from the center of HD209458.

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# The Shaped Pupil Concept

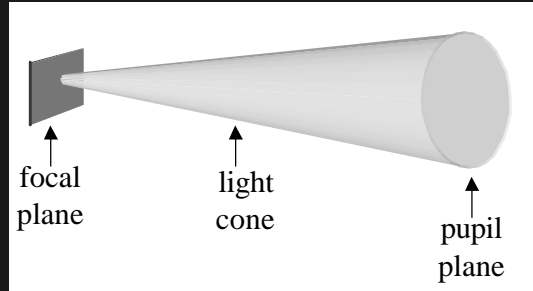
Consider a telescope. Light enters the front of the telescope—the *pupil plane*.

The telescope focuses the light passing through the pupil plane from a given direction at a certain point on the *focal plane*, say  $(0, 0)$ .

However, the wave nature of light makes it impossible to concentrate all of the light at a point. Instead, a small disk, called the *Airy disk*, with diffraction rings around it appears.

These diffraction rings are bright relative to any planet that might be orbiting a nearby star and so would completely hide the planet. The Sun, for example, would appear  $10^{10}$  times brighter than the Earth to a distant observer.

By placing a mask over the pupil, one can control the shape and strength of the diffraction rings. The problem is to find an optimal shape so as to put a very deep *null* very close to the Airy disk.



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# The Shaped Pupil Concept

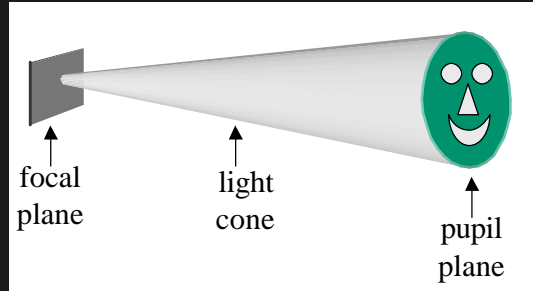
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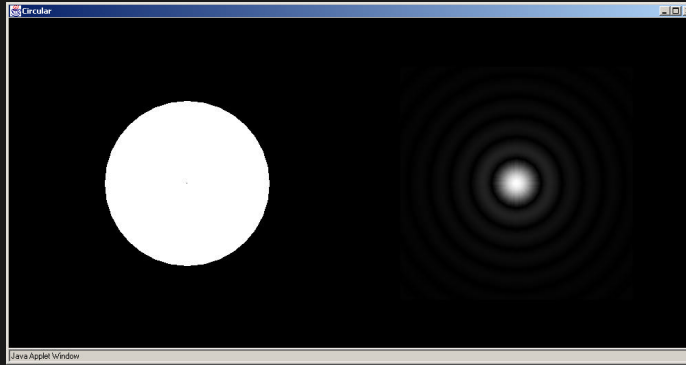
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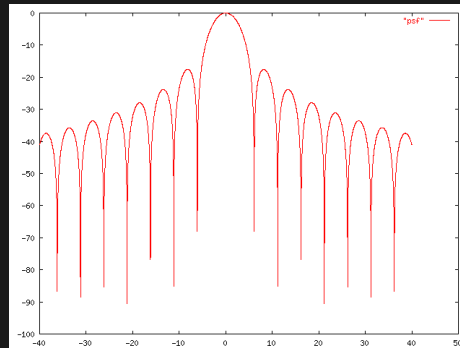
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# Airy Disk and Diffraction Rings

A conventional telescope has a circular opening as depicted by the left side of the figure. Visually, a star then looks like a small disk with rings around it, as depicted on the right.



The rings grow progressively dimmer as this log-plot shows:



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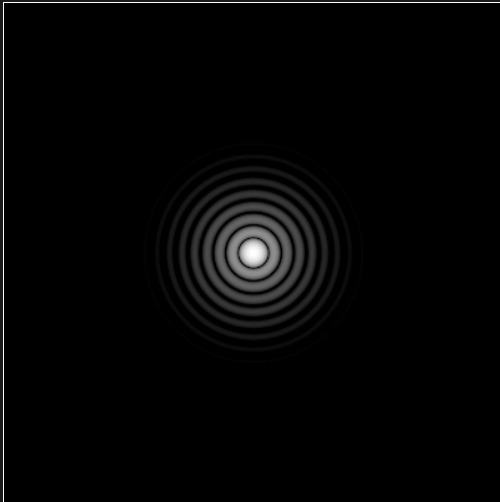
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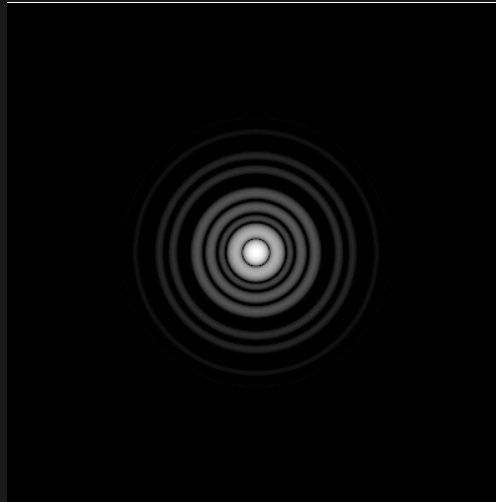
# Central Obstructions are an Example of a Shaped Pupil

Logarithmically scaled plots of 2-D point spread functions for apertures with and without a 30.3% central obstruction. White is 1 and black is  $10^{-4}$ .

Without (refractor):



With (Questar):



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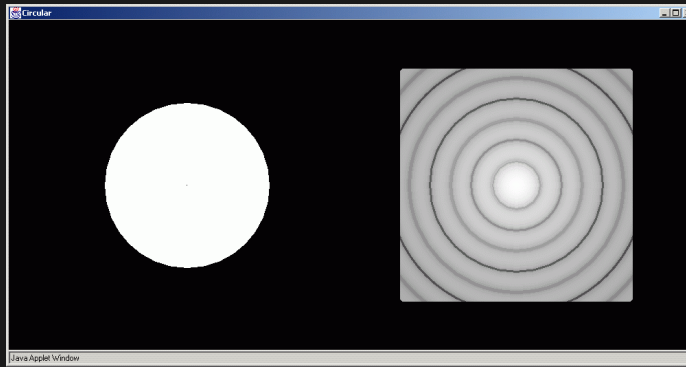
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# Airy Disk and Diffraction Rings—Log Scaling



Here's the unobstructed Airy disk from the previous slide plotted using a logarithmic brightness scale with  $10^{-11}$  set to black:



The problem is to find an aperture mask, i.e. a pupil plane mask, that yields a  $10^{-10}$  dark zone somewhere near the first diffraction ring. A *hard problem!* Such a dark zone would appear almost black in this log-scaled image.

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# Mathematical Details

The light from a distant point source, i.e. a star or a planet, is a coherent plane wave.

A coherent plane wave passing the pupil plane produces an interference pattern at the focal plane given by the 2-D Fourier transform of the indicator function of the mask:

$$E(\xi, \zeta) = \iint e^{-ik(x\xi+y\zeta)/f} 1_{\mathcal{M}}(x, y) dy dx.$$

Here,  $k$  is the wave number ( $2\pi/\lambda$ ),  $f$  is the focal length of the instrument, and  $\mathcal{M}$  denotes the mask set.

We assume that  $\mathcal{M}$  is symmetric wrt to the axes so that  $E$  is real.

What is measured at the image plan is the square of the magnitude of the electric field:

$$P(\xi, \zeta) = |E(\xi, \zeta)|^2$$

Note that  $E(0, 0)$  is the area of the mask.



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# The Optimization Problem

The problem is to maximize light throughput, i.e. the open area of the mask, subject to the constraint that the intensity of the light in a specified dark zone  $\mathcal{S}$  is at most  $10^{-10}$  as bright as at the center of the star's Airy disk:

$$\text{maximize: } E(0, 0)$$

$$\text{subject to: } -10^{-5}E(0, 0) \leq E(\xi, \zeta) \leq 10^{-5}E(0, 0), \quad x \in \mathcal{S}$$
$$\mathcal{M} \subset \mathcal{A}$$

Here,  $\mathcal{A}$  denotes the shape of the underlying mirror—typically, a circle or an ellipse.



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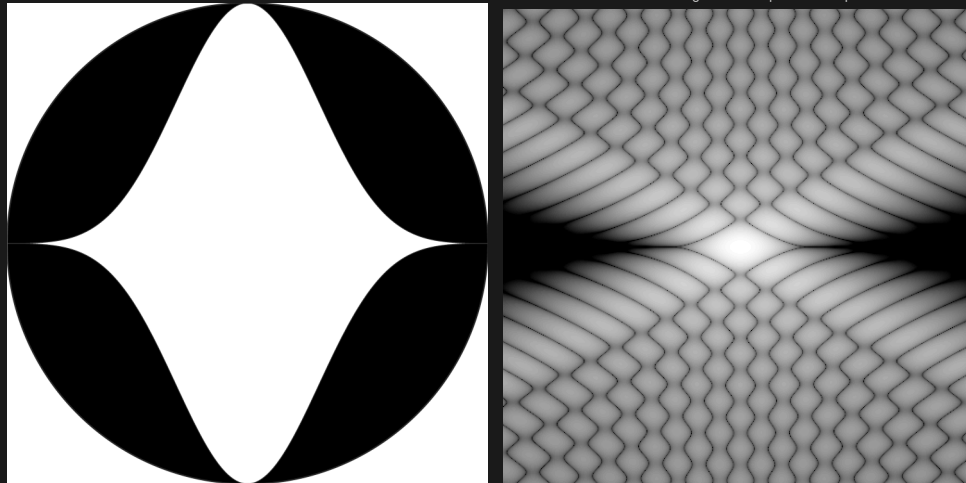
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# Kasdin-Spergel Prolate Spheroidal Mask

Early on, Kasdin/Spergel realized that Slepian's prolate spheroidal wave function solves a related problem and provides the desired contrast on most of the  $\xi$ -axis:  $\mathcal{S} = \{(\xi, 0) : |\xi| \geq 4\}$ .



Unfortunately, the discovery zone is too narrow close in to the Airy disk to be of use.



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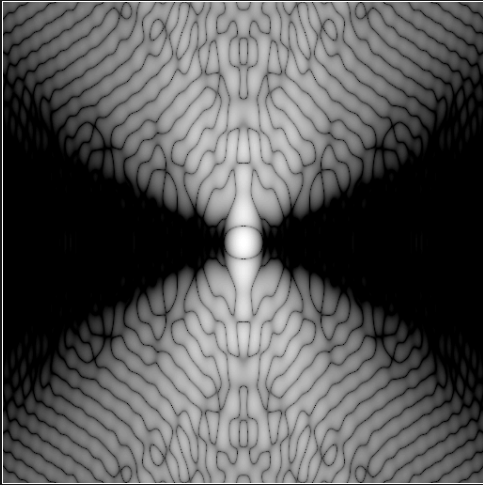
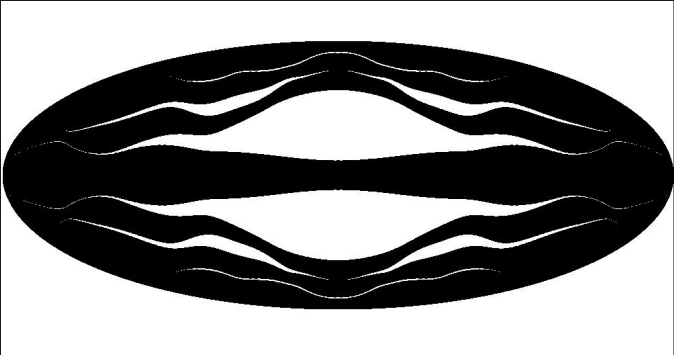
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# A Better Mask via Numerical Optimization



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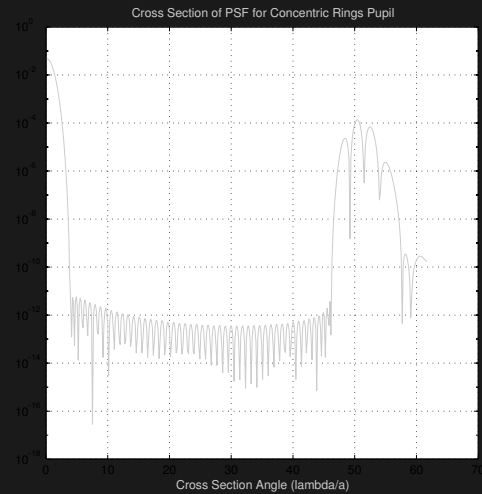
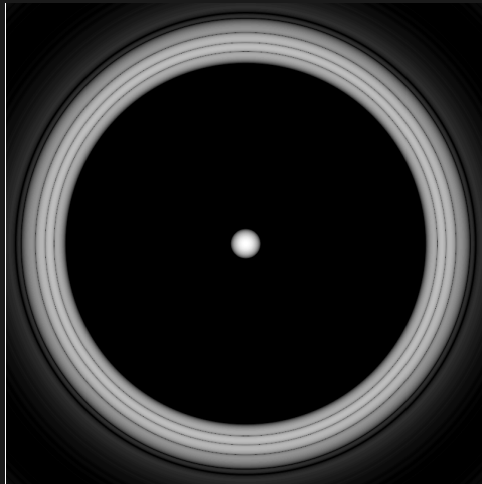
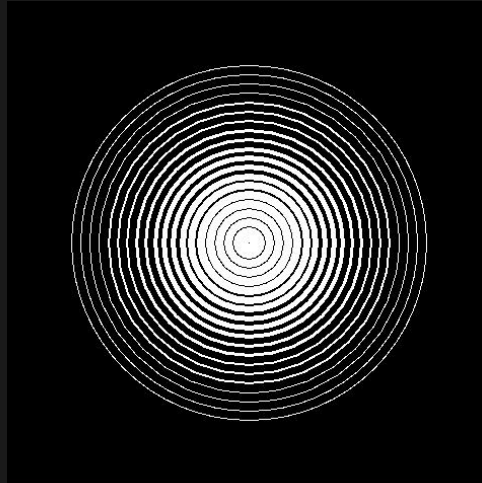
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# Azimuthally Symmetric Mask



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# Statistical Issues

So far we've only considered light as a wave. But, all detectors in fact detect, i.e. count, individual photons.

Let  $\eta = (\xi, \zeta)$  denote points in the image plane.

Partition the image plane into discrete array of *pixels*.

Identify pixels by their centers.

Let

$$\begin{aligned} X(\eta') &= \# \text{ of photons "intended" for pixel } \eta' \\ &\sim \text{Poisson}(\mu(\eta')) \\ Y(\eta) &= \# \text{ of photons actually arriving at pixel } \eta \\ &\sim \text{Poisson} \left( \sum_{\eta'} \mu(\eta') P(\eta - \eta') \right) \end{aligned}$$

The underlying parameters,  $\mu$  (indexed over the pixels), are unknown and are to be estimated.

Note that the *point-spread function* (psf)  $P$  is just a normalization of the square of the electric field discussed earlier.



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## Maximum Likelihood Estimator

The fundamental problem is to determine the deconvolved image  $\mu$ .

The most common technique is to look for the *maximum likelihood estimator* (MLE). That is, we seek  $\mu$  that maximizes the log-likelihood function:

$$\log \mathbb{P}_\mu(Y = y) = - \sum_{\eta'} \mu(\eta') + \sum_{\eta} y(\eta) \log \left( \sum_{\eta'} \mu(\eta') P(\eta - \eta') \right) + c$$

The maximum is achieved by  $\mu_{\text{MLE}}$  defined by the following nonlinear system of equations:

$$\frac{y}{\mu_{\text{MLE}} * P} * P = 1,$$

where  $*$  denotes the 2-D convolution:

$$f * g(\eta) = \sum_{\eta'} f(\eta') g(\eta - \eta').$$

Unfortunately, there is no closed form solution to this defining relation and so an algorithm is required...

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# The EM Algorithm

$$\begin{aligned}\mu^{(0)} &= y \\ \mu^{(k+1)} &= \mu^{(k)} \left( \frac{y}{\mu^{(k)} * P} * P \right).\end{aligned}$$

In the astronomy community, this algorithm is called the *Richardson–Lucy deconvolution* algorithm.

Shepp and Vardi were the first to apply the EM-algorithm to problems in medical imaging.

The Richardson–Lucy deconvolution algorithm is famous for its success in restoring the images taken with the Hubble Space Telescope's original flawed optics.



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## Integration Time

Fix attention on a certain pixel. Suppose there is a planet there and that its Poisson arrival rate is the same as the side-lobe arrival rate of the star. Call it  $\mu$ . It is proportional to the mask's open area,  $A$ , and integration (exposure) time  $t$ :  $\mu = ctA$ .

The *signal* is the expected number of photons:  $\mu$ .

The *noise* is the standard deviation of the planet plus the star:  $\sqrt{2\mu}$ .

The *signal-to-noise ratio* is:  $S/N = \sqrt{\mu/2} = \sqrt{cAt/2}$ .

Hence, integration time required to achieve a given  $S/N$  is:

$$t = \frac{2(S/N)^2}{cA}.$$

That is, doubling open area implies halving integration time.

## Hypothesis Tests

Null hypothesis: there is no planet in a certain subset of the dark zone.

What integration (exposure) time is needed to achieve a specified  $p$ -value of the test when a planet is present?

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# Field Test



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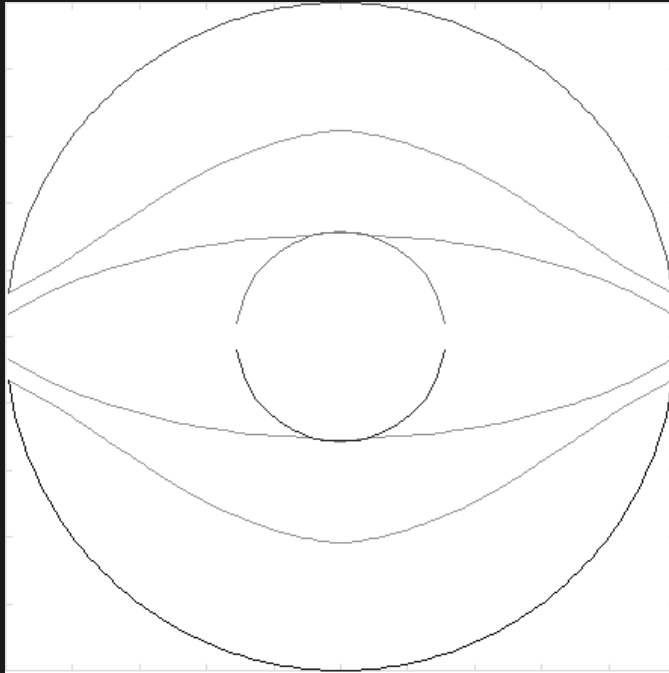
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# Dim Double Splitter Mask

A mask was made for a 3.5" Questar. The mask was cut from paper with scissors (a crude tool at best) according to the template shown, backed with cardboard, and framed with 4" PVC endcap.



The outer circle represents the full aperture, the inner circle the central obstruction, and the remaining arcs the mask opening.



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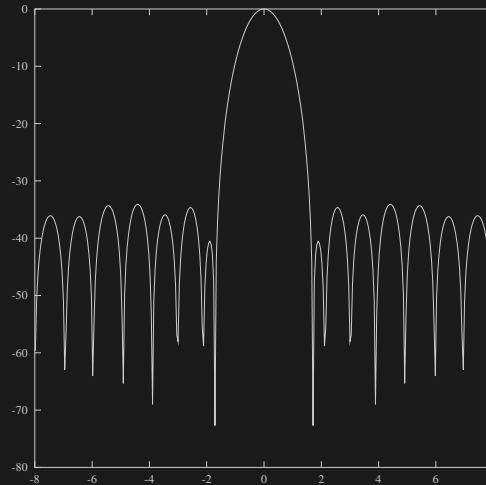
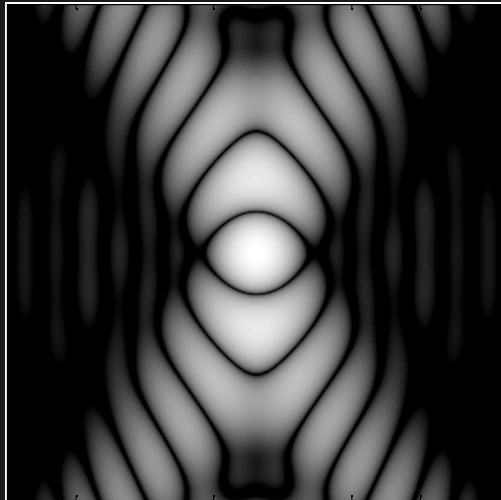
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# Computed PSF



Logarithmically scaled plot of the 2-D point spread function and a graph of its  $x$ -axis slice. White is 0 dB and black is  $-40$  dB. Throughput is 18.2%.



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# 31 Leonis

31 Leonis is a dim double.

Primary/secondary visual magnitude: 4.37/13.6

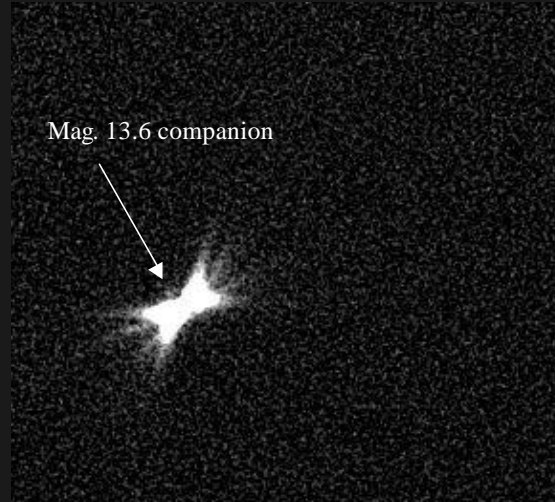
Luminance difference = 9.2 = -36.8 dB

Separation:  $7.9'' = 6.9\lambda/D$  (at 500nm). Position Angle:  $44^\circ$

Without mask:



With mask:



The secondary is to the upper left of the primary in the mask image.



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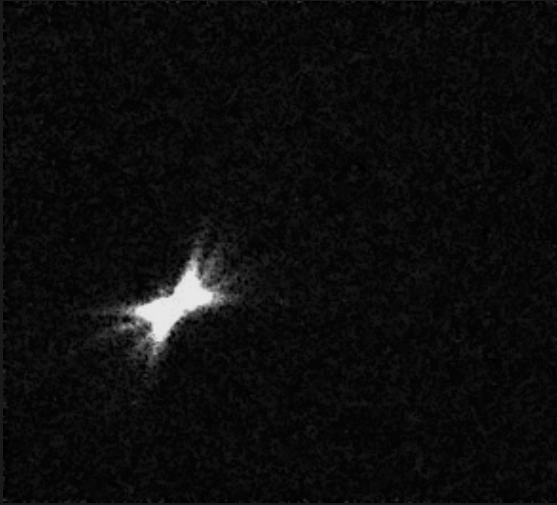
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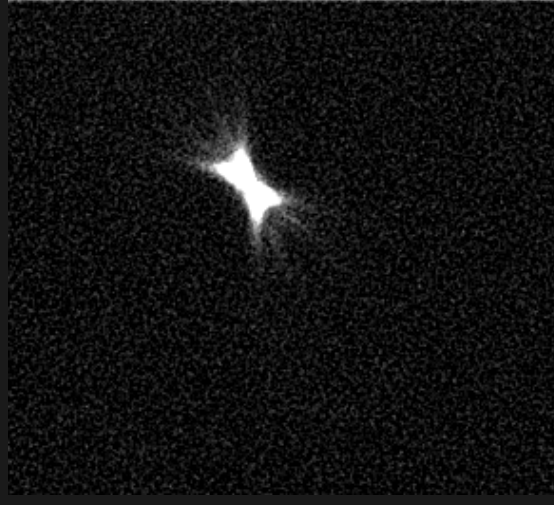
# Is it real?

We took another image with the mask rotated about  $90^\circ$ . The rotated mask shows no hint of a secondary:

Original orientation:



Rotated:



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# Conclusions

- Detection of extrasolar terrestrial planets orbiting nearby stars is technically very difficult but may well be practical within the foreseeable future.
  
- A space-based telescope with an elliptical mirror and a shaped aperture provides the contrast needed to detect and perhaps characterize such planets.



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