

Shaped Pupils

Robert J. Vanderbei

March 23, 2004
JPL Site Visit



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Masks and Things...



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Electric Field

The image-plane *electric field* $E()$ produced by an on-axis plane wave and an apodized aperture defined by an *apodization function* $A()$ is given by

$$E(\xi, \zeta) = \int_{-1/2}^{1/2} \int_{-1/2}^{1/2} e^{-2\pi i(x\xi + y\zeta)} A(x, y) dy dx$$
$$\vdots$$
$$E(\rho) = 2\pi \int_0^{1/2} J_0(2\pi r \rho) A(r) r dr,$$

where J_0 denotes the 0-th order Bessel function of the first kind.

The unitless pupil-plane “length” r is given as a multiple of the aperture D .

The unitless image-plane “length” ρ is given as a multiple of focal-length times wavelength over aperture ($f\lambda/D$) or, equivalently, as an angular measure on the sky, in which case it is a multiple of just λ/D .

The *point spread function* (psf) is the square of the electric field.

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Performance Metrics



Inner and Outer Working Angles

$$\rho_{iwa} \quad \rho_{owa}$$

Contrast:

$$E^2(\rho) / E^2(0)$$

Airy Throughput:

$$\frac{\int_0^{\rho_{iwa}} E^2(\rho) 2\pi \rho d\rho}{(\pi(1/2)^2)} = 8 \int_0^{\rho_{iwa}} E^2(\rho) \rho d\rho.$$

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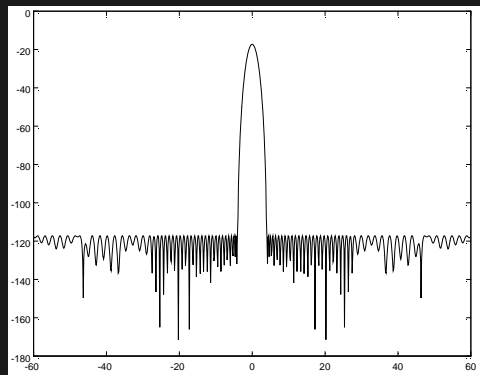
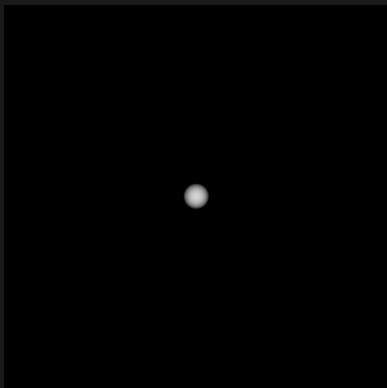
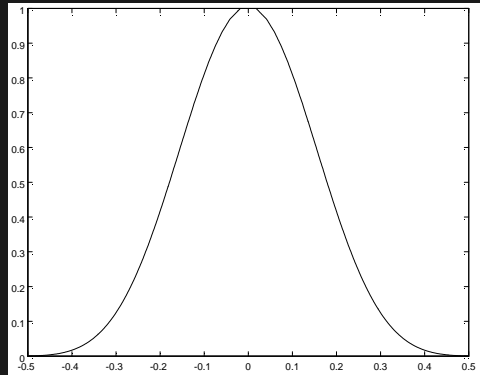
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Apodization

$$\rho_{iwa} = 4 \quad \mathcal{I}_{\text{Airy}} = 9\%$$

Excellent dark zone. **Unmanufacturable.**



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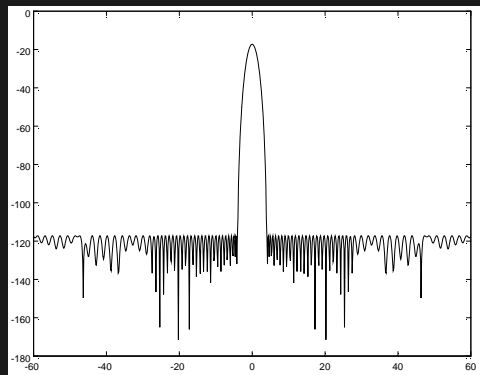
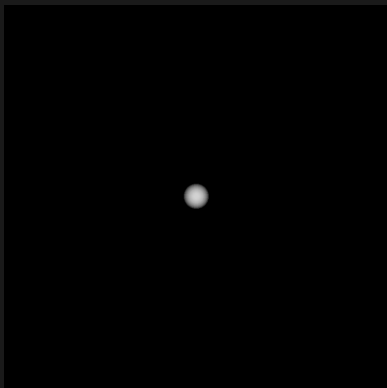
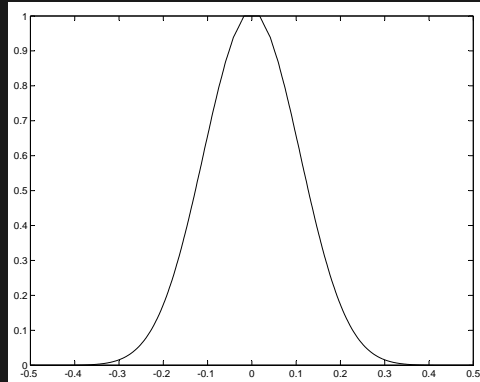
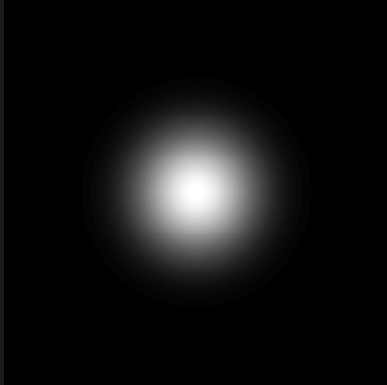
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Apodization

$$\rho_{iwa} = 4 \quad \mathcal{T}_{\text{Airy}} = 9\%$$

Excellent dark zone. **Unmanufacturable.**



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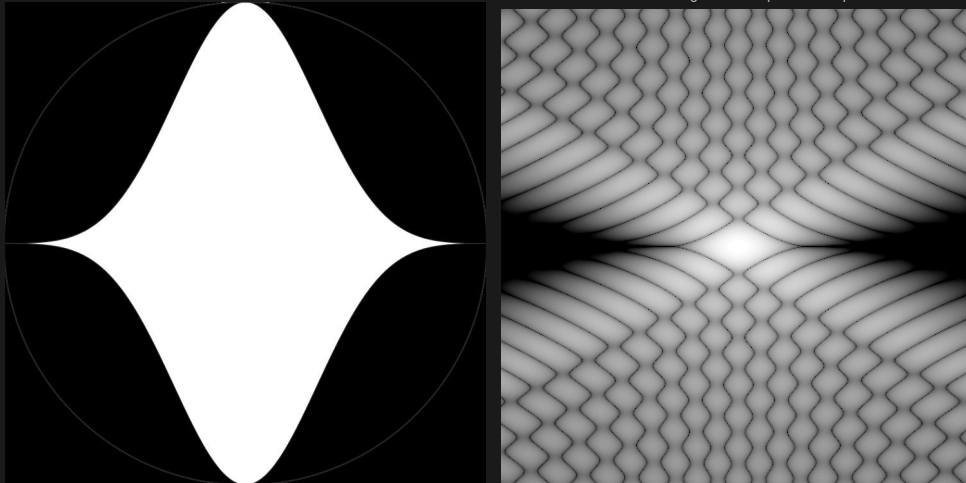
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One Pupil w/ On-Axis Constraints

$$\rho_{iwa} = 4 \quad \mathcal{T}_{\text{Airy}} = 43\%$$

Small dark zone...Many rotations required



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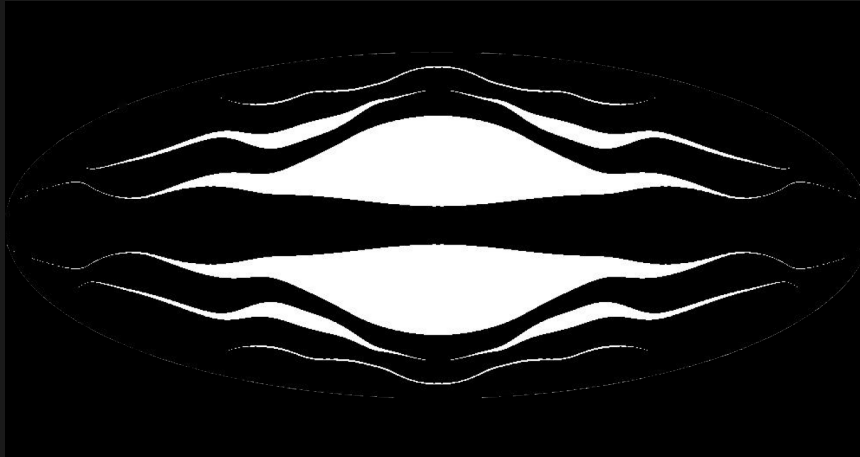
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Multiple Pupil Mask



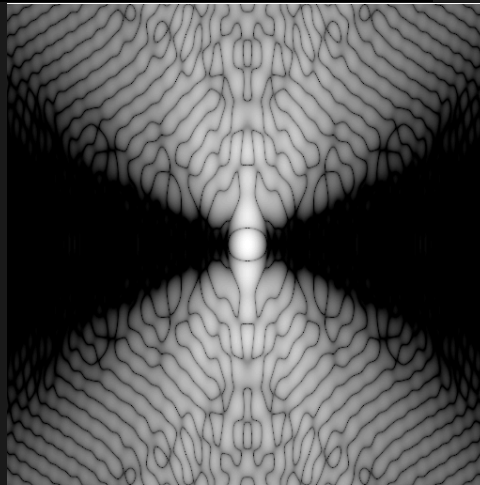
$$\rho_{iwa} = 4 \quad \mathcal{T}_{\text{Airy}} = 30\%$$

Throughput relative to ellipse

Large C/O

Easy to make

Very few rotations



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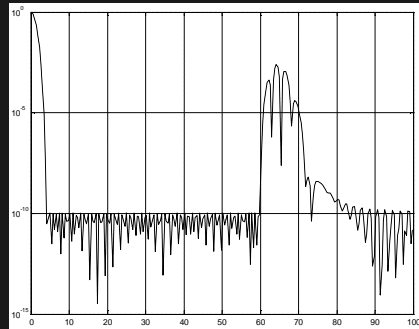
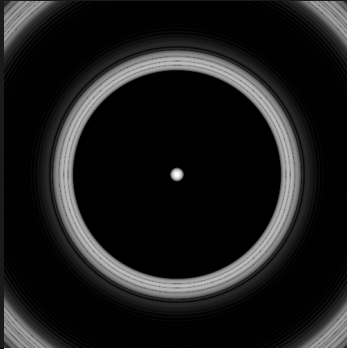
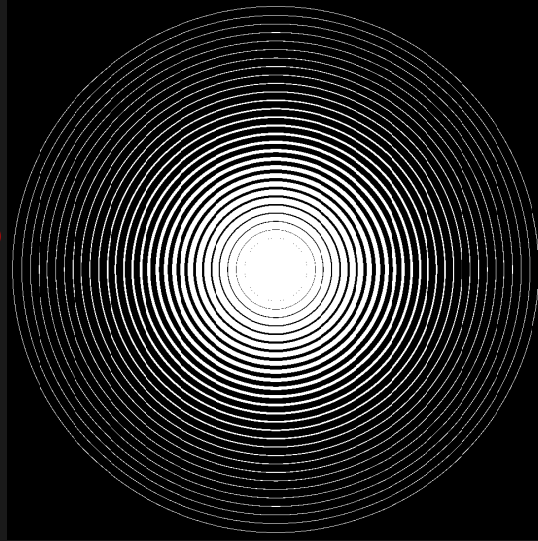
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Concentric Ring Mask

$$\rho_{iwa} = 4 \quad \rho_{owa} = 60 \quad \mathcal{T}_{\text{Airy}} = 9\%$$

Impossible to make
No rotations



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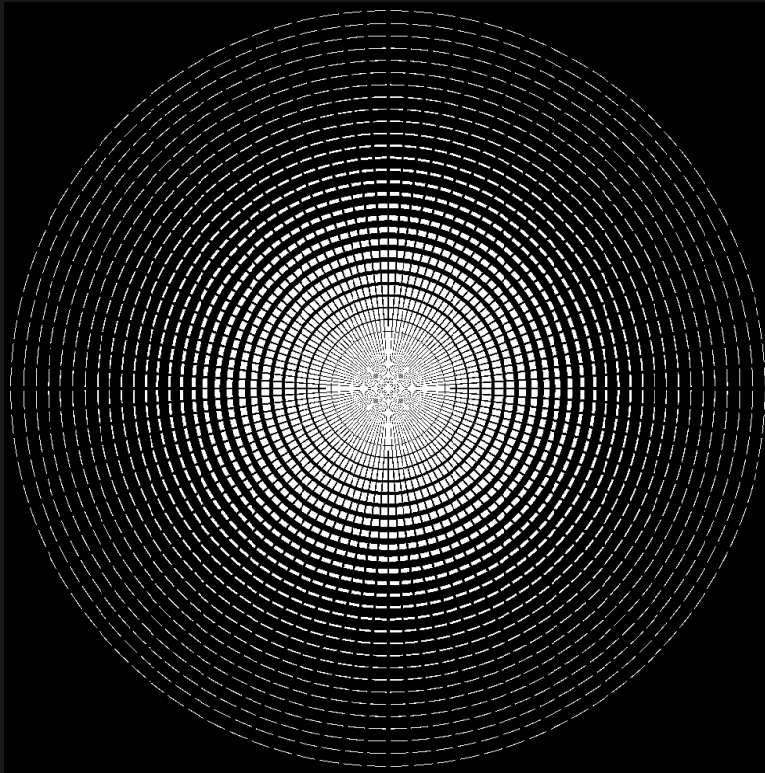
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Spiderweb Mask: 100 Vanes

$$\rho_{iwa} = 4 \quad \rho_{owa} = 40 \quad \mathcal{T}_{\text{Airy}} = 9(1 - \theta)\%$$

Manufacturable?



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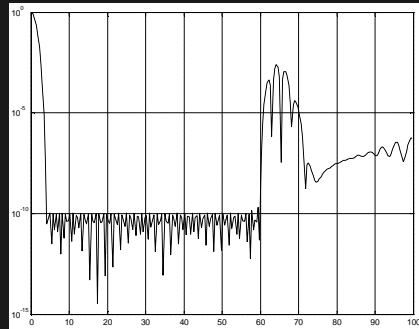
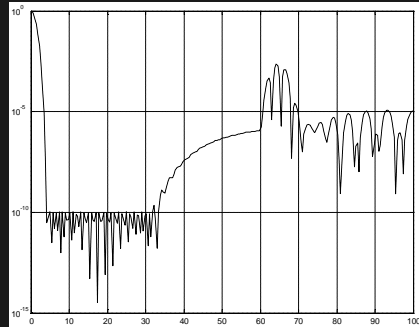
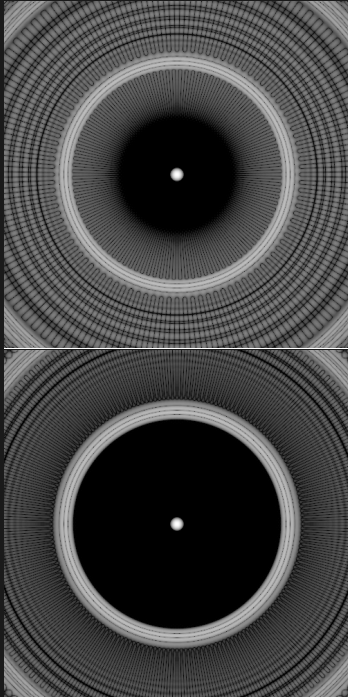
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Spiderweb Mask: 100 and 180 Vanes

$$\rho_{iwa} = 4 \quad \rho_{owa} = 40 \quad \mathcal{T}_{\text{Airy}} = 9(1 - \theta)\%$$



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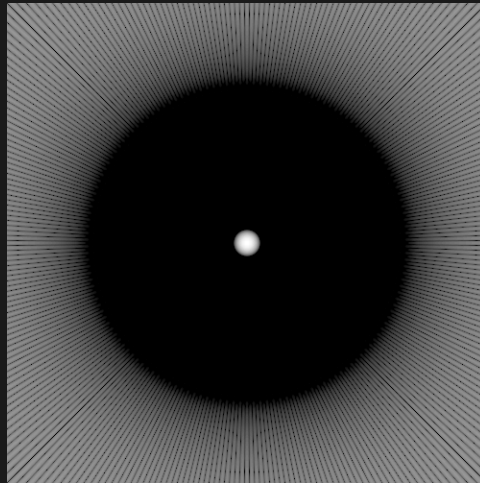
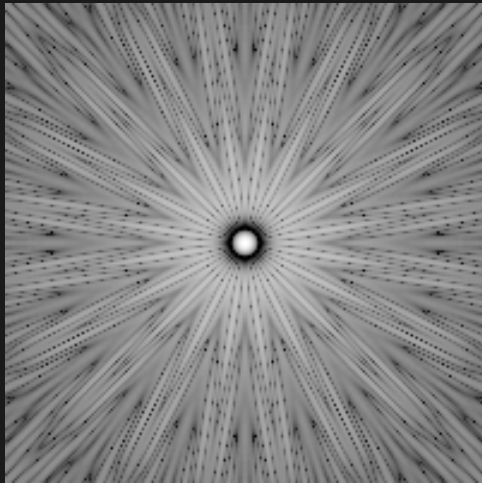
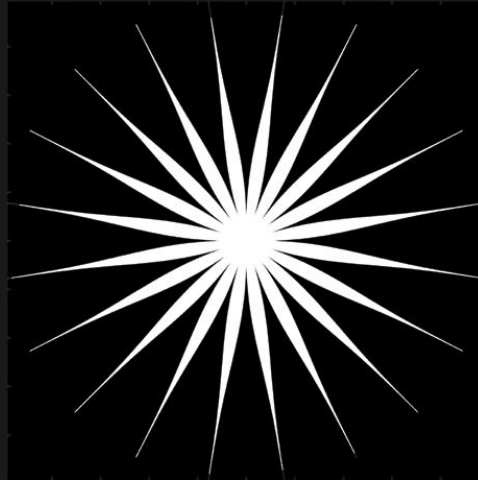
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Starshaped Masks: 20 and 150 Points

Easier to make?

Any apodization works:

$$\alpha(r) = \frac{2\pi}{N} A(r)$$



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Barcode Mask



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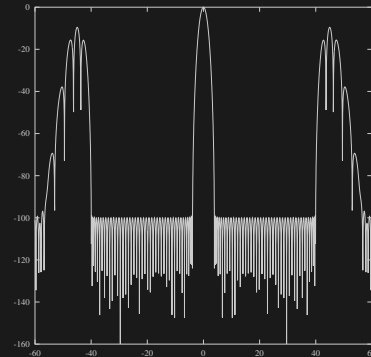
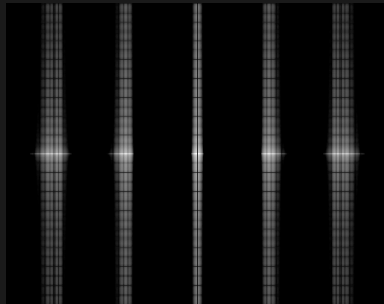
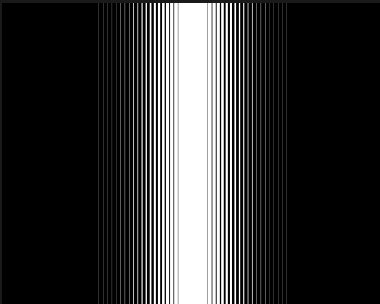
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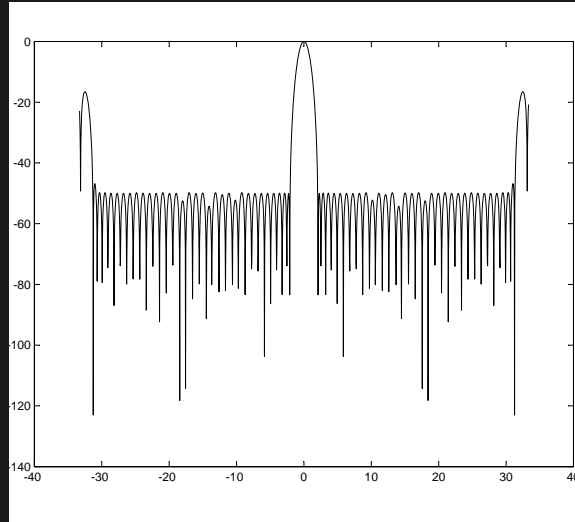
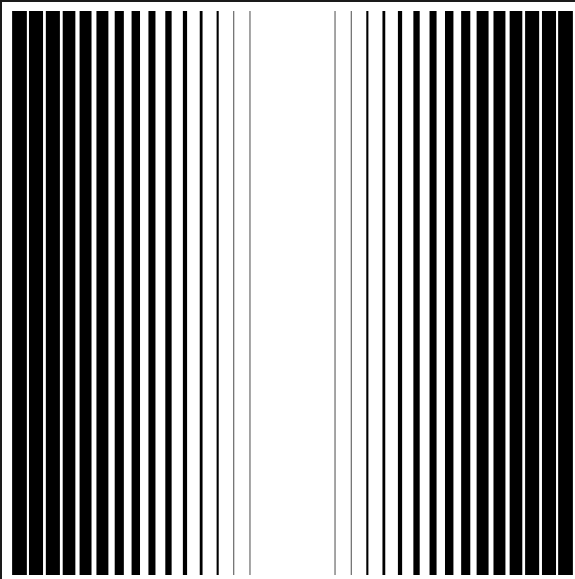
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$$\rho_{iwa} = 4 \quad \rho_{owa} = 40 \quad \mathcal{T}_{\text{Airy}} = 25\% \quad \text{Contrast} = 10^{-10}$$



Barcode Mask as Stepping Stone...

$$\rho_{iwa} = 2 \quad \rho_{owa} = 25 \quad \mathcal{T}_{\text{Airy}} = 39\% \quad \text{Contrast} = 10^{-5}$$



A stepping stone to Checkerboard designs...



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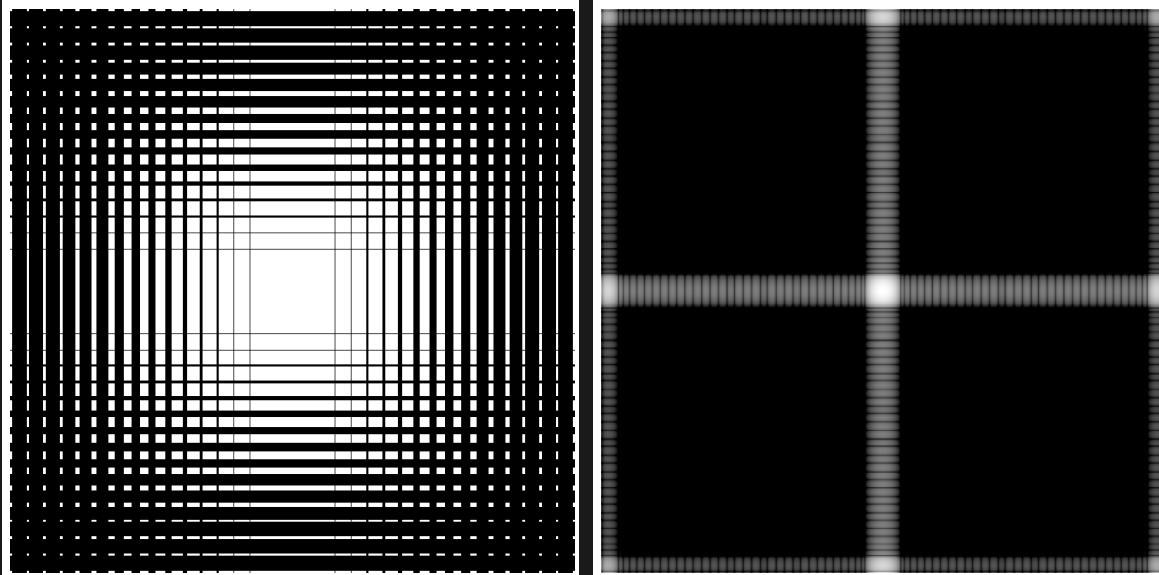
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Checkerboard Mask (Two Barcodes)

$$\rho_{iwa} = 2\sqrt{2} \quad \rho_{owa} = 25 \quad \mathcal{T}_{\text{Airy}} = 15\% \quad \text{Contrast} = 10^{-10}$$



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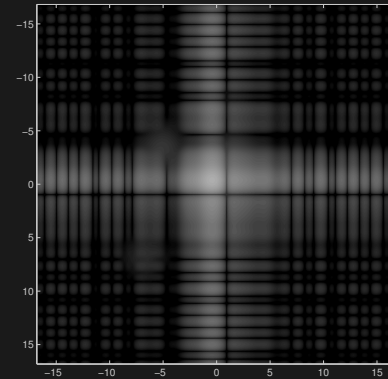
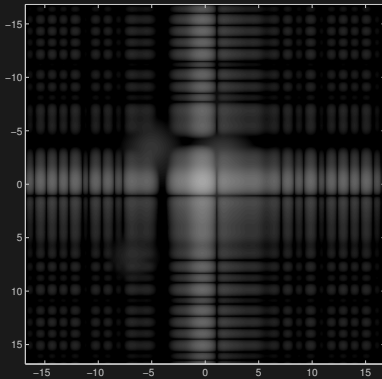
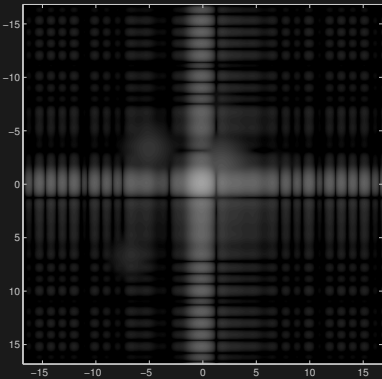
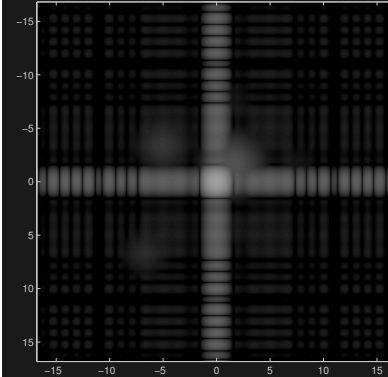
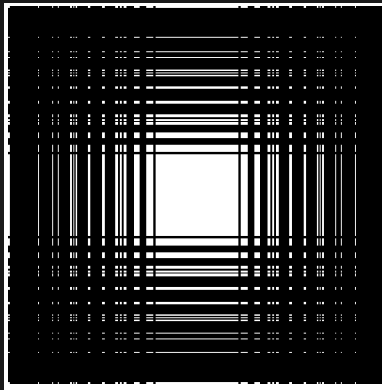
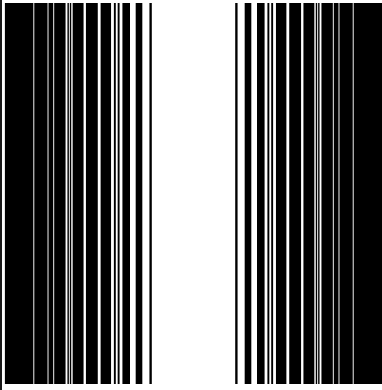
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Image Plane Occulter $0.6\lambda/D$ + Checkerboard Lyot Mask

$$\rho_{iwa} = 1.4\sqrt{2} \quad \rho_{owa} = 21 \quad \mathcal{T}_{\text{Airy}} = 6.2\% \quad \text{Contrast} = 10^{-10}$$



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... with 2% Central Obstruction

$$\rho_{iwa} = 1.4 \quad \rho_{owa} = 21 \quad \mathcal{T}_{\text{Airy}} = 3.5\% \quad \text{Contrast} = 10^{-10}$$



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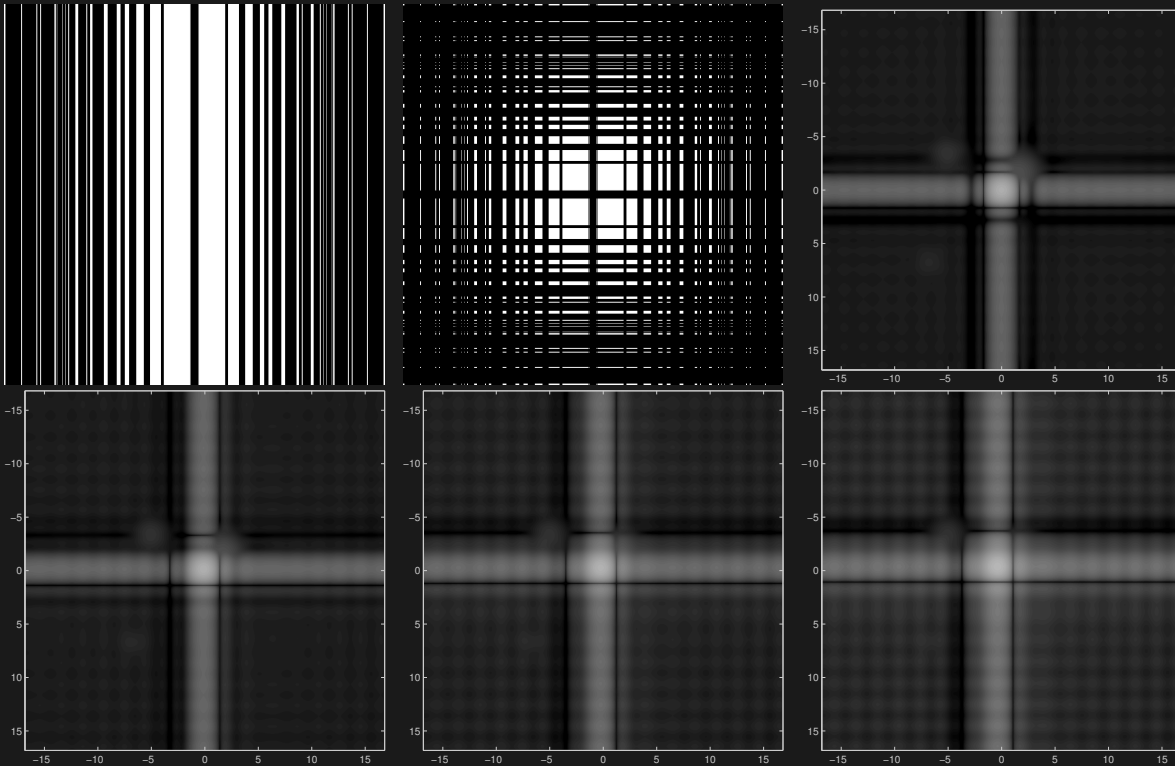
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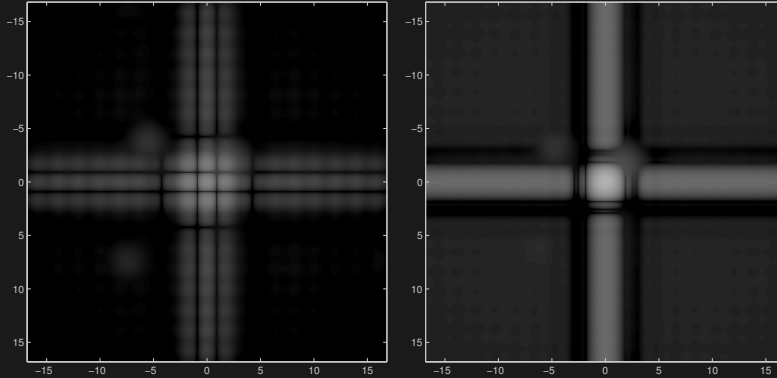
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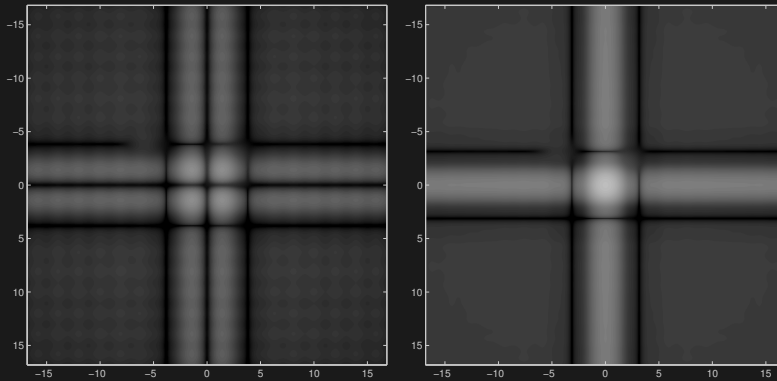


... Wavelength Dependence

$\lambda = 90\%$, 105% of design value



$\lambda = 85\%$, 110% of design value



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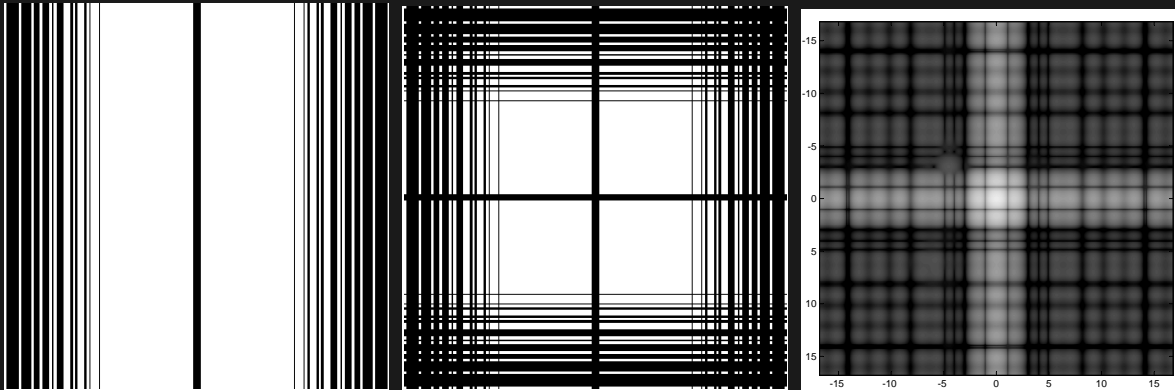
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... with Higher Throughput

$$\rho_{iwa} = 4 \quad \rho_{owa} = 21 \quad \mathcal{T}_{\text{Airy}} = 40\% \quad \text{Contrast} = 10^{-10}$$



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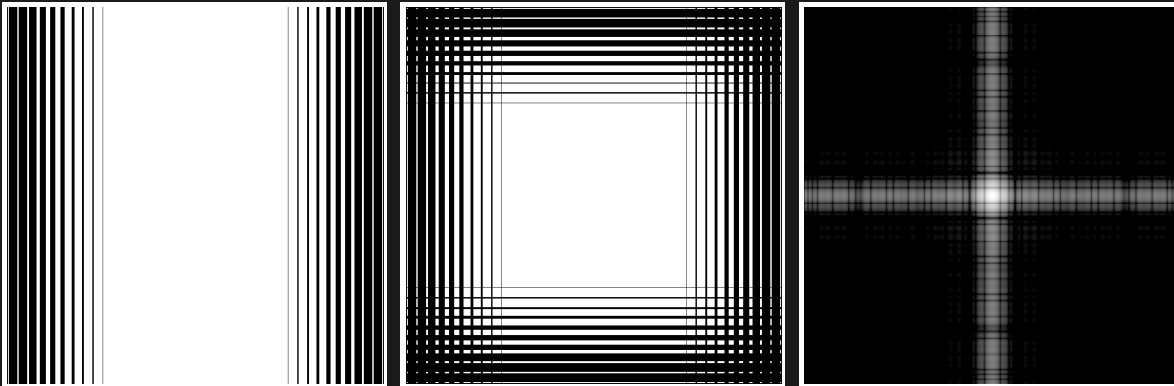
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... with Neither Occulter Nor C/O

$$\rho_{\text{iwa}} = 4 \quad \rho_{\text{owa}} = 25 \quad \mathcal{T}_{\text{Airy}} = 38\% \quad \text{Contrast} = 10^{-10}$$

Note: central lobe has side-lobes.



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Other Stuff...

- Pupil Mapping (aka PIAA)
- Apodization and Variable Phase-Shift
- Phase-Knife Coronagraph
- Higher Order Bessel

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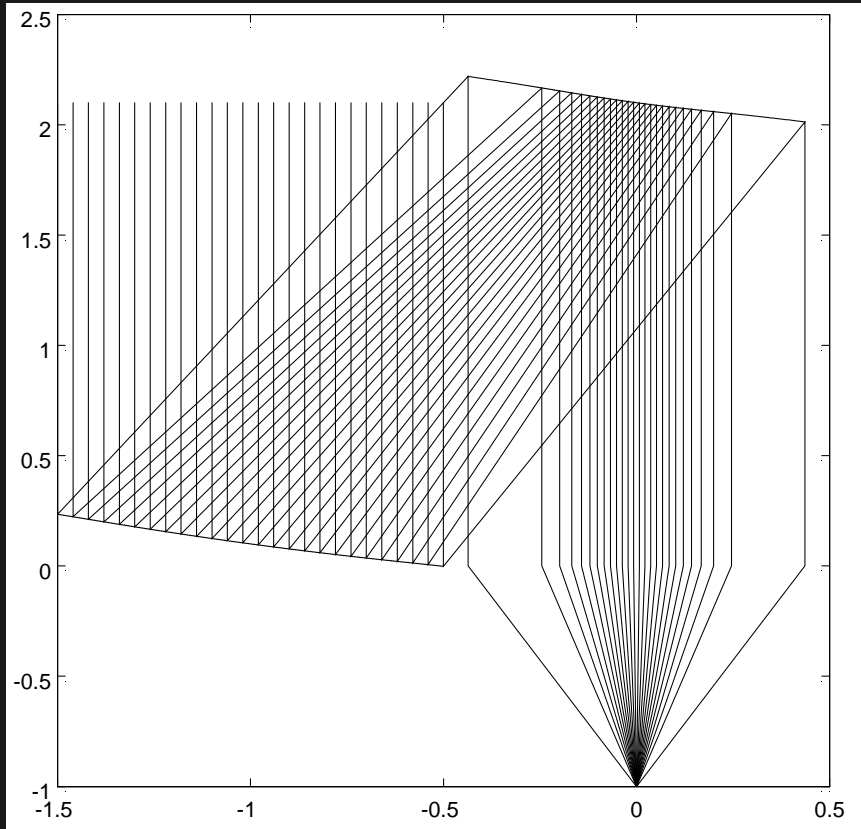
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Pupil Mapping (aka PIAA)

w/ Wes Traub



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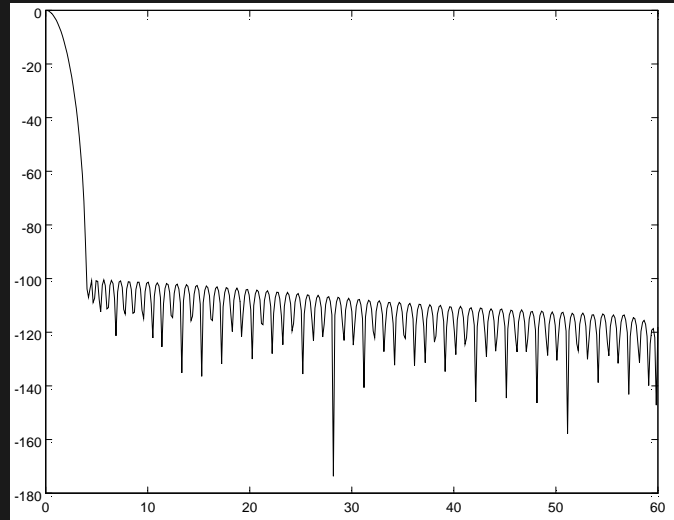
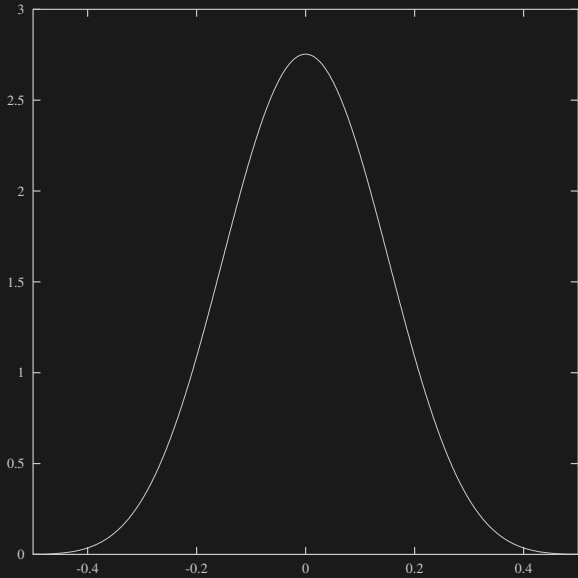
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No Light is Lost

$$\rho_{iwa} = 4 \quad \mathcal{T}_{\text{Airy}} = 100\%$$



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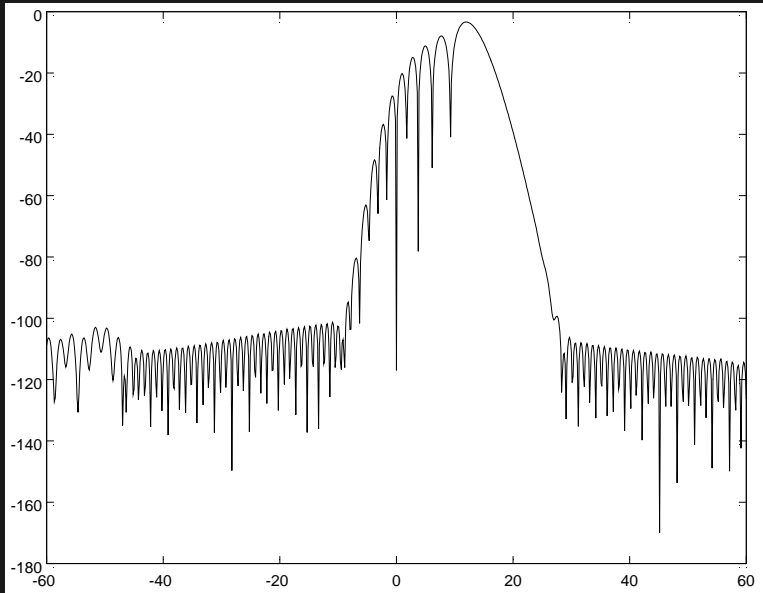
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Effective Magnification = $2.4\times$

Psf for an off-axis source at $5\lambda/D$:

- has its peak at $12\lambda/D$
- is **smear**ed and
- **extended stellar source gets smear**ed



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Apodization and Variable Phase-Shift

w/ Marc Kuchner

Our previous computations of optimal apodizations assumed that the *electric field* in the image plane is given by:

$$E(\xi) = \int_{-1/2}^{1/2} e^{-2\pi i x \xi} A(x) dx = 2 \int_0^{1/2} \cos(2\pi x \xi) A(x) dx$$

However, apodization introduces phase lag—the stronger the attenuation, the longer the lag. Hence, the correct expression for the electric field is

$$E(\xi) = \int_{-1/2}^{1/2} e^{-2\pi i x \xi + ik \log(A(x))} A(x) dx$$

where k is a constant giving the rate that lag appears as attenuation is increased.

Note that, unlike the first expression, this complex expression does not reduce to a real-valued one.



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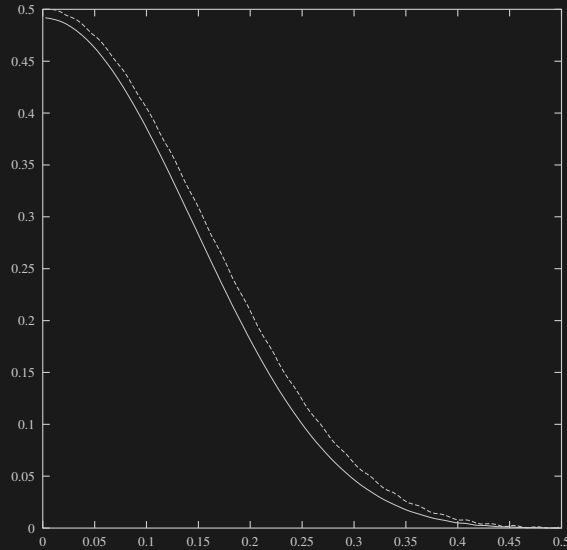
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Optimal Apodization with and without Phase Shift



Upper apodization — without phase shift. Computed using $\rho_{iwa} = 4$ and $\rho_{owa} = 40$.

Lower apodization — with phase shift. Computed using $\rho_{iwa} = 4.5$ and $\rho_{owa} = 40$.

Infeasible with a tighter inner working angle.



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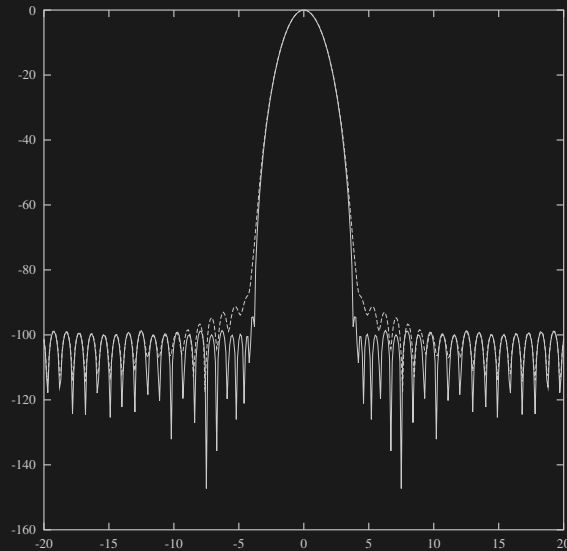
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PSF Designed without Phase Shift



Here is shown the PSF for the apodization computed without accounting for phase shift.

The upper curve is the PSF one gets with the phase shift term using $k = 0.1$.



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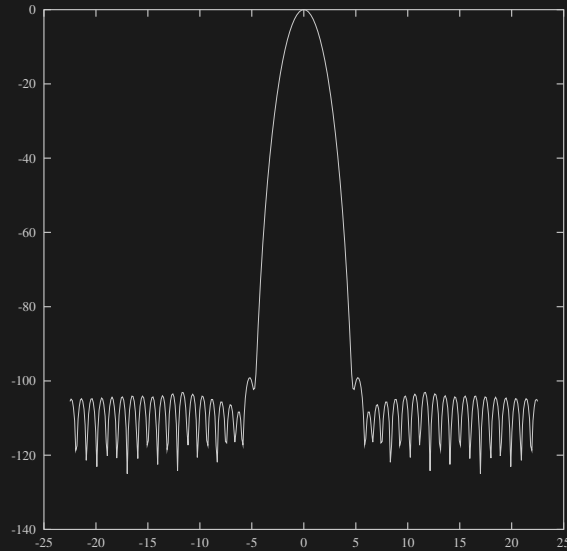
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PSF Designed with Phase Shift



Here is shown the PSF for the apodization computed with accounting for phase shift. Inner working angle is $4.5\lambda/D$.



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Phase-Knife Coronagraph

Image-plane phase mask:

$$M(\xi, \zeta) = \text{sgn}(\xi)\text{sgn}(\zeta)1_{|\xi| \leq R}1_{|\zeta| \leq R}$$

Fourier transform:

$$\widehat{M}(x, y) = \frac{-1}{\pi^2 xy} (1 - \cos(2\pi Rx)) (1 - \cos(2\pi Ry))$$

Lloyd et.al. show that, for $R = \infty$, this is a perfect coronagraph. That is, **no on-axis light** gets past the Lyot stop.

We show that for $R = 200$, on-axis light is only attenuated only to the 10^{-5} level.

Making a **polychromatic** phase-shifter is not easy.



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Higher Order Bessel



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