Optimal Shaped Pupil Coronagraphs for Extrasolar Planet Finding

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Objective

Create a system Point Spread Function (PSF) with the needed *contrast* allowing planet discovery at the smallest *inner working distance (IWD)* in the shortest *integration time.*

Outline

- Point Spread Function for Equal-Area Apertures
- Performance Metrics
- Apodized Pupils Comparisons
- Optimal Shaped Pupil Designs

Note: We assume systematic error is separately reduced to below background.

Background

Irradiance $I(\xi, \zeta)$ is an intensity I_o times the point spread function $P(\xi, \zeta)$:

$$I(\xi, \zeta) = I_o P(\xi, \zeta)$$

The intensity I_o depends only on the source strength and the aperture area $A = a^2$.

The point spread function (PSF) for a $a\sqrt{r} \times a/\sqrt{r}$ rectangular design space is given by

$$P(\xi, \zeta) = \left| \int_{-\frac{1}{2\sqrt{r}}}^{\frac{1}{2\sqrt{r}}} \int_{-\frac{\sqrt{r}}{2}}^{\frac{\sqrt{r}}{2}} \mathcal{A}(y, z) e^{-2\pi i(\xi y + \zeta z)} dy dz \right|^2,$$

where the function $\mathcal{A}()$ denotes the apodization function.

We distinguish purely open/closed pupils from smooth apodizations:

$$\mathcal{A} \in \{0, 1\} \implies \textit{pupil mask}$$
$$\mathcal{A} \in [0, 1] \implies \textit{smooth apodization.}$$

Optimization Criteria

We compare and optimize different designs with equal pre-apodized aperture area.

Comparisons are made on four performance metrics:

- Contrast
- Integration Time
- Inner Working Distance
- Discovery Space

Contrast

Brown and Burrows (Icarus 1990) introduced the *contrast quotient*:

$$Q = \frac{I_{\text{planet}} P(0,0)}{I_{\text{star}} P(\xi_{\text{planet}}, \zeta_{\text{planet}}) + I_{\text{scatter}},} = \frac{I_{\text{planet}} P(0,0)}{I_{\text{bkgrd}}}$$

where I_{planet} denotes the planet's irradiance, I_{star} denotes the star's irradiance, and $(\xi_{\text{planet}}, \zeta_{\text{planet}})$ denotes the image-plane coordinates of the planet relative to an on-axis parent star.

It is estimated that $I_{\text{planet}}/I_{\text{star}}$ might be as small as 10^{-10}. Hence, to have a unit contrast ratio, $Q = 1$, we require

$$P(\xi_{\text{planet}}, \zeta_{\text{planet}}) \leq 10^{-10} P(0,0).$$

Integration Time

Detection in Known Background

Assume Poisson arrival statistics for planet signal and background:

$$S/N = \frac{I_{\text{planet}} t \Delta P}{\sqrt{I_{\text{bkgrd}} t \Delta S}} \qquad \Longrightarrow \qquad t_1 = \frac{(S/N)^2 P(0,0) \Delta S}{I_{\text{planet}} Q \Delta P^2},$$

where ΔP denotes the area under the PSF in a neighborhood ΔS of $(0,0)$.

Photometry in Unknown Background (Characterization)

$$t_2 = \frac{(S/N)^2 P(0)}{I_{\text{planet}} \left[\Delta S \Delta^2 P - (\Delta P)^2 \right]} \left\{ \frac{\Delta S^2 \Delta^3 P - (\Delta P)^3}{P(0,0) \left[\Delta S \Delta^2 P - (\Delta P)^2 \right]} + \frac{\Delta S}{Q} \right\}$$

Inner Working Distance (IWD)

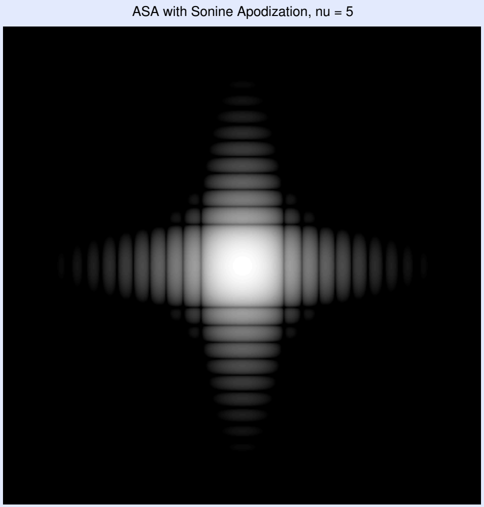
The smallest angular separation from the star for which the PSF contrast reaches the required value of 10^{-10}.

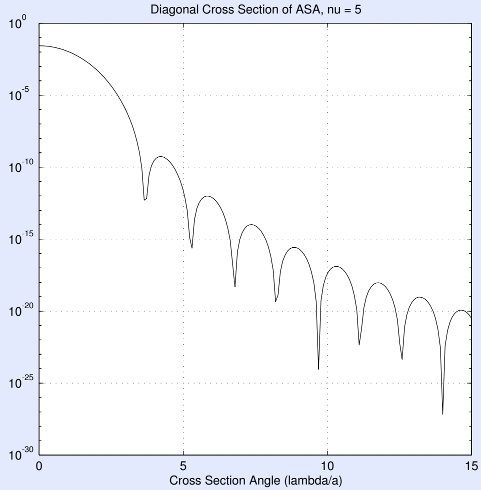
Discovery Space

The azimuthal region of the image plane within which discovery is possible; that is, where 10^{-10} contrast is achieved.

Apodized Pupils

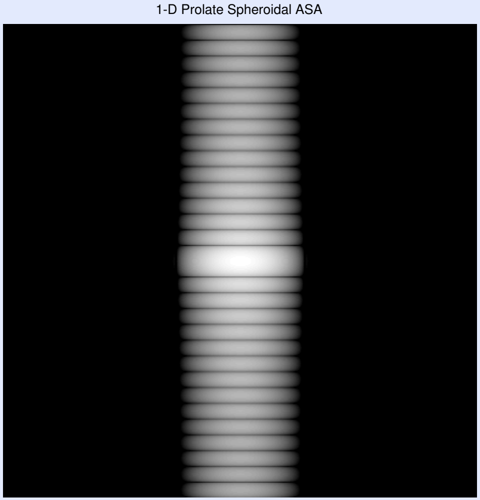
Apodized Square Aperture (ASA) (Nisenson & Papaliolios, ApJ 2001)

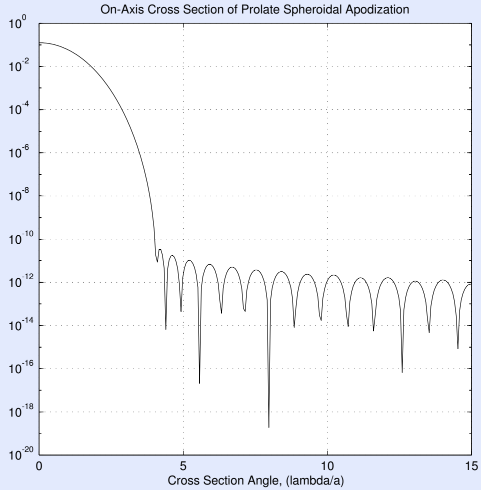


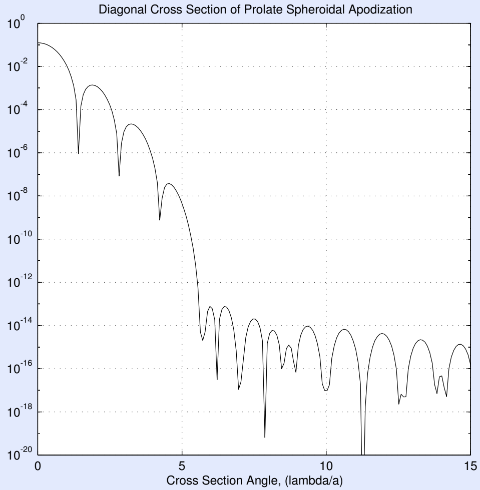


Left The PSF for an ASA with a sonine apodization ($\nu = 5$) plotted on a logarithmic scale with black areas 10^{-10} below brightest. *Right* Cross section of ASA on diagonal showing inner working distance of $5\lambda/a$. Single-exposure integration time of $t_1 = 25.5$.

Prolate Spheroidal Apodization (Slepian, J.Opt.Soc.Am. 1965)



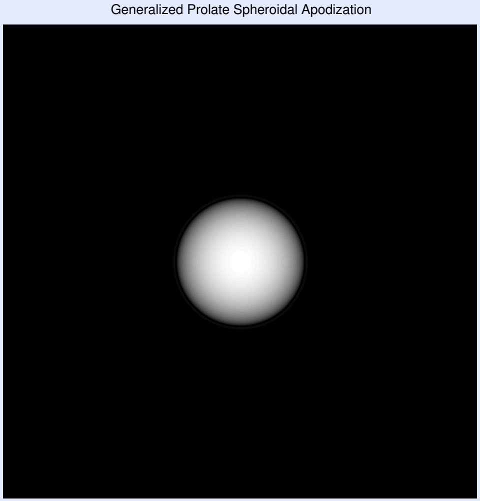


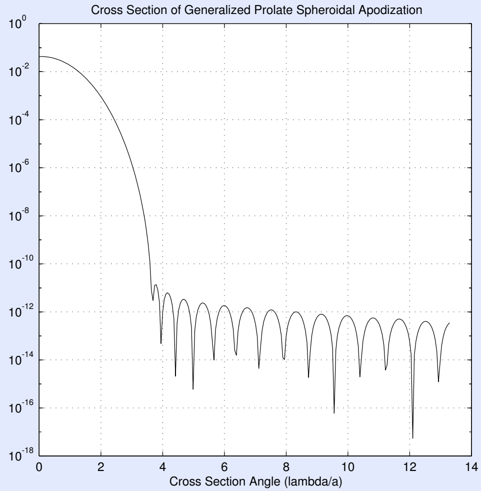


On-diagonal, IWD $\approx 5\lambda/a$. Single-exp. integration time of $t_1 = 9.5$.

Generalized Prolate Spheroidal (GPS) Apodization

Note: Circular Telescope





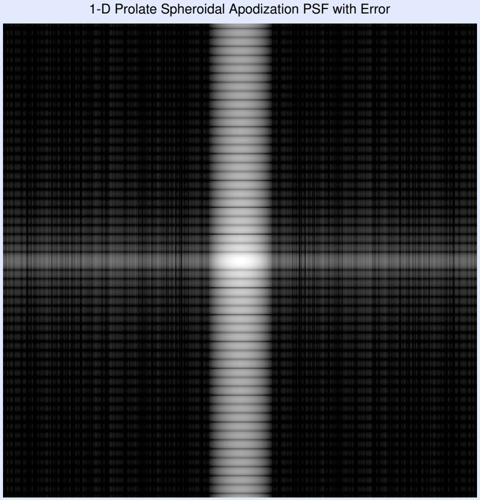
IWD $= 3.5\lambda/a$. Single-exposure integration time of $t_1 = 17$.

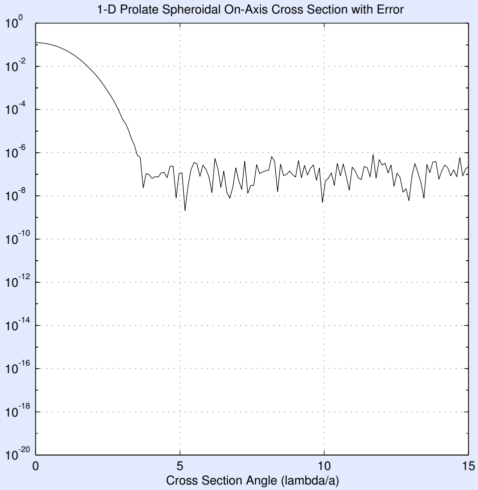
Integration Times for Apodizations

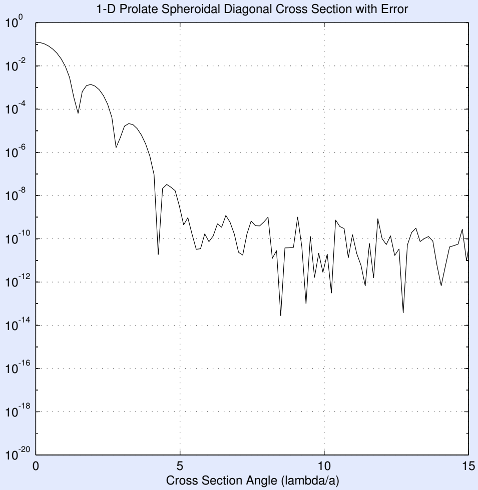
Type	$\frac{t_1 I_p}{(S/N)^2}$	$\frac{t_2 I_p}{(S/N)^2}$	*IWD*	*Disc. Space*
Sonine, $\nu = 5$	25.5	48	5 (diag), 13 (axis)	$< 1/2$
Sonine, $\nu = 4$	19	35	7 (diag) , 15 (axis)	$< 1/2$
1-D Prolate	9.5	16.5	5 (diag), 4 (axis)	1/2
Gen. Prolate	17	34	3.5	full

Integration time comparisons for four different pupil apodizations in equal area apertures. Integration times are for a single exposure and have been normalized by planet irradiance and signal-to-noise ratio. The integration time t_1 assumes a known background level and is for planet discovery only. The integration time t_2 simultaneously estimates planet and background irradiance by using a region of the image plane slightly larger than the full width of the main lobe.

Prolate Spheroidal with White Apod. Error of 6×10^{-12} Hz^{-2}



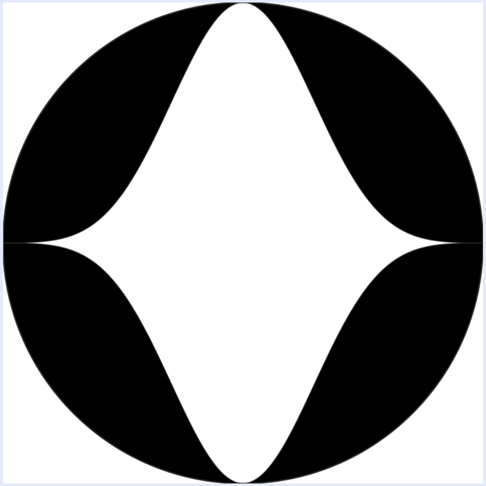


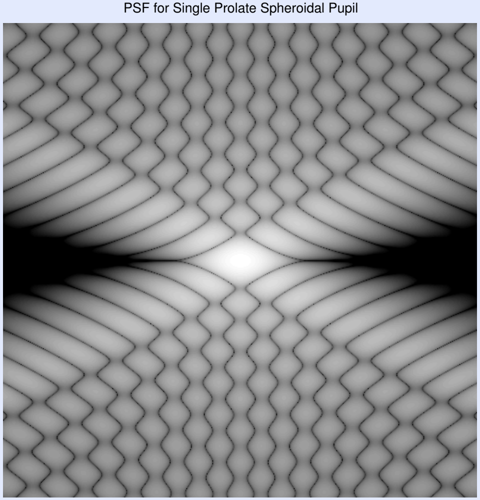


Four orders of magnitude degradation.

Shaped Pupil Masks

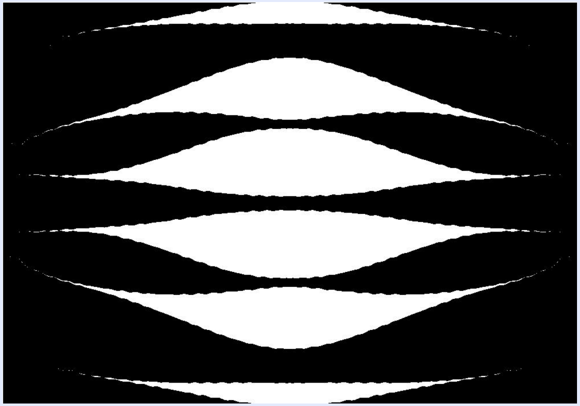
Prolate Spheroidal Mask

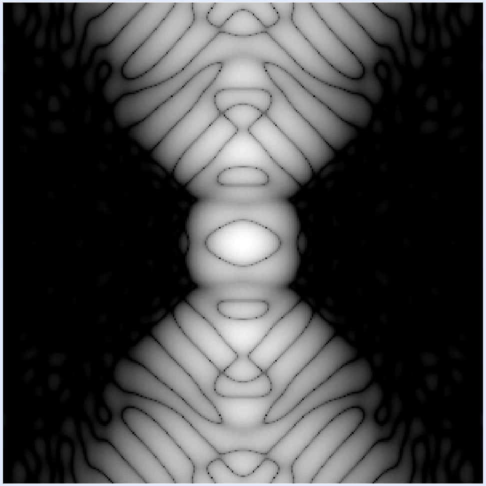


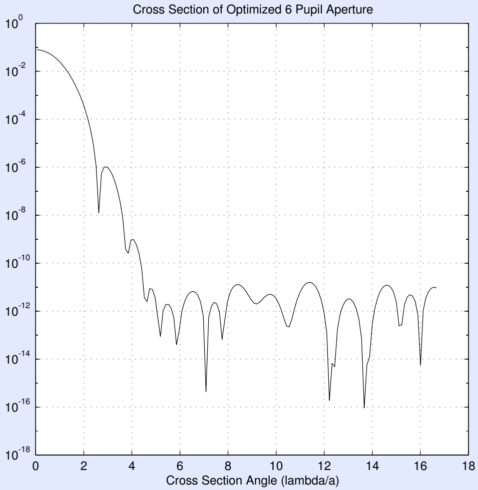


IWD $= 4\lambda/a$. Single-exposure integration time of 4.6.

6-Pupil Rectangular

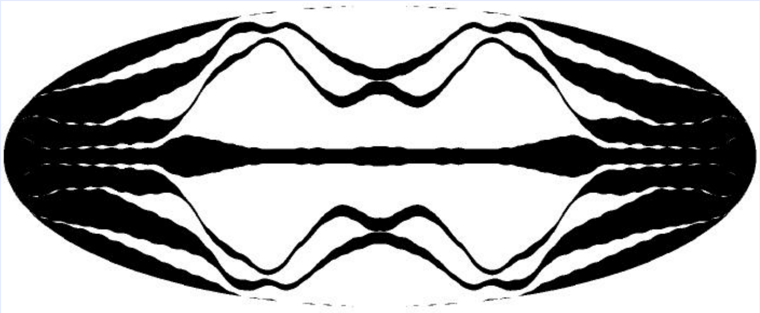


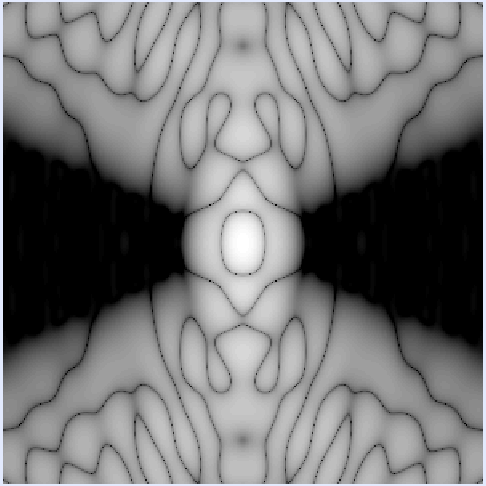


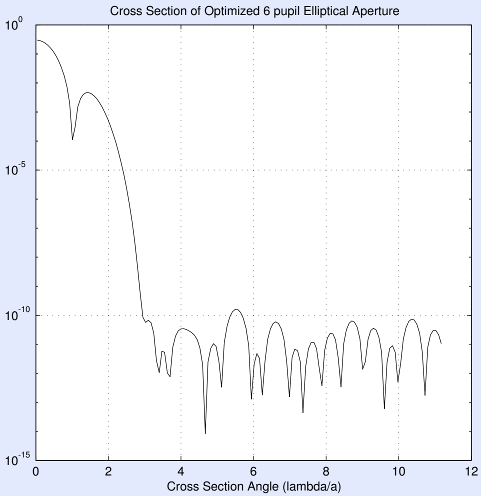


IWD $= 4.2\lambda/a$. Single-exposure integration time of $t_1 = 15$.

6-Pupil Elliptical

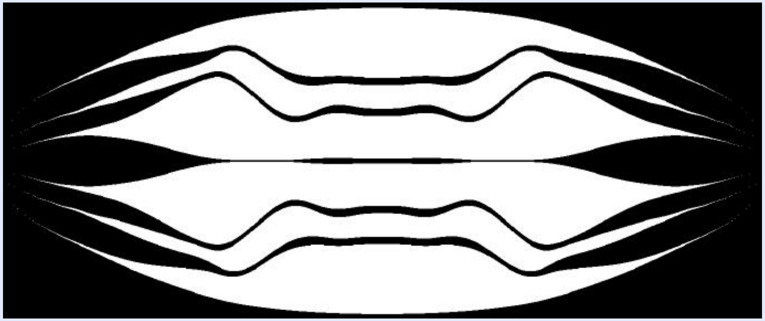


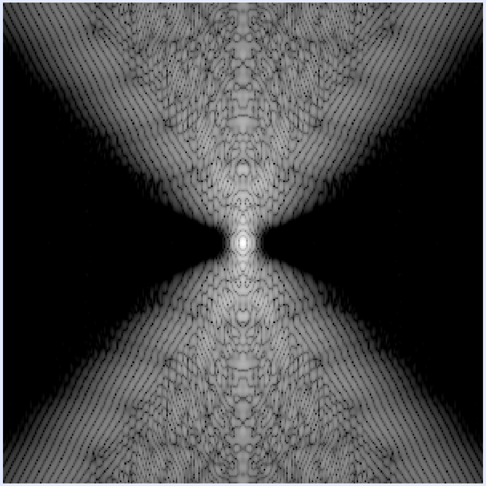


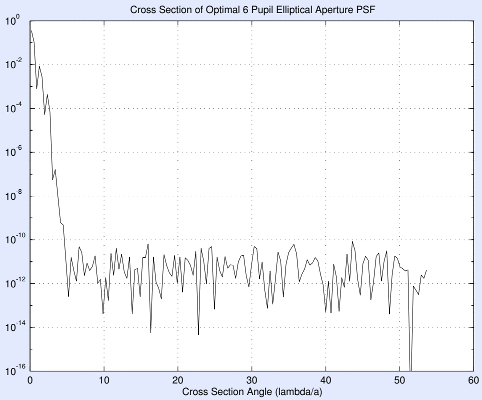


IWD $= 2.8\lambda/a$. Single-exposure integration time of $t_1 = 6.5$.

Another 6-Pupil Elliptical







Larger OWD than the previous mask. IWD $= 4.5\lambda/a$. Single-exposure integration time of $t_1 = 4.8$.

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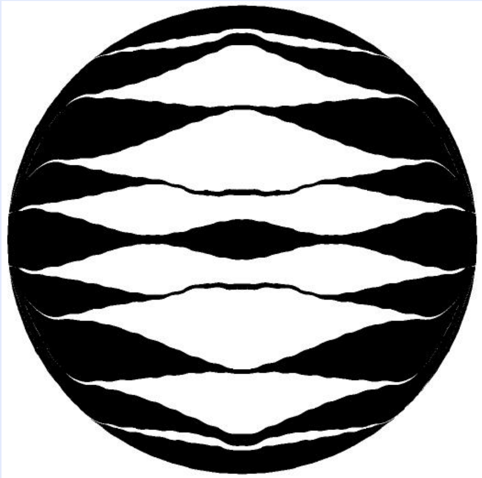
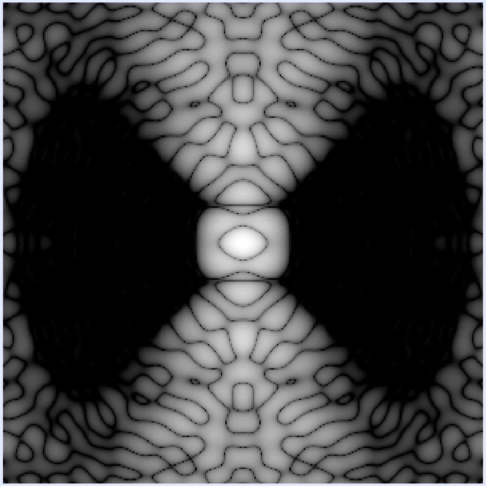
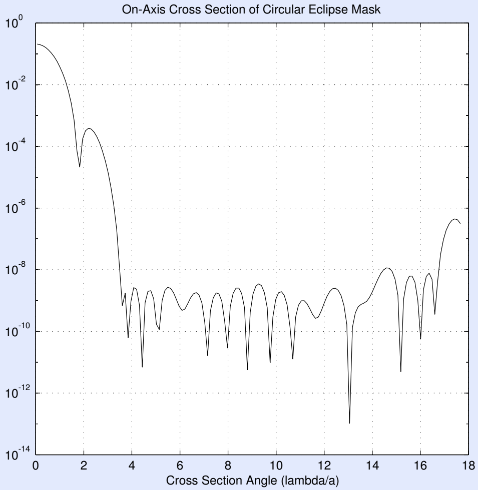
8 Pupil Circular Eclipse-Class (Black = 10^{-8})

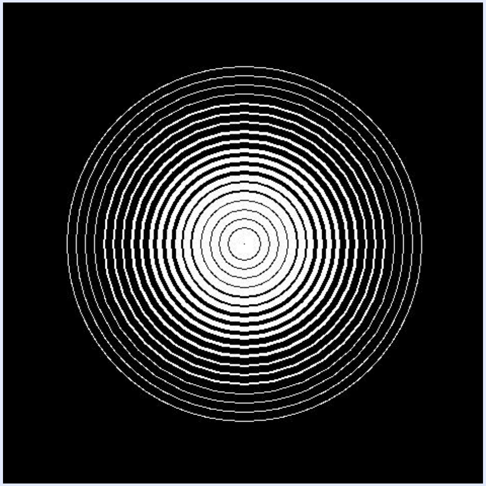


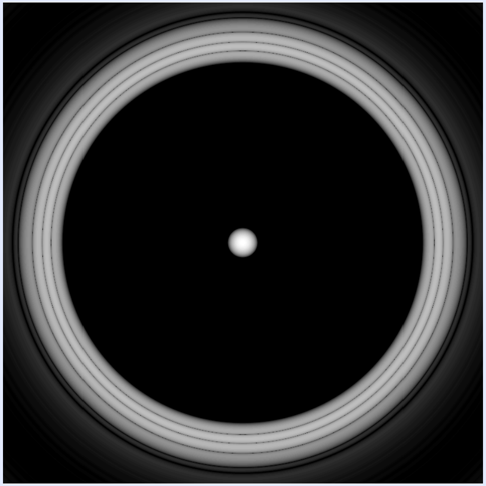


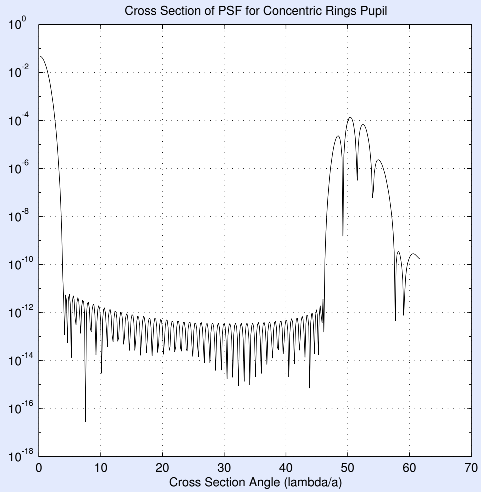


IWD = $3.5\lambda/a$. Single-exposure integration time of $t_1 = 7.9$.

Azimuthally Symmetric Mask







IWD $= 3.5\lambda/a$. Single-exposure integration time of $t_1 = 18$.

Integration Times for Masks

Type	$\frac{t_1 I_p}{(S/N)^2}$	$\frac{t_2 I_p}{(S/N)^2}$	*IWD*	*Discovery Space*
single prolate	4.6	8	4	$\ll 1/2$
6 pupil rectangle	15	21	4.2	1/2
6 pupil ellipse small OWD	6.5	9.5	2.8	1/4
6 pupil ellipse large OWD	4.8	7	4.5	1/4
8 pupil Eclipse	7.9	11	3.5	1/2
Concentric Rings	18	25	3.5	*full*

Integration time comparisons for three different shaped pupil apodizations in equal area apertures. Integration times are for a single exposure and have been normalized by planet irradiance and signal-to-noise ratio. The integration time t_1 assumes a known background level and is for planet discovery only. The integration time t_2 simultaneously estimates planet and background irradiance by using a region of the image plane slightly larger than the full width of the main lobe.

Conclusions

SHAPED PUPIL MASKS RULE! Here's why:

- Better integration time, inner working distance, discovery space.
- Easier to manufacture.
- Less susceptibility to space environment.
- Flexible design.
  - On or off axis.
  - Discovery vs. characterization.
- Advantages of tailoring the PSF.
  - High PSF contrast implies high speckle contrast.
  - Insensitivity to jitter and stellar size.
- The best shaped pupils are as good as or better than the best graded apodizations.