An Interior–Point Algorithm for Nonconvex Nonlinear Programming

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Level(3) ^{SM} Communications

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1 Outline

- The Basic Interior-Point Paradigm (for LP/QP)
- Modifications for Convex Optimization
- Modifications for Nonconvex Optimization
- Computational Results

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The Basic Interior-Point Paradigm (for LP/QP)

2 Introduce Slack Variables

- Start with an optimization problem—for now, the simplest NLP:

$$\begin{aligned} &\text{minimize} && f(x) \\ &\text{subject to} && h_i(x) \geq 0, \qquad i = 1, \ldots, m \end{aligned}$$

- Introduce slack variables to make all inequality constraints into nonnegativities:

$$\begin{aligned} &\text{minimize} && f(x) \\ &\text{subject to} && h(x) - w = 0, \\ & && w \geq 0 \end{aligned}$$

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3 Associated Log-Barrier Problem

- Replace nonnegativity constraints with logarithmic barrier terms in the objective:

$$\begin{aligned} \text{minimize} \quad & f(x) - \mu \sum_{i=1}^{m} \log(w_i) \\ \text{subject to} \quad & h(x) - w = 0 \end{aligned}$$

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First-Order Optimality Conditions

- Incorporate the equality constraints into the objective using Lagrange multipliers:

$$L(x, w, y) = f(x) - \mu \sum_{i=1}^{m} \log(w_i) - y^T(h(x) - w)$$

- Set all derivatives to zero:

$$\begin{aligned} \nabla f(x) - \nabla h(x)^T y &= 0 \\ -\mu W^{-1} e + y &= 0 \\ h(x) - w &= 0 \end{aligned}$$

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- Rewrite system:

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6 Apply Newton's Method

- Apply Newton's method to compute search directions, Δx, Δw, Δy:

$$\begin{bmatrix} H(x,y) & 0 & -A(x)^T \\ 0 & Y & W \\ A(x) & -I & 0 \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta w \\ \Delta y \end{bmatrix} = \begin{bmatrix} -\nabla f(x) + A(x)^T y \\ \mu e - WYe \\ -h(x) + w \end{bmatrix}.$$

  Here,

$$H(x,y) = \nabla^2 f(x) - \sum_{i=1}^{m} y_i \nabla^2 h_i(x)$$

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$$A(x) = \nabla h(x)$$

- Note: $H(x,y)$ is positive semidefinite if f is convex, each h_i is concave, and each $y_i \geq 0$.

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7 Reduced KKT System

- Use second equation to solve for Δw. Result is the reduced KKT system:

$$\begin{bmatrix} -H(x,y) & A^T(x) \\ A(x) & WY^{-1} \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} = \begin{bmatrix} \nabla f(x) - A^T(x)y \\ -h(x) + \mu Y^{-1} e \end{bmatrix}$$

- Iterate:

$$x^{(k+1)} = x^{(k)} + \alpha^{(k)} \Delta x^{(k)}$$
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Modifications for Convex Optimization

8 Need to Use Shorter Steps

For convex nonquadratic optimization, it does not suffice to choose the steplength α simply to maintain positivity of nonnegative variables.

- Consider, e.g., minimizing

$$f(x) = (1 + x^2)^{1/2}.$$

- The iterates can be computed explicitly:

$$x^{(k+1)} = -(x^{(k)})^3$$

- Converges if and only if $|x| \leq 1$.
- Reason: away from 0, function is too linear.

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9 Merit Function

A merit function is used to guide the choice of steplength α.

We use the Fiacco–McCormick merit function

$$\Psi_{\beta,\mu}(x,w) = f(x) - \mu \sum_{i=1}^{m} \log(w_i) + \tfrac{\beta}{2}\|h(x) - w\|_2^2.$$

Define the dual normal matrix:

$$N(x,y,w) = H(x,y) + A^T(x) W^{-1} Y A(x).$$

Theorem 1 *Suppose that $N(x,y,w)$ is positive definite.*

1. *For β sufficiently large, $(\Delta x, \Delta w)$ is a descent direction for $\Psi_{\beta,\mu}$.*
2. *If current solution is primal feasible, then $(\Delta x, \Delta w)$ is a descent direction for the barrier function.*

Note: minimum required value for β is easy to compute.

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Modifications for NonConvex Optimization

Nonconvex Optimization: Diagonal Perturbation

- If $H(x, y)$ is not positive semidefinite then $N(x, y, w)$ might fail to be positive definite.
- In such a case, we lose the descent properties given in previous theorem.
- To regain those properties, we perturb the Hessian: $\tilde{H}(x, y) = H(x, y) + \lambda I$.
- And compute search directions using $\tilde{H}$ instead of H.

Notation: let $\tilde{N}$ denote the dual normal matrix associated with $\tilde{H}$.

Theorem 2 *If $\tilde{N}$ is positive definite, then $(\Delta x, \Delta w, \Delta y)$ is a descent direction for*

1. *the primal infeasibility, $\|h(x) - w\|$;*
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Nonconvex Optimization: Jamming

Theorem 3 *If the problem is convex and and the current solution is not optimal and ..., then for any slack variable, say w_i, we have $w_i = 0$ implies $\Delta w_i \geq 0$.*

- To paraphrase: for convex problems, as slack variables get small they tend to get large again. This is an antijamming theorem.
- A recent example of Wächter and Biegler shows that for nonconvex problems, jamming really can occur.
- Recent modification:
  - if a slack variable gets small and
  - its component of the step direction contributes to making a very short step,
  - then increase this slack variable to the average size of the variables the "mainstream" slack variables.
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Nonconvex Optimization: Jamming

Theorem 3 *If the problem is convex and and the current solution is not optimal and ..., then for any slack variable, say* w_i*, we have* $w_i = 0$ *implies* $\Delta w_i \geq 0$*.*

- To paraphrase: for convex problems, as slack variables get small they tend to get large again. This is an antijamming theorem.
- A recent example of Wächter and Biegler shows that for nonconvex problems, jamming really can occur.
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13 Modifications for General Problem Formulations

- Bounds, ranges, and free variables are all treated implicitly as described in Linear Programming: Foundations and Extensions (LP:F&E).
- Net result is following reduced KKT system:

$$\begin{bmatrix} -(H(x,y)+D) & A^T(x) \\ A(x) & E \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} = \begin{bmatrix} \Phi_1 \\ \Phi_2 \end{bmatrix}$$

- Here, D and E are positive definite diagonal matrices.
- Note that D helps reduce frequency of diagonal perturbation.
- Choice of barrier parameter μ and initial solution, if none is provided, is described in the paper.
- Stopping rules, matrix reordering heuristics, etc. are as described in LP:F&E.

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14 Computational Results

Compared:

- LOQO version 5.03 (20000528)
- SNOPT version 5.3-2 (Sep 1999), driver 19981124
- NITRO last week's version

Input/output interface: AMPL with Hessian info (recent enhancement due to hard work of David Gay).

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The CUTE/Schittkowski Set—Efficiency and Robustness

Size	Probs		SNOPT	NITRO	LOQO	LOQO tuned
Small	557	Num solved	535	476	532	16
$n < 100$		Tot time	87.22	191.99	69.32	9.21
Medium	63	Num solved	53	52	51	8
$n < 1000$		Tot time	551.46	690.33	254.47	189.49
Large	76	Num solved	38	60	71	1
$n < 10000$		Tot time	16032.99	13786.36	7357.81	0.63
Very Large	65	Num solved	11	13	49	12
$10000 \leq n$		Tot time	246026.22	249.71	49986.83	2744.93

16 The CUTE/Schittkowski Set—The Largest Ones

Out of the 65 very large problems, we looked at those which both LOQO and SNOPT solved. There were 8 such problems. From this 8, we have the following total times:

Number	SNOPT	LOQO
8	226844.88	1127.01

We also looked at those which both LOQO and NITRO solved. There were 13 such problems. From this 13, we have the following total times:

Number	NITRO	LOQO
13	249.71	240.11

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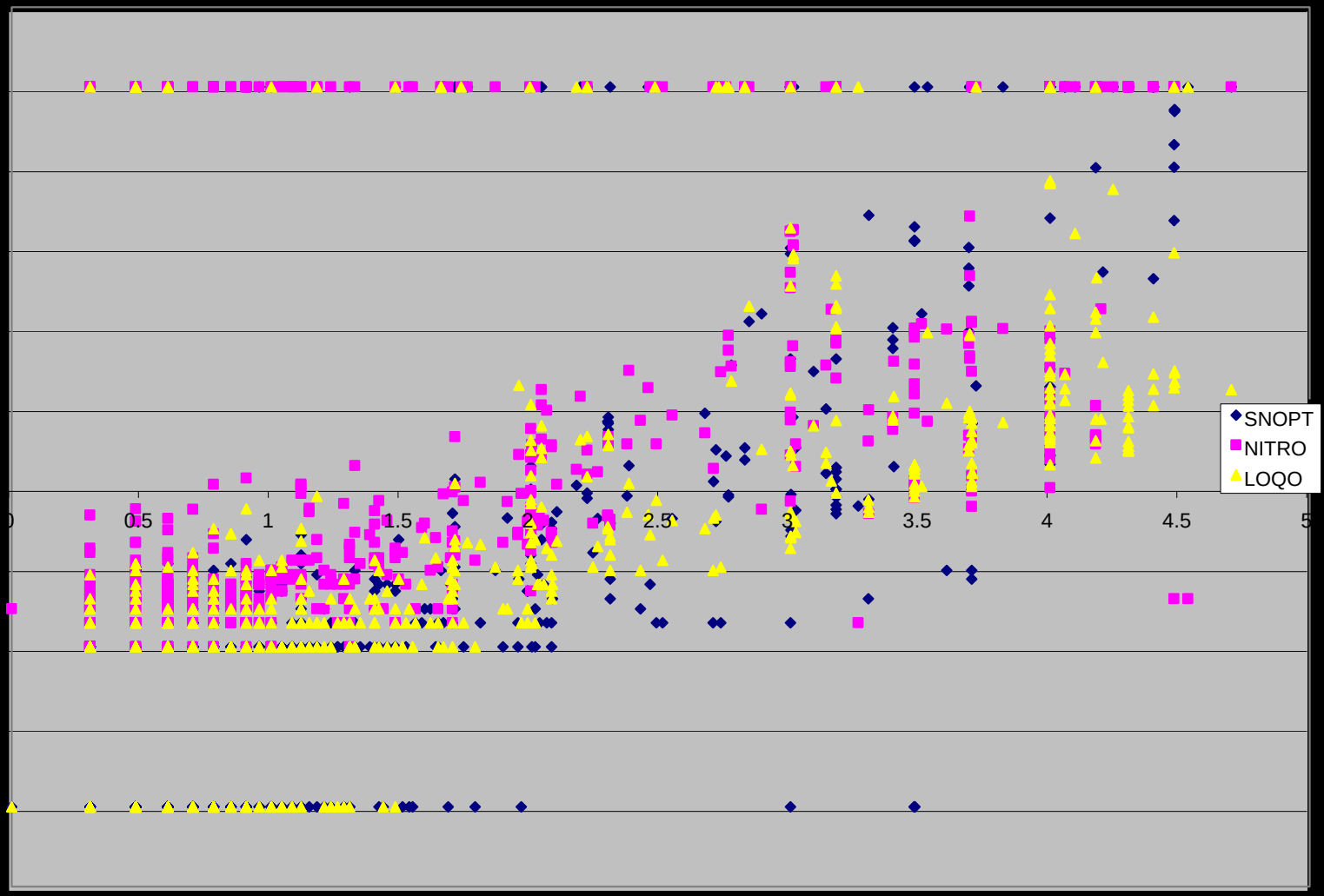
17 Log-Log Plot of Time vs. Size



18 LOQO Failures

We have identified 5 issues that account for almost all of the LOQO failures:

- Failure to satisfy a constraint qualification. This means no KKT point.
- Unbounded set of optimal solutions. Interior-point methods go to the analytic center, which lies at infinity if the set is unbounded.
- Infeasible or unbounded problem. In nonconvex nonlinear programming, this is harder to detect than in LP.
- Nondifferentiability. If one of the nonlinear functions is not differentiable then one should anticipate trouble, especially if the nondifferentiability happens at the optimal solution.
- Bad Scaling. What can I say, shoot the modeler?

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19 Nondifferentiability—s332.mod

In the Schittkowski model s332.mod, the following expressions appear:

$$\begin{aligned} y_i &= \arctan(\left|(1/t_i - x_1)/(\ln t_i + x_2)\right|), \\ p &= \max y_i, \\ p &\leq 30. \end{aligned}$$

Since $\arctan|x| = |\arctan x|$, the displayed expressions above are equivalent to

$$-30 \leq \arctan((1/t_i - x_1)/(\ln t_i + x_2)) \leq 30.$$

- This system is clearly differentiable, while the original system is not.
- All three codes failed on the problem as initially posed.
- But when restated, all solved the problem easily.
- SNOPT in .04 secs, NITRO in .53 secs, and LOQO in .1 secs.

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As we saw, LOQO solves most problems with the defaults but tuning did help in some cases. We only adjusted 5 tuning parameters. They are:

- bndpush. Most problems specify initial values for the variables. But, of course, they don't tell us how to initialize the slack variables that we add. `bndpush` gives a lower bound on the initial values for these slacks.
- sigfig. By default, LOQO asks for 8 digits of agreement between the primal and dual objective functions. For some problems this is just too much to expect.
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20 Tuning

As we saw, LOQO solves most problems with the defaults but tuning did help in some cases. We only adjusted 5 tuning parameters. They are:

- bndpush. Most problems specify initial values for the variables. But, of course, they don't tell us how to initialize the slack variables that we add. `bndpush` gives a lower bound on the initial values for these slacks.
- sigfig. By default, LOQO asks for 8 digits of agreement between the primal and dual objective functions. For some problems this is just too much to expect.
- inftol. By default, LOQO asks for relative infeasibilities to be less than `1.0e-6` before declaring a primal or dual solution feasible. Again, sometimes this is too much to ask for.
- iterlim. By default, LOQO stops after a max of 200 iterations. Sometimes letting it go longer is a good thing.
- convex. Some problems are known to be convex. Asserting that that is the case sometimes helps.