

Consider the equation for the trajectory of a particle along a circle of radius r centered at (a, b) :

$$(x_t - a)^2 + (y_t - b)^2 = r^2. \quad (1)$$

There are three unknowns: a , b , and r . To compute these unknowns from the path, we need two more equations. Let's differentiate (and divide by 2):

$$(x_t - a)\dot{x}_t + (y_t - b)\dot{y}_t = 0. \quad (2)$$

And one more time:

$$(x_t - a)\ddot{x}_t + (y_t - b)\ddot{y}_t + \dot{x}_t^2 + \dot{y}_t^2 = 0. \quad (3)$$

We want to solve for the three unknowns in terms of the values and derivatives of the trajectory at time t . First, we solve (2) for $y_t - b$ in terms of $x_t - a$:

$$y_t - b = -(x_t - a) \frac{\dot{x}_t}{\dot{y}_t} \quad (4)$$

Next, we substitute this formula into (1) and solve for $x_t - a$:

$$\begin{aligned} x_t - a &= \frac{r}{\sqrt{1 + (\dot{x}_t/\dot{y}_t)^2}} \\ &= \frac{\dot{y}_t r}{\sqrt{\dot{x}_t^2 + \dot{y}_t^2}} \end{aligned}$$

From this, we see that

$$y_t - b = -\frac{\dot{x}_t r}{\sqrt{\dot{x}_t^2 + \dot{y}_t^2}} \quad (5)$$

We then plug these formulas for $x_t - a$ and $y_t - b$ into (3) to get an equation involving only unknown r :

$$\frac{\dot{y}_t r \ddot{x}_t}{\sqrt{\dot{x}_t^2 + \dot{y}_t^2}} - \frac{\dot{x}_t r \ddot{y}_t}{\sqrt{\dot{x}_t^2 + \dot{y}_t^2}} + \dot{x}_t^2 + \dot{y}_t^2 = 0.$$

Solving for r and squaring both sides, we get

$$r^2 = \frac{(\dot{x}_t^2 + \dot{y}_t^2)^3}{(\dot{y}_t \dot{x}_t - \dot{x}_t \dot{y}_t)^2}.$$