Hang Glider: Range Maximization

The problem is to compute the flight inputs to a hang glider so as to provide a maximum range flight. This problem first appears in [1].

The hang glider has weight W (glider plus pilot), a lift force L acting perpendicular to its velocity v_r relative to the air, and a drag force D acting in a direction opposite to v_r. Denote by x the horizontal position of the glider, by v_x the horizontal component of the absolute velocity, by y the vertical position, and by v_y the vertical component of absolute velocity.

The airmass is not static: there is a thermal just 250 meters ahead. The profile of the thermal is given by the following upward wind velocity:

$$u_a(x) = u_m e^{-\left(\frac{x}{R}-2.5\right)^2} \left(1 - \left(\frac{x}{R} - 2.5\right)^2\right).$$

We take $R = 100$ m and $u_m = 2.5$ m/s. Note, MKS units are used throughout. The upwind profile is shown in Figure 1.

Letting η denote the angle between v_r and the horizontal plane, we have the following equations of motion:

$$\begin{aligned} \dot{x} &= v_x, & \dot{v}_x &= \tfrac{1}{m}(-L\sin\eta - D\cos\eta), \\ \dot{y} &= v_y, & \dot{v}_y &= \tfrac{1}{m}(L\cos\eta - D\sin\eta - W) \end{aligned}$$

with

$$\eta = \arctan\left(\frac{v_y - u_a(x)}{v_x}\right), \qquad v_r = \sqrt{v_x^2 + (v_y - u_a(x))^2},$$

$$L = \frac{1}{2}c_L \rho S v_r^2, \quad D = \frac{1}{2}c_D(c_L)\rho S v_r^2, \quad W = mg.$$

The glider is controlled by the lift coefficient c_L. The drag coefficient is assumed to depend on the lift coefficient as

$$c_D(c_L) = c_0 + k c_L^2$$

where $c_0 = 0.034$ and $k = 0.069662$. In addition, there is an upper limit on the lift coefficient:

$$c_L \le c_{L\,\max} := 1.4.$$

Other constants are:

$$\begin{aligned} m &= 100 && \text{mass of glider and pilot} \\ S &= 14 && \text{wing area} \\ \rho &= 1.13 && \text{air density} \\ g &= 9.81 && \text{acc due to gravity.} \end{aligned}$$

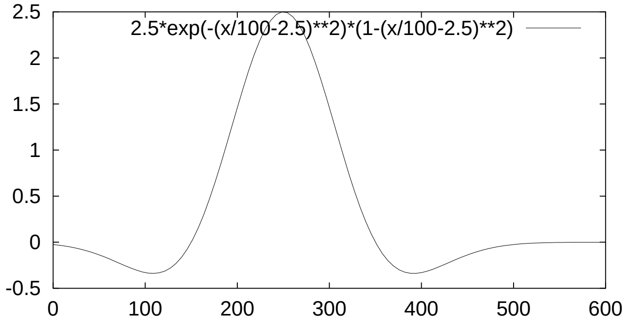

FIGURE 1. Updraft profile

The boundary conditions are:

$$
\begin{aligned}
x(0) &= 0, & & \\
y(0) &= 1000, & y(T) &= 900, \\
v_x(0) &= 13.23, & v_x(T) &= 13.23, \\
v_y(0) &= -1.288, & v_y(T) &= -1.288.
\end{aligned}
$$

The total time T for the flight is, of course, a variable. The objective is to maximize $x(T)$.

The optimal solution depicted in Figures 2–7 was obtained using a uniform discretization of the time domain into 150 discrete points. Derivatives were approximated by differences at the midpoints of each discrete time interval. The optimal range is 1248.26 m and takes 98.4665 s to fly.

REFERENCES

[1] Bulirsch, R., Nerz, E., Pesch, H. & von Stryk, O. (1993), Combining direct and indirect methods in optimal control: Range maximization of a hang glider, *in* R. Bulirsch, A. Miele, J. Stoer & K. Well, eds, ‘”Optimal Control: Calculus of Variations, Optimal Control Theory and Numerical Methods’, Birkhauser Verlag, Basel, Boston, Berlin, pp. 273–288. 1

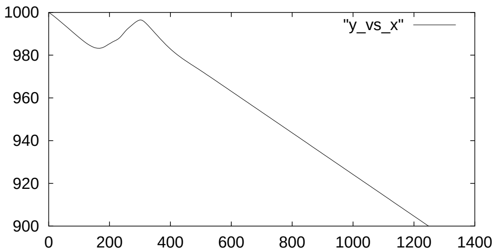

FIGURE 2. y vs x

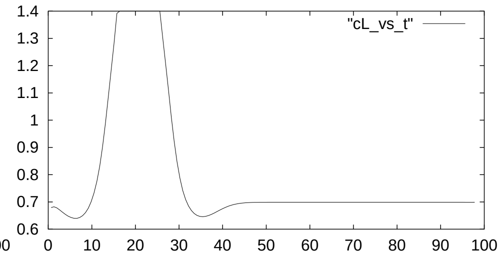

FIGURE 5. cL vs t

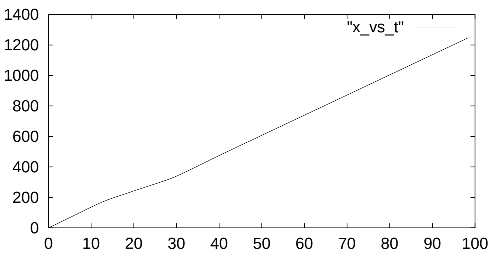

FIGURE 3. x vs t

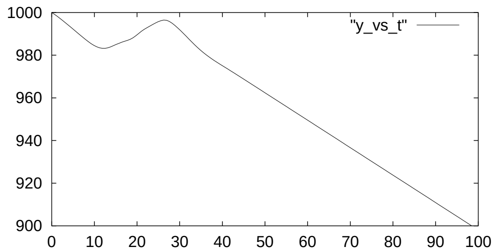

FIGURE 6. y vs t

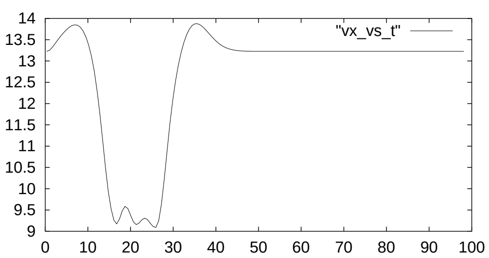

FIGURE 4. v_x vs t

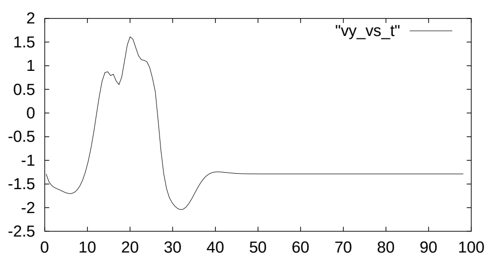

FIGURE 7. v_y vs t