(1) Consider the following dictionary which arises in solving a problem using the primal–dual one–phase simplex method:

$$\begin{array}{rcrcrcrcr}
\zeta & = & -3 & & & - & (-1+2\mu)x_1 & - & (3-\mu)x_3 \\
\hline
x_2 & = & -1 & + & \mu & + & x_1 & - & x_3 \\
x_4 & = & -4 & + & 3\mu & + & 3x_1 & - & 2x_3 \\
x_5 & = & 2 & & & + & x_1 & + & x_3
\end{array} .$$

(a) For which values of μ is the current dictionary optimal?
(b) For the next pivot in the primal–dual one–phase simplex method, identify the entering and the leaving variable.

(2) Consider a linear programming problem that has an optimal dictionary in which exactly k of the original slack variables are nonbasic. Show that, by ignoring feasibility preservation of intermediate dictionaries, this dictionary can be arrived at in exactly k pivots. Hint: Don't forget to allow for the fact that some pivot elements might be zero.