

Robert J. Vanderbei

# The Earth Is Not Flat

## An Analysis of a Sunset Photo

Can a photo of the sunset  
over Lake Michigan reveal the  
shape of our planet?



I will show you how we can...

measure something *BIG* (the size of the Earth)

by first measuring something *small* (my height),  
and measuring an *angle* (off from a photograph)

and then doing some *geometry*.



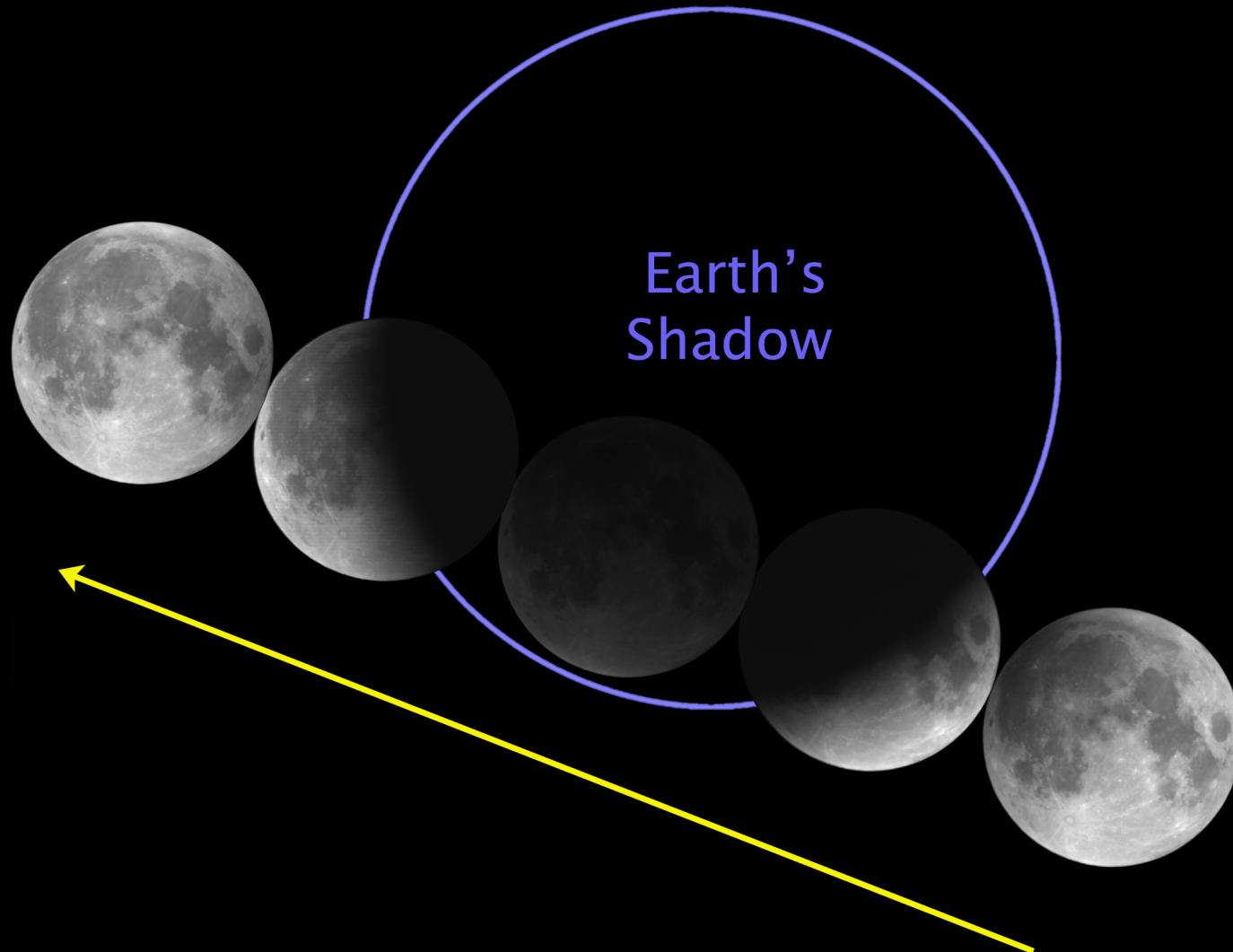
The Earth is a big sphere. How do we know?  
Several ways. One way is to look at a Lunar eclipse...



Photo taken March 3, 2007, at about 8pm.  
Check out what astronomer Fritz Zwicky has to say about it:  
click [here](#) and search for “bastard”



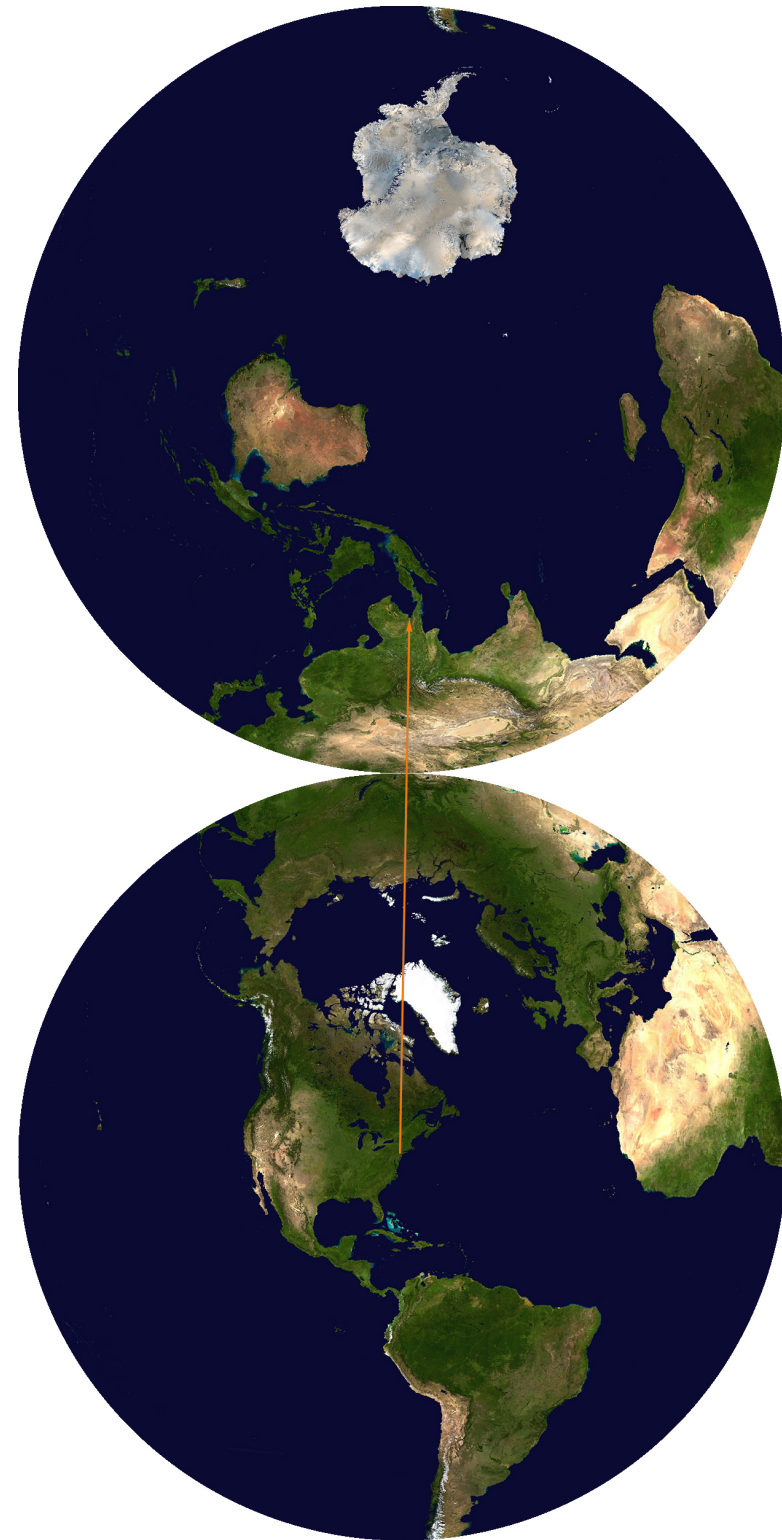
From a lunar eclipse, we can determine that the Earth is about 3 or 4 times larger than the Moon. But, how big is the Earth?



Next total lunar eclipse (barely) visible from here in NJ is at 6am on March 3, 2026.

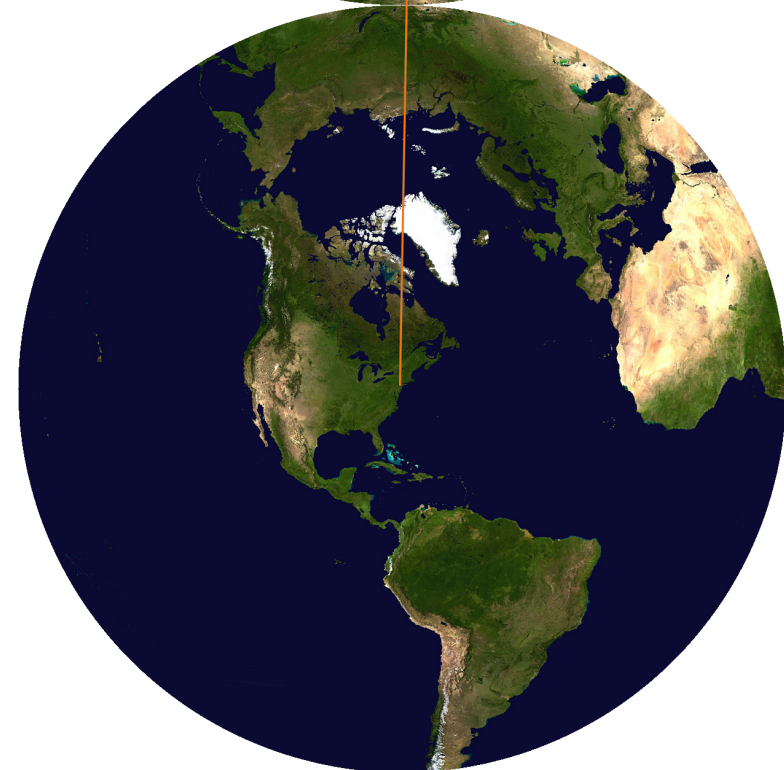
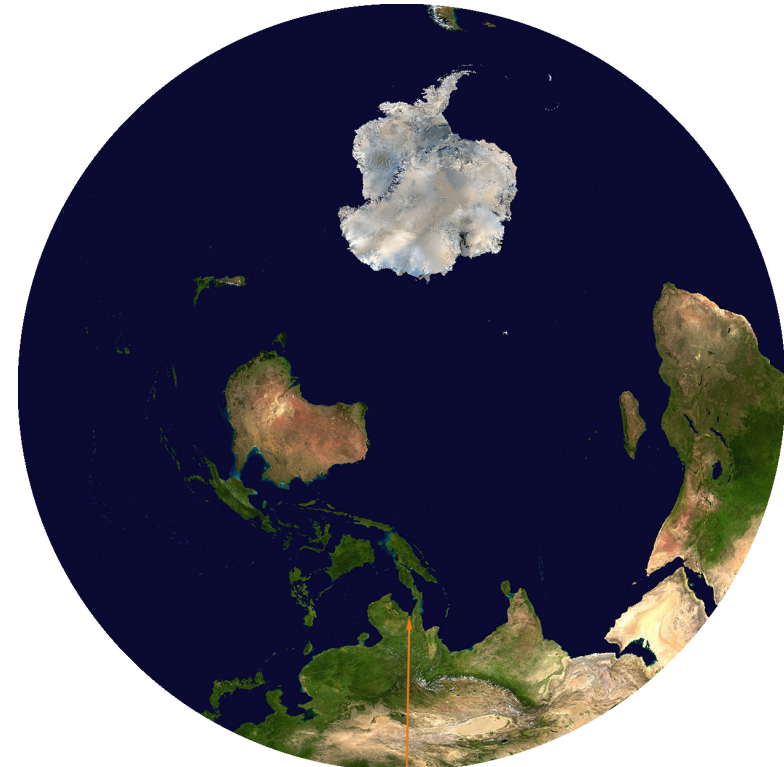


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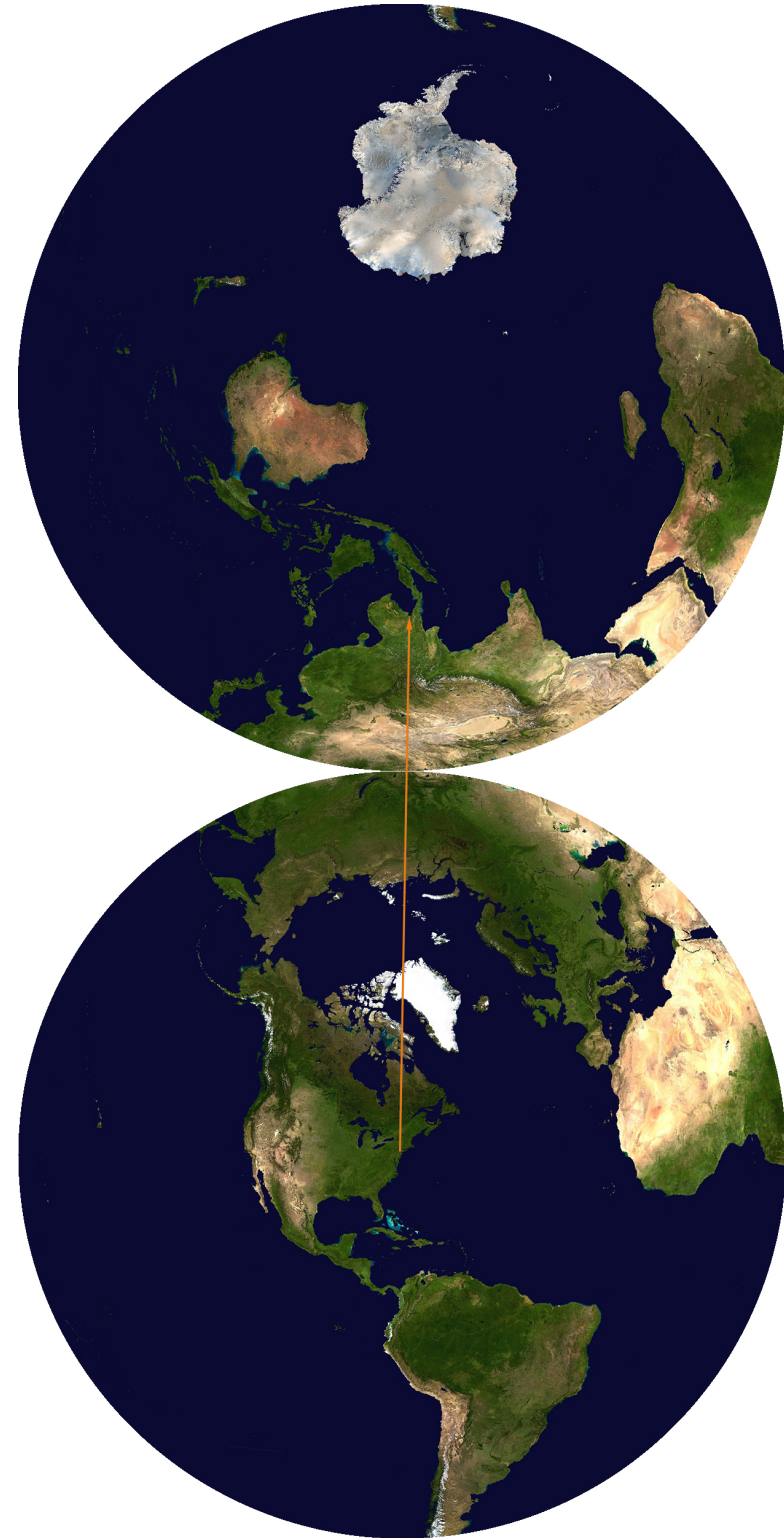


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You'll get the right answer:

radius = 3,960 miles.

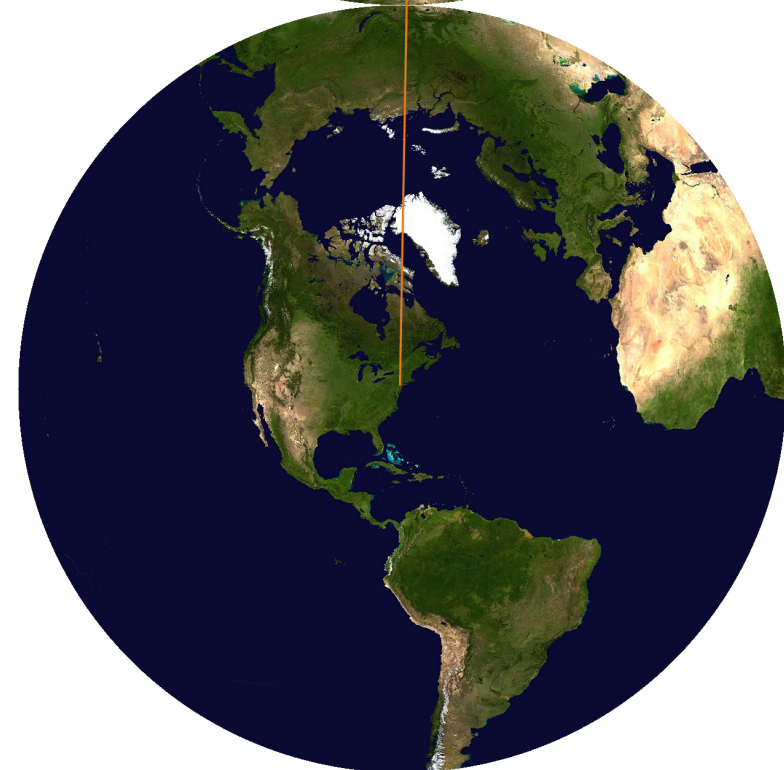


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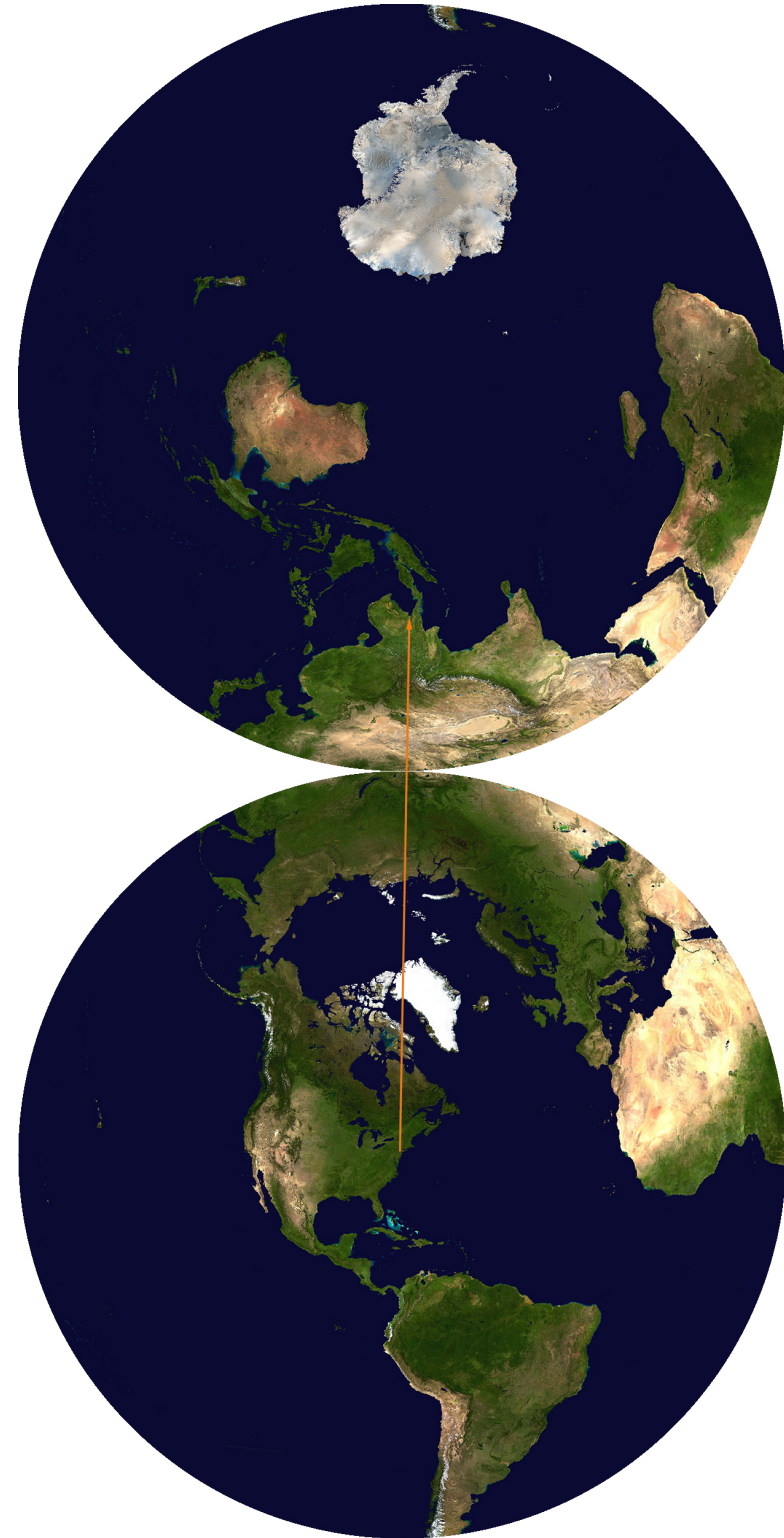
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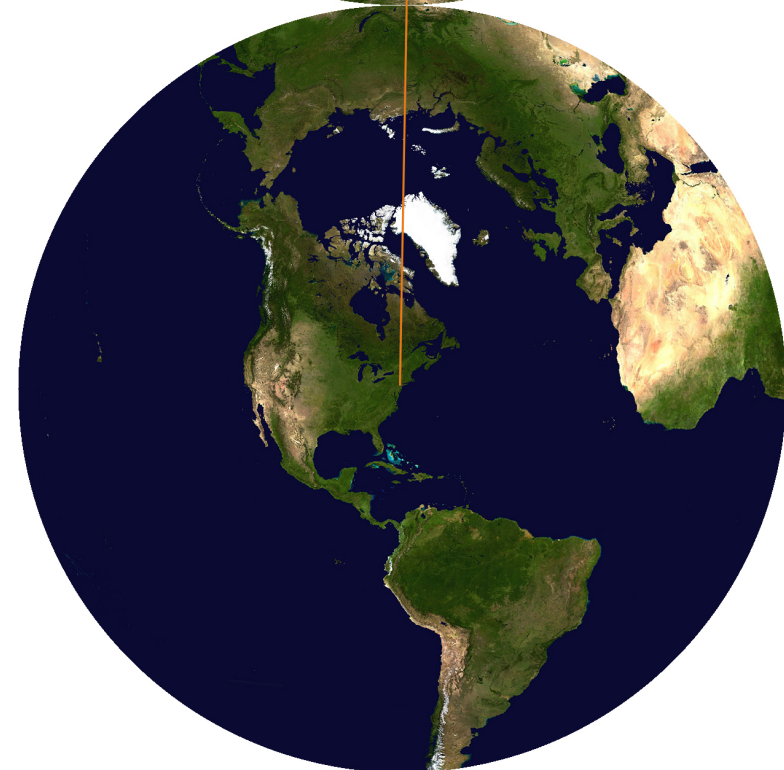
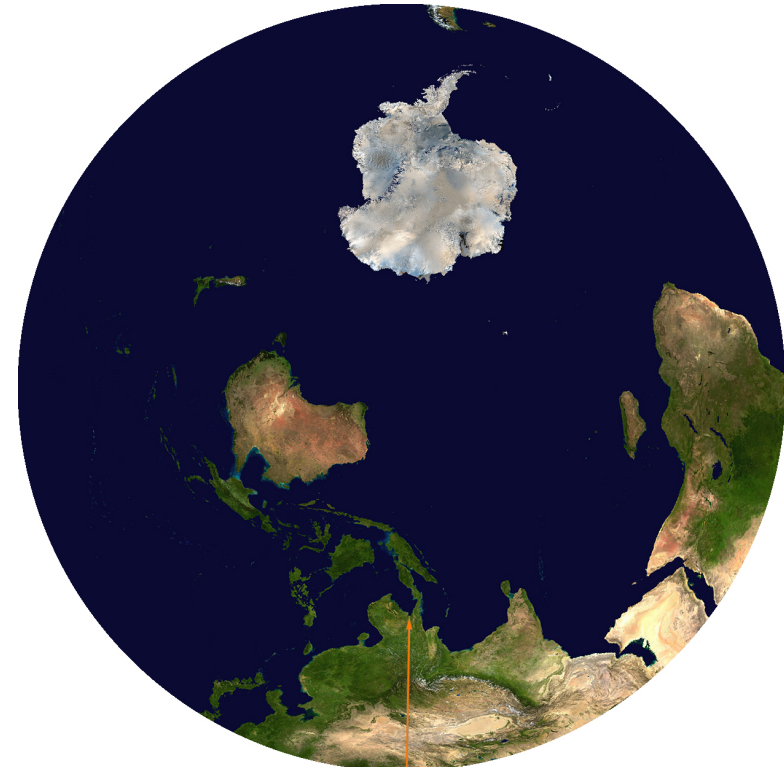
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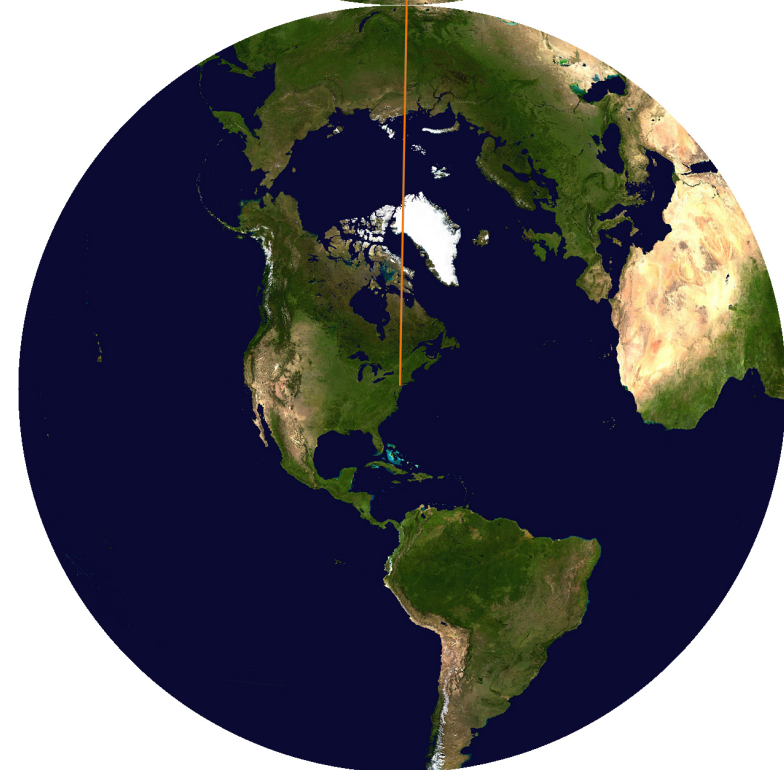
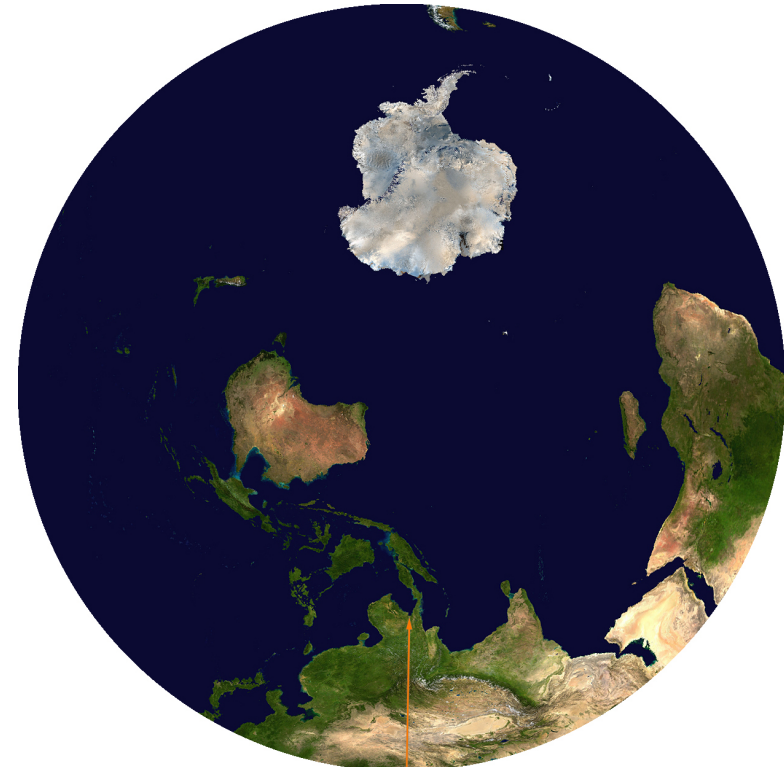
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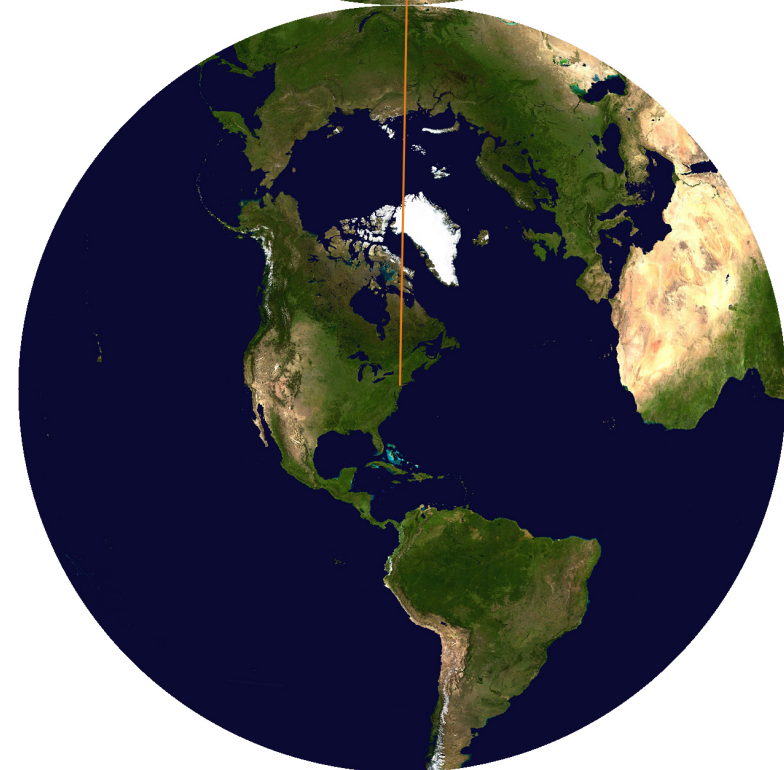
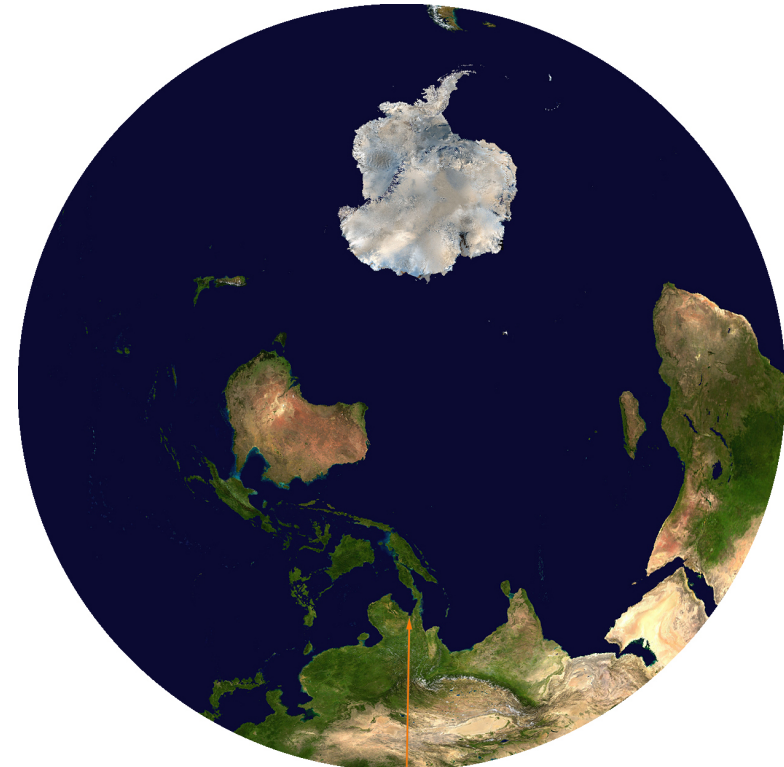
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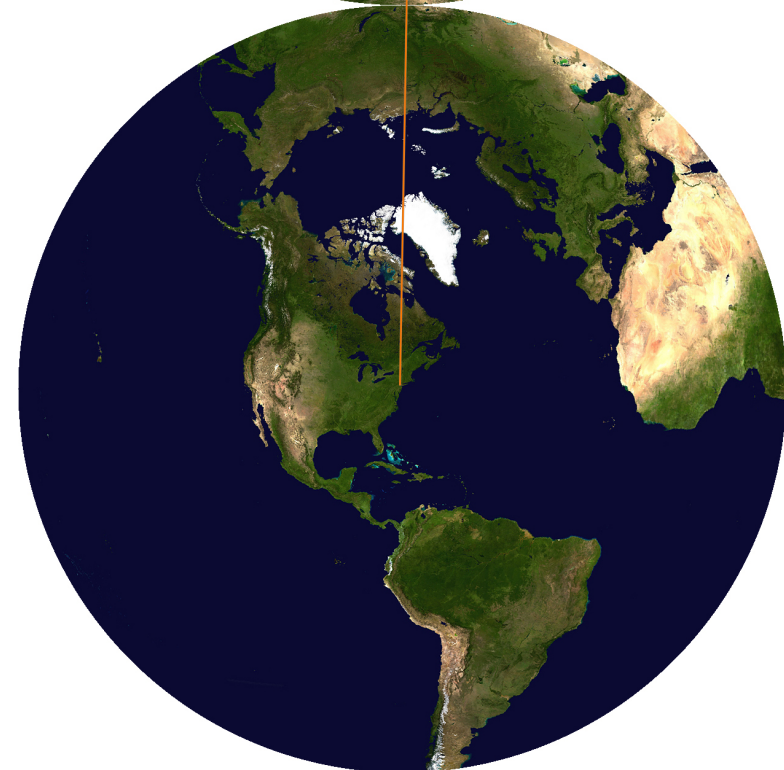
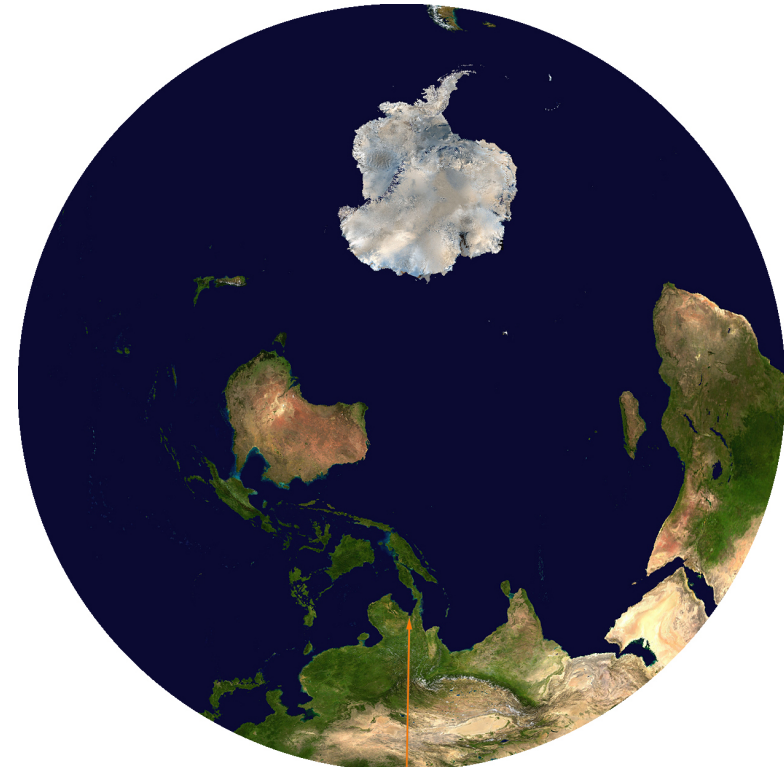
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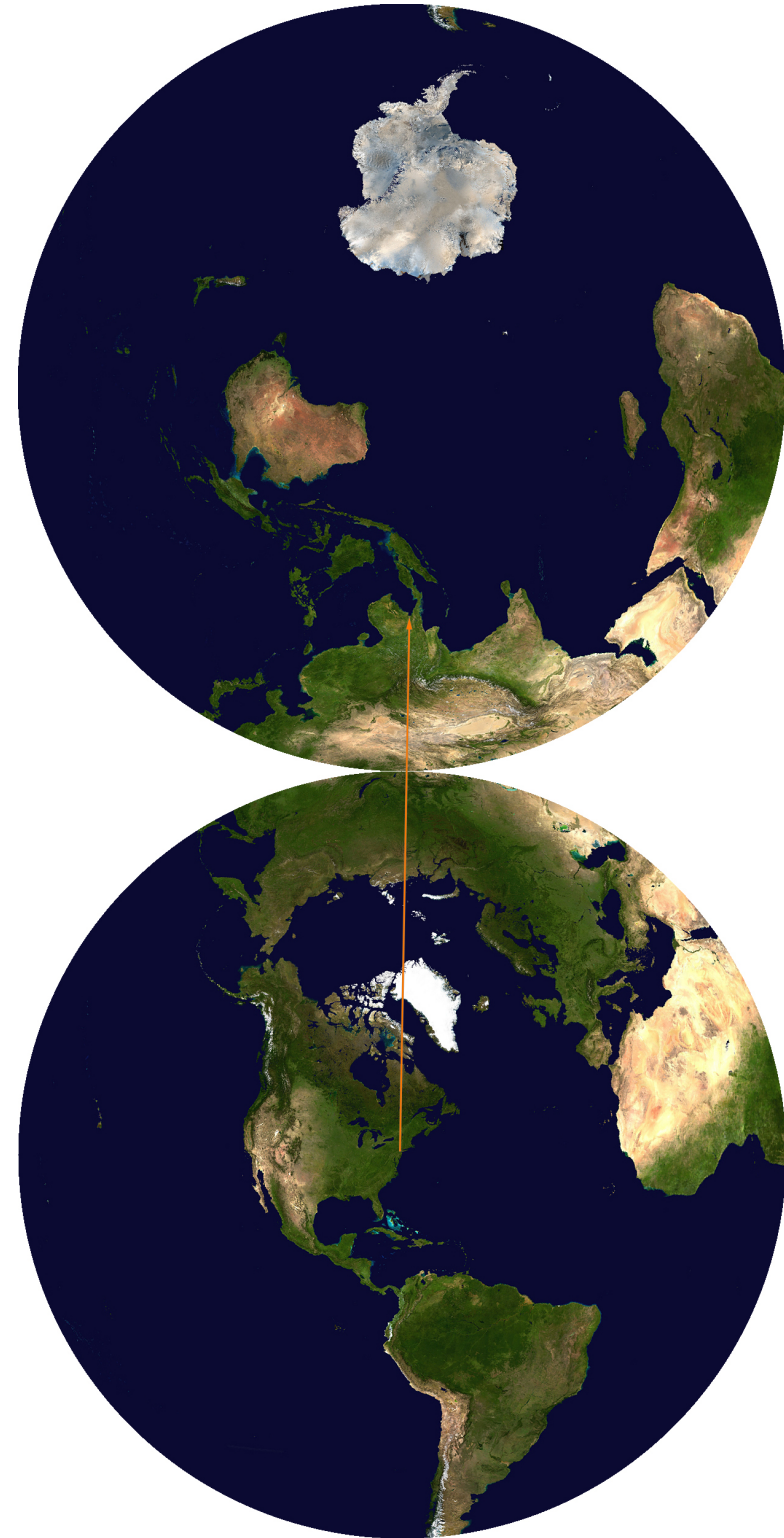
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So, the distance I flew is *about*

$17 \text{ hours} \times 600 \text{ miles/hour} = 10,200 \text{ miles.}$



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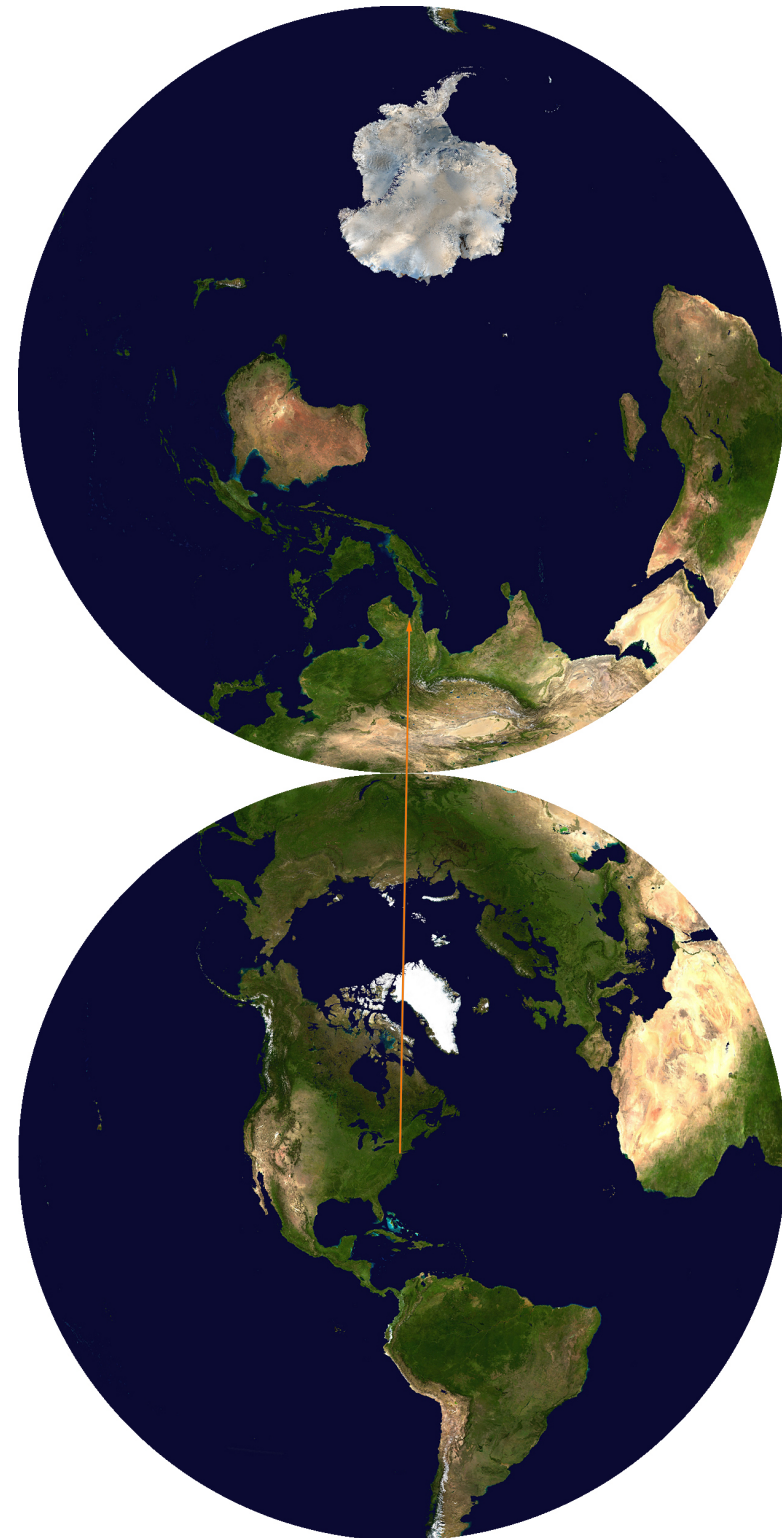
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Bangkok's latitude is about  $10^\circ$  north and NJ is about  $40^\circ$  north. So, the trip wasn't halfway around. It was just  $130/360 \approx 0.36$  around. So, the circumference is then about  $10,200/0.36 \approx 28,000$  miles and the radius is about  $28,000/2\pi = 4,500$  miles.



IS THERE AN EASIER WAY?

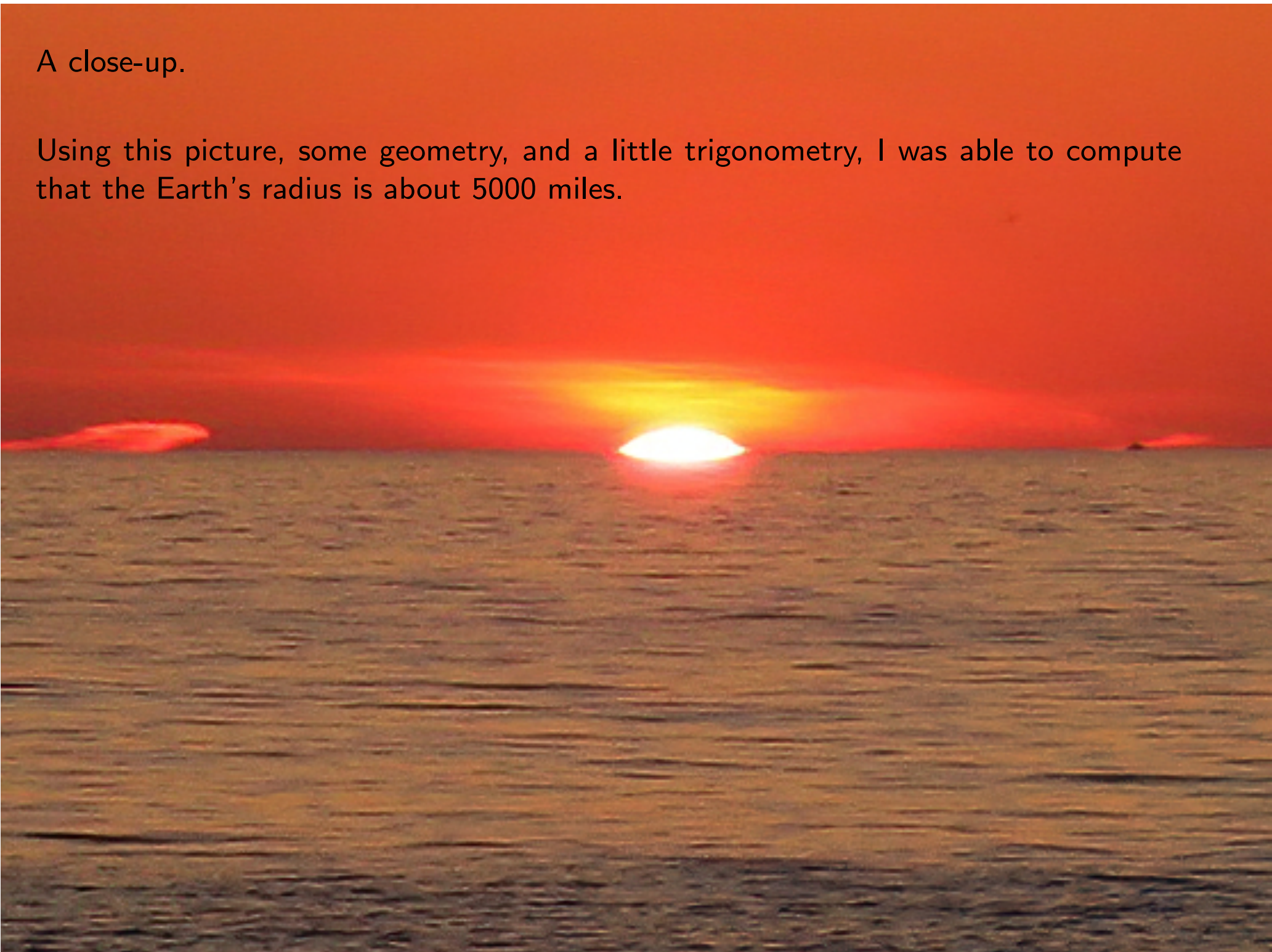


A picture I took of a sunset over Lake Michigan.



A close-up.

Using this picture, some geometry, and a little trigonometry, I was able to compute that the Earth's radius is about 5000 miles.

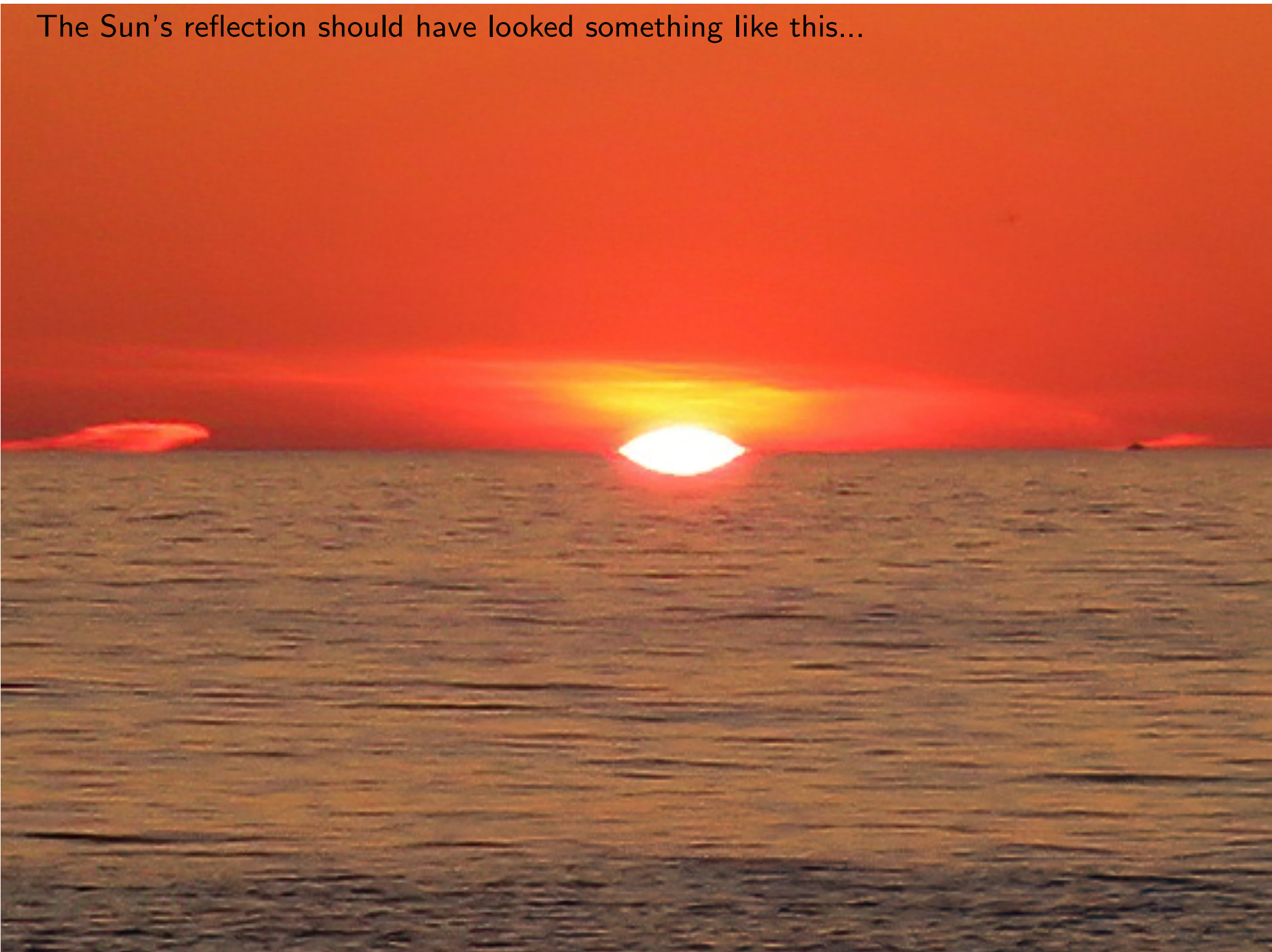


A smooth lake is supposed to act like a mirror.



Credit: Lorene Lavora 

The Sun's reflection should have looked something like this...



Or not!

What's going on?



Lake Michigan is not a flat mirror.



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Its surface is curved  
because the Earth is a  
sphere.



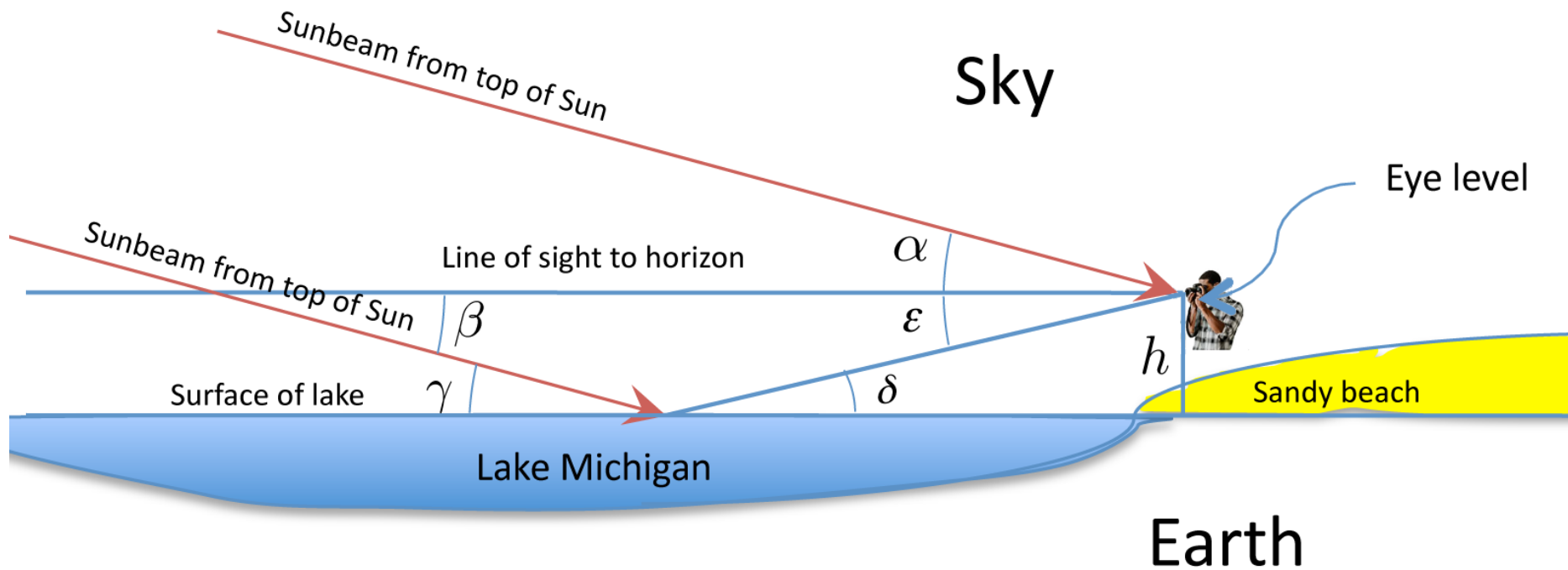
Lake Michigan is not a flat mirror.

Its surface is curved because the Earth is a sphere.

That's why we can't see the shore on the opposite side—it's below the horizon!



# Geometry — If the Earth Were Flat!



- $\alpha = \beta$  alternate interior angles are equal
- $\beta = \gamma$  alternate interior angles are equal
- $\gamma = \delta$  angle of incidence equals angle of reflection (from Physics!)
- $\delta = \epsilon$  alternate interior angles are equal

Therefore,

$$\alpha = \epsilon.$$

The reflection dips just as far below the horizon as the Sun stands above the horizon.



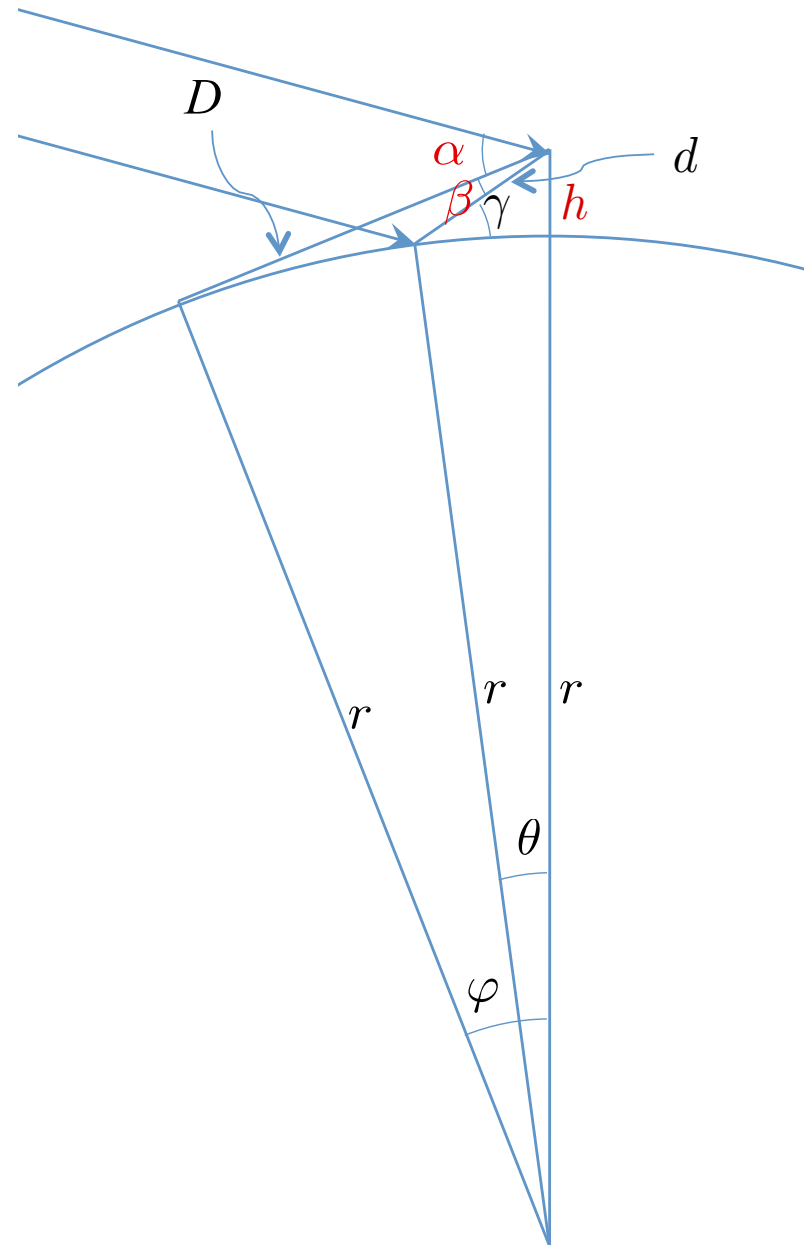
# Geometry — The Earth Is Not Flat!

Draw a picture.

Label everything of possible relevance.

Identify what we know:

- $\alpha$  Angle between horizon and top of Sun (measured from photo).
- $\beta$  Angle between horizon and “top” of Sun in reflection (measured from photo).
- $h$  Height of “eye-level” above “water-level”.



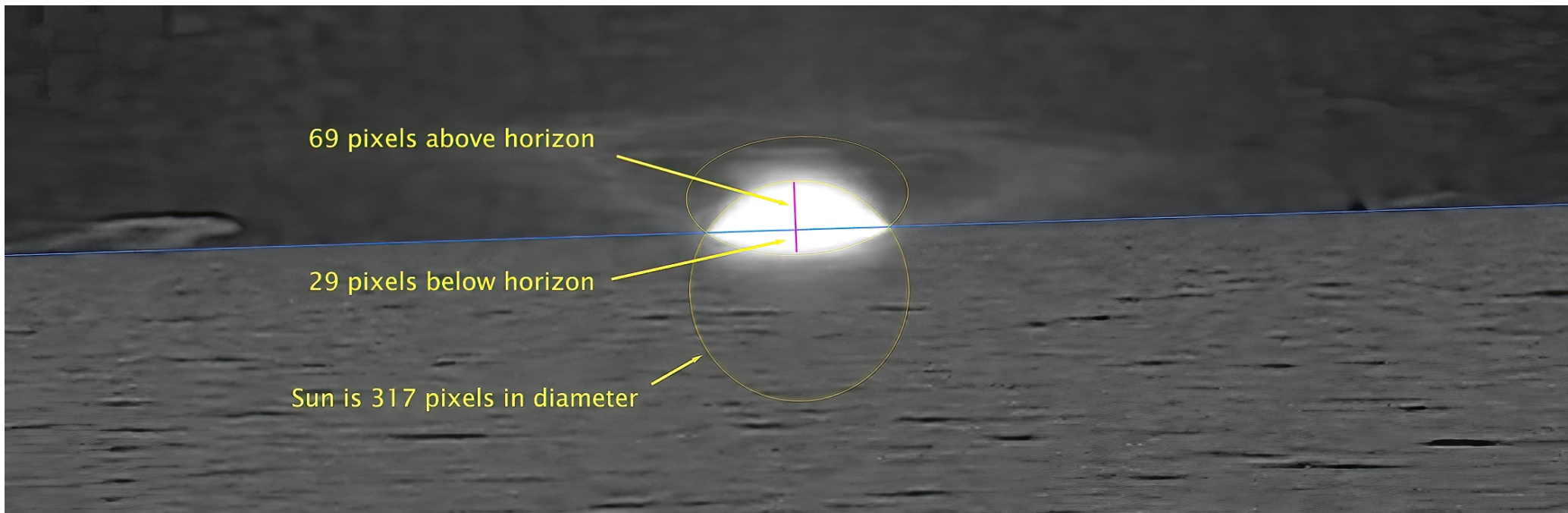








## What We Know (Measure!)



The Sun is  $1/2^\circ$  in diameter. Therefore,  $1^\circ$  equals  $2 \times 317 = 634$  pixels.  
And so,

$$\alpha = 69 \text{ pixels} \times \frac{1 \text{ degree}}{634 \text{ pixels}} = 0.1088 \text{ degrees}$$

and

$$\beta = 29 \text{ pixels} \times \frac{1 \text{ degree}}{634 \text{ pixels}} = 0.0457 \text{ degrees.}$$

And, we assume that eye level is

$$h = 7 \text{ feet}$$



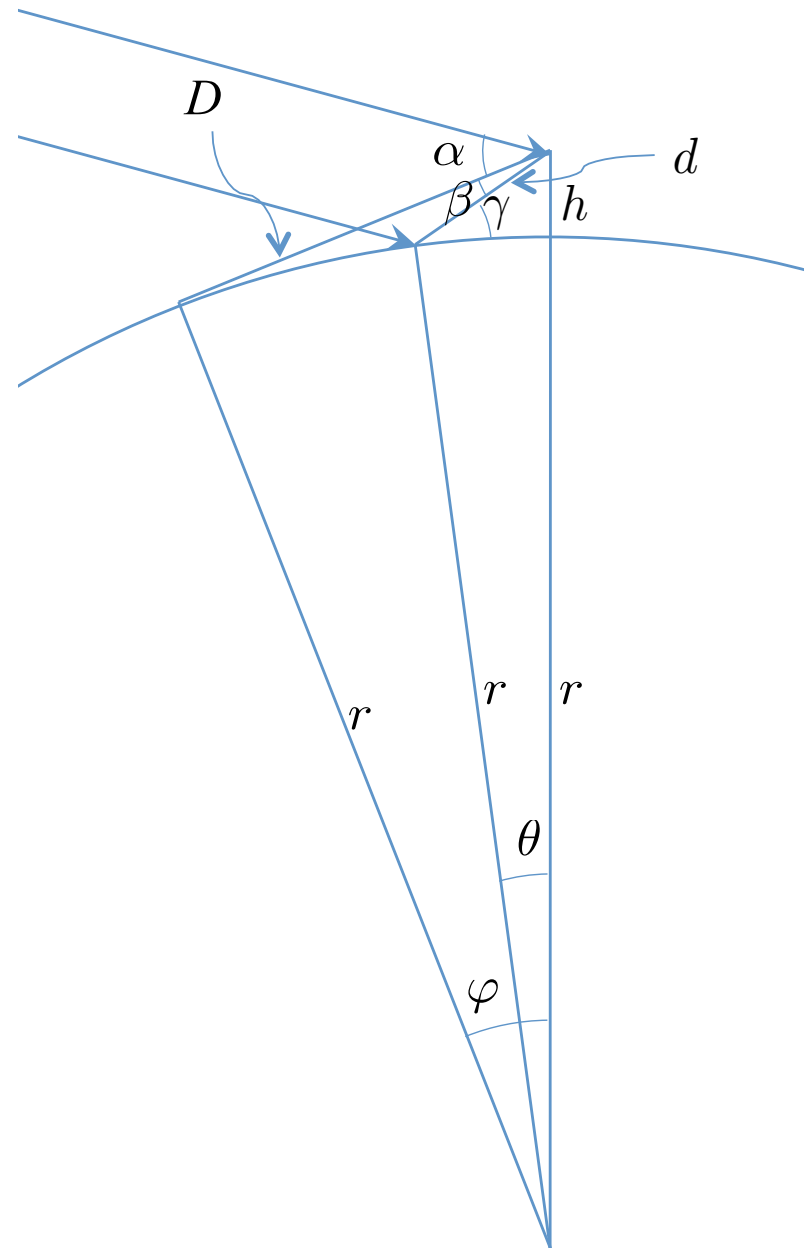
What We Need To Figure Out:

Angles:

- $\gamma$  Angle of reflection off water.
- $\theta$  Angle between observer (me) and point of reflection.
- $\varphi$  Angle between observer (me) and point of horizon.

Distances (lengths):

- $d$  Distance to point of reflection.
- $D$  Distance to horizon.
- $r$  Radius of Earth.



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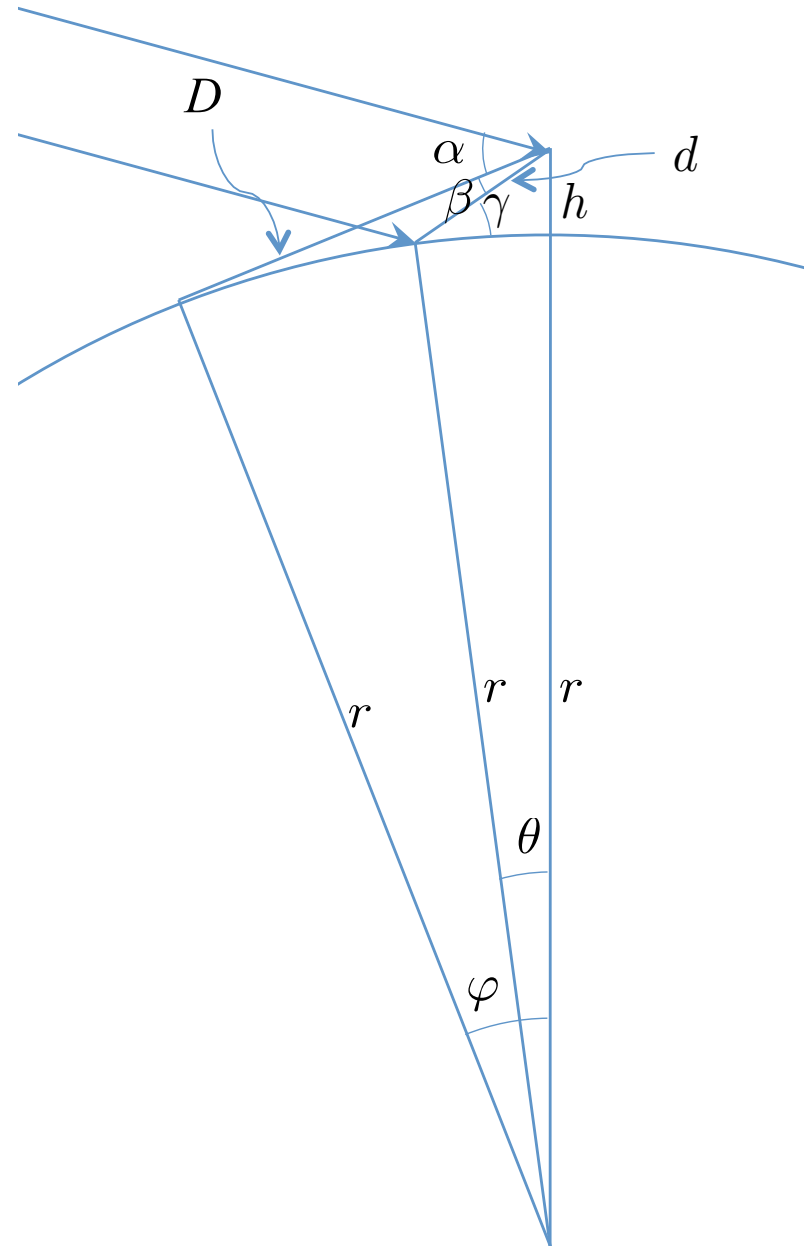
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- $r$  Radius of Earth. ← This one is key

That's SIX UNKNOWNs.

We need SIX EQUATIONS.



Equation 1:

The sum of the angles around a quadrilateral is  $360^\circ$ .

Hence,

$$(\varphi - \theta) + 90 + \beta + (270 - \gamma) = 360.$$

Simplifying, we get

$$\varphi + \beta = \theta + \gamma.$$

