Extreme Optics
and
The Search for Earth-Like Planets

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ORFE 522 — Fun (Pre-Thanksgiving) Lecture

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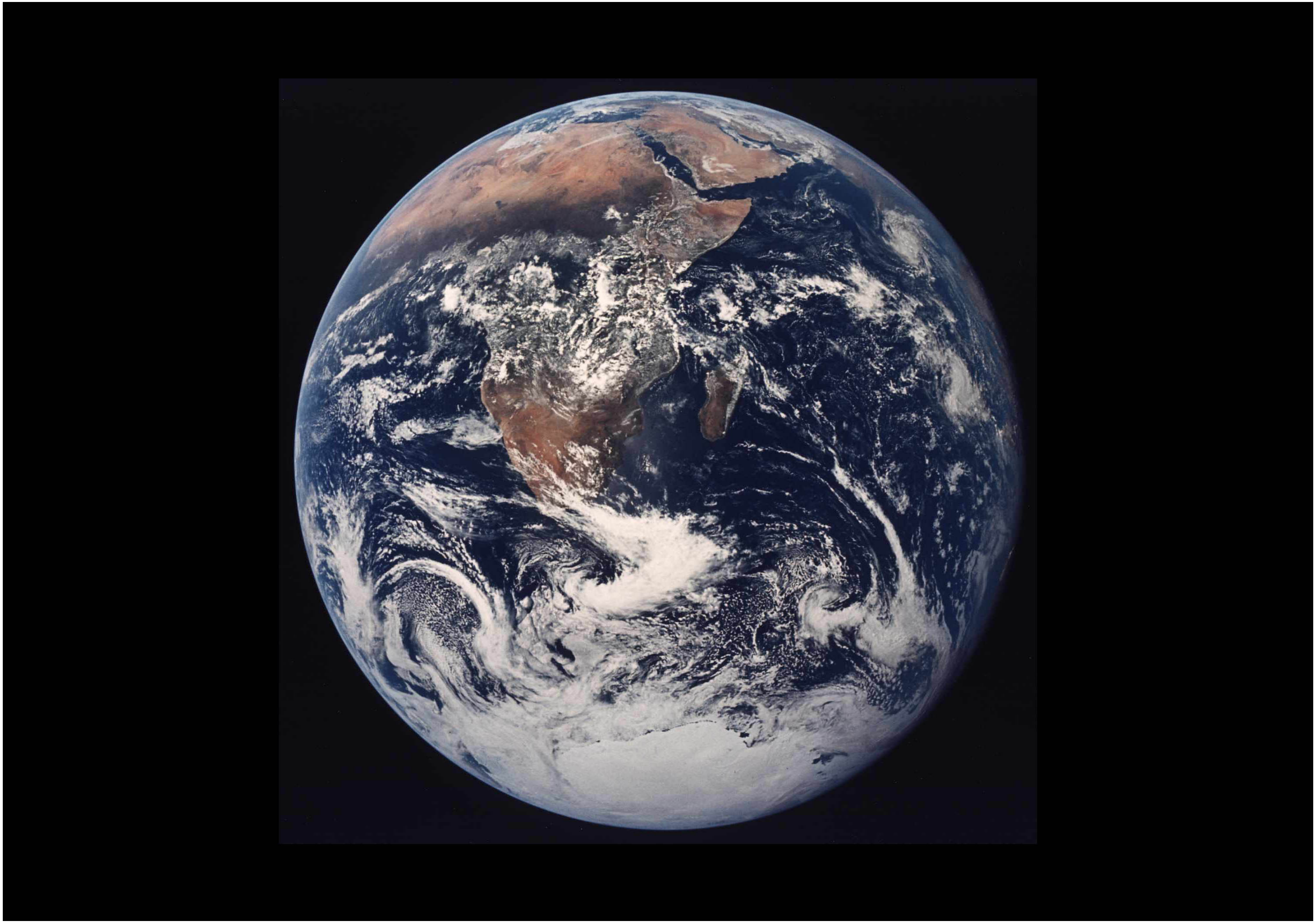
ABSTRACT

- NASA/JPL plans to build and launch a space telescope to look for Earth-like planets.

- I will describe the detection problem and explain why it is hard.

- Optimization is key to several design concepts.

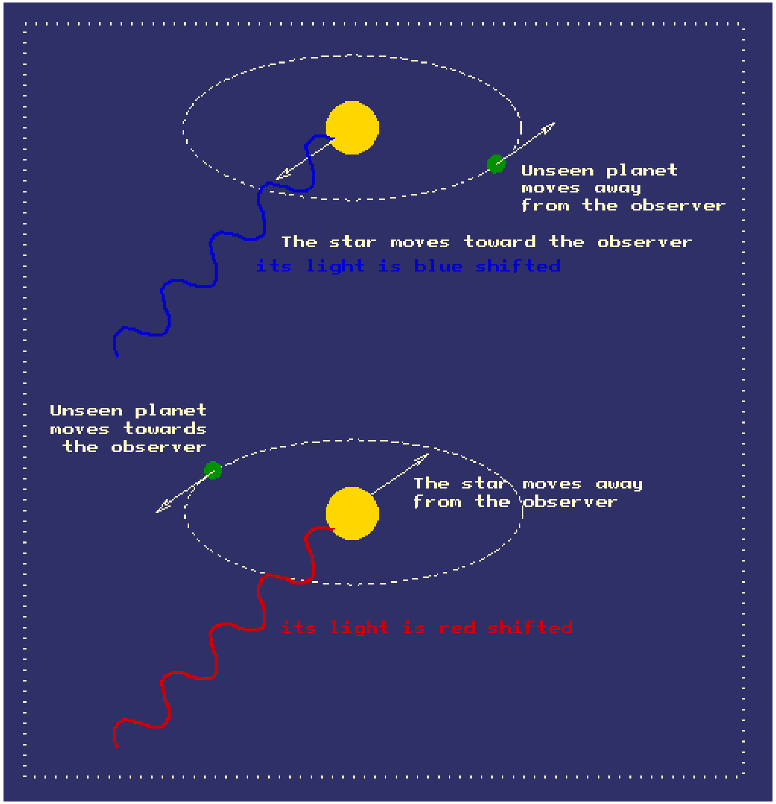
Are We Alone?



Wobble Methods

Radial Velocity.
For edge-on systems.
Measure periodic doppler shift.

Astrometry.
Best for face-on systems.
Measure circular wobble against background stars.



The Transit Method

A few planets have been discovered using the **Transit Method**.

On June 8, 2004, Venus transited in front of the Sun.

I took a picture of this event with my small telescope.

If we on Earth are lucky to be in the right position at the right time, we can detect similar transits of exosolar planets.

A few exosolar planets have been discovered this way.

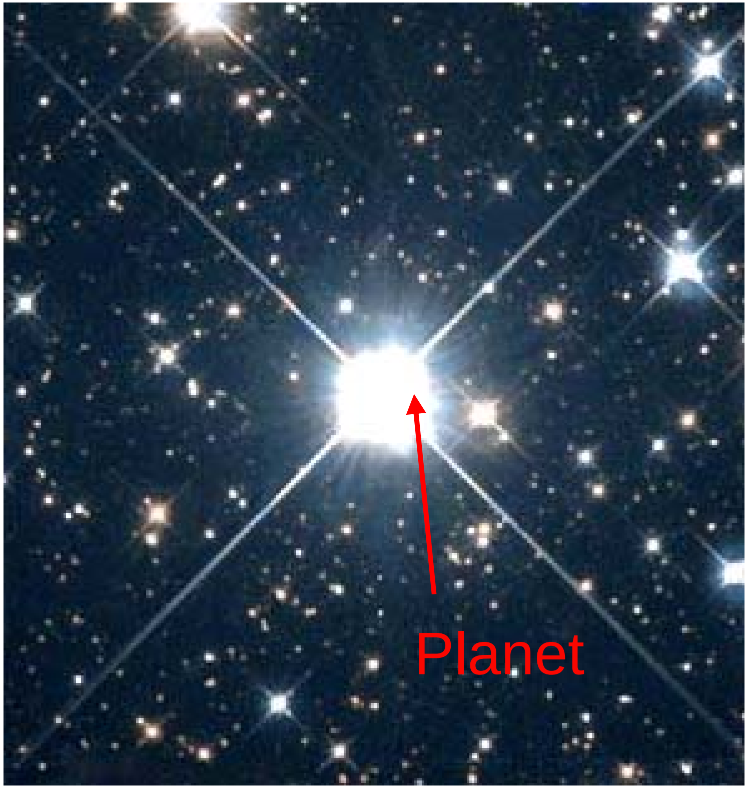
Terrestrial Planet Finder Telescope

- NASA/JPL space telescope.
- Launch date: ~~2014~~...well, sometime in our lifetime.
- DETECT: Search 150-500 nearby (5-15 pc distant) Sun-like stars for Earth-like planets.
- CHARACTERIZE: Determine basic physical properties and measure "biomarkers", indicators of life or conditions suitable to support it.

Why Is It Hard?

- **Contrast.** Star $= 10^{10} \times$ Planet

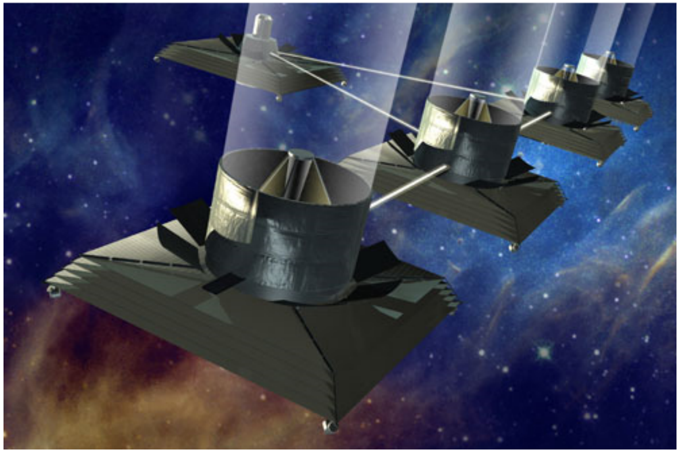
- **Angular Separation.** 0.1 arcseconds.



Early Design Concepts

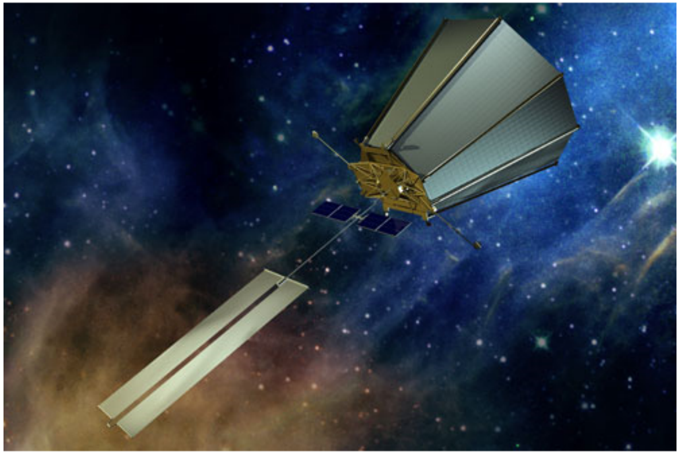
Space-based infrared nulling interferometer (TPF-I).

TPF-Interferometer



Visible-light telescope with an elliptical mirror (3.5 m x 8 m) and an **optimized** diffraction control system (TPF-C).

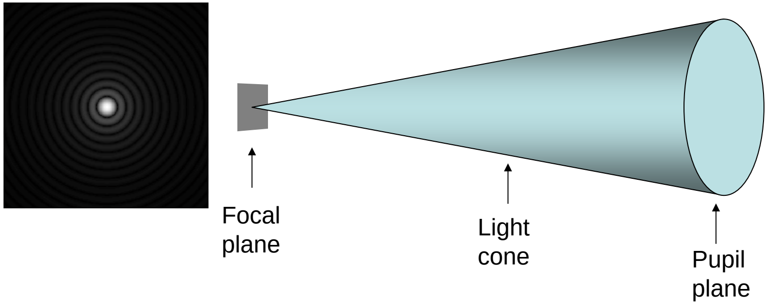
TPF-Coronagraph



Diffraction Control via Shaped Pupils

Consider a telescope. Light enters the front of the telescope—the **pupil plane**.

The telescope focuses the light passing through the pupil plane from a given direction at a certain point on the **focal plane**, say $(0, 0)$.



However, a point source produces not a point image but an **Airy pattern** consisting of an **Airy disk** surrounded by a system of **diffraction rings**.

These diffraction rings are too bright. The rings would completely hide the planet.

By placing a mask over the pupil, one can control the shape and strength of the diffraction rings. The problem is to find an optimal shape so as to put a very deep **null** very close to the Airy disk.

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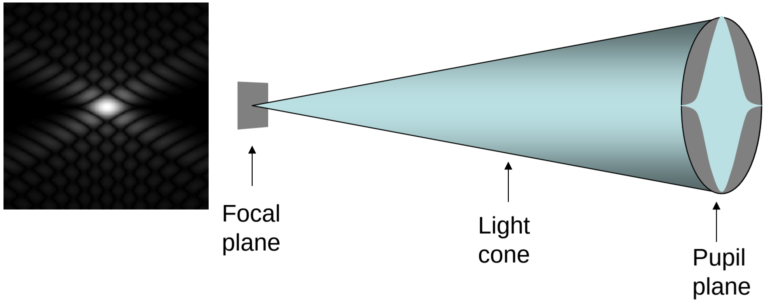

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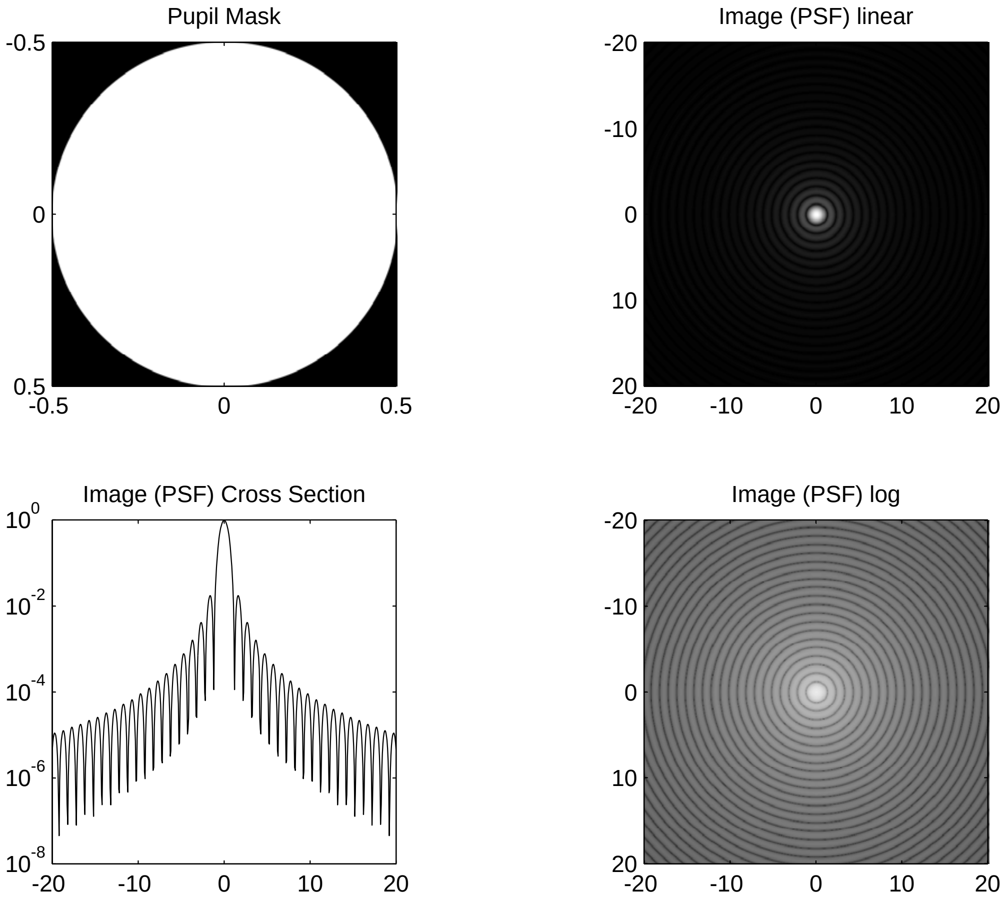
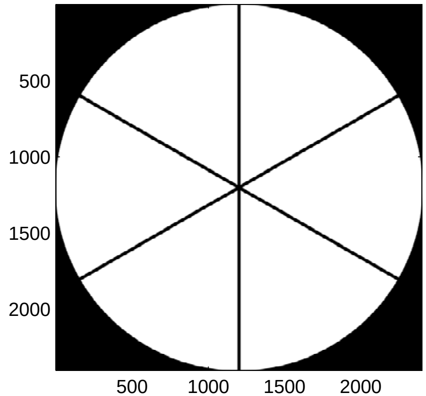
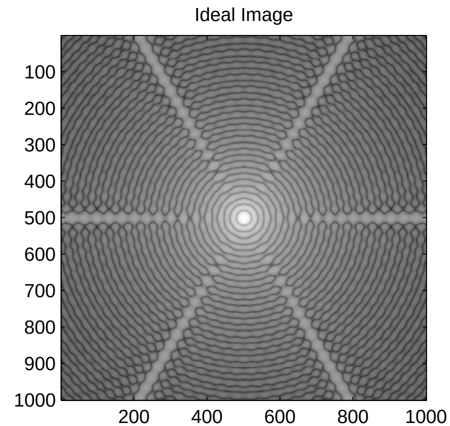
The Airy Pattern



Spiders are an Example of a Shaped Pupil





Note the six bright radial **spikes**

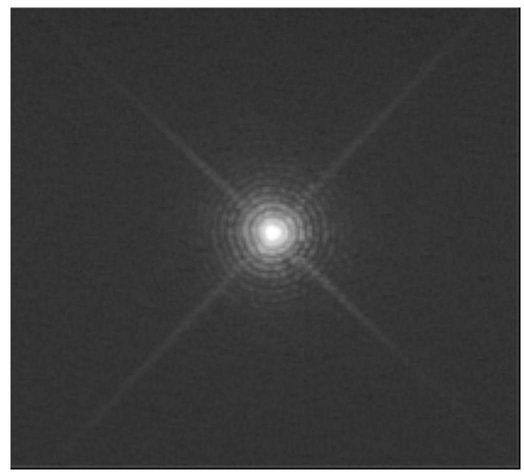
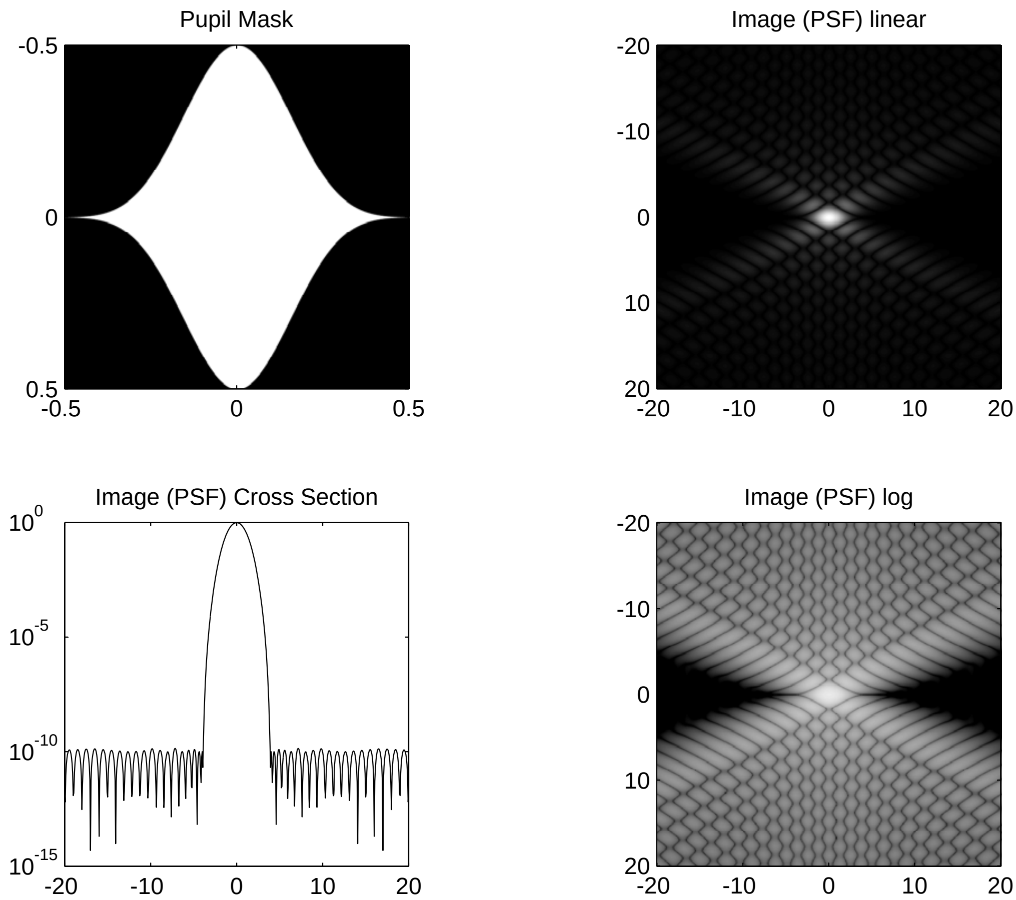


Image of Vega taken with my “big” 250mm telescope.

The Seven Sisters with Spikes

Pleiades image taken with small refractor equipped with **dental floss** spiders.

The Spergel-Kasdin-Vanderbei Pupil



Telescopes Designed for High Contrast

aka Coronagraphs

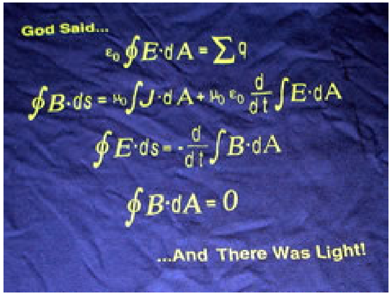
Problem Class: Maximize a linear functional
of a "design" function
given constraints on its Fourier transform.

The Physics of Light

Light consists of photons.

Photons are wave packets.

Diffraction is a wave property.



Maxwell's equations for the Electro-Magnetic field.

$\Downarrow$

Wave equation for electric field (and magnetic field).

$\Downarrow$

Huygens wavelet model

$\Downarrow$

Fresnel/Fraunhofer approximation (Fourier transform!)

$\Downarrow$

Ray optics

Electric Field—Fraunhofer Model

Input: Perfectly flat wavefront (electric field is unity).

Pupil: Described by a mask/tint function $A(x, y)$.

Output: Electric field $E()$:

$$E(\xi, \zeta) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i(x\xi + y\zeta)} A(x, y) dy dx$$

$$\vdots$$

$$E(\rho) = 2\pi \int_{0}^{1/2} J_0(r\rho) A(r) r dr,$$

where J_0 denotes the 0-th order Bessel function of the first kind.

The **intensity** is the magnitude of the electric field **squared**.

The unitless pupil-plane "length" r is given as a multiple of the aperture D.

The unitless image-plane "length" ρ is given as a multiple of focal-length times wavelength over aperture ($f\lambda/D$) or, equivalently, as an angular measure on the sky, in which case it is a multiple of just λ/D. (Example: $\lambda = 0.5\mu$m and $D = 10$m implies $\lambda/D = 10$mas.)

Performance Metrics

Inner and Outer Working Angles

$$\rho_{\text{iwa}} \qquad \rho_{\text{owa}}$$

Contrast:

$$E^2(\rho)/E^2(0)$$

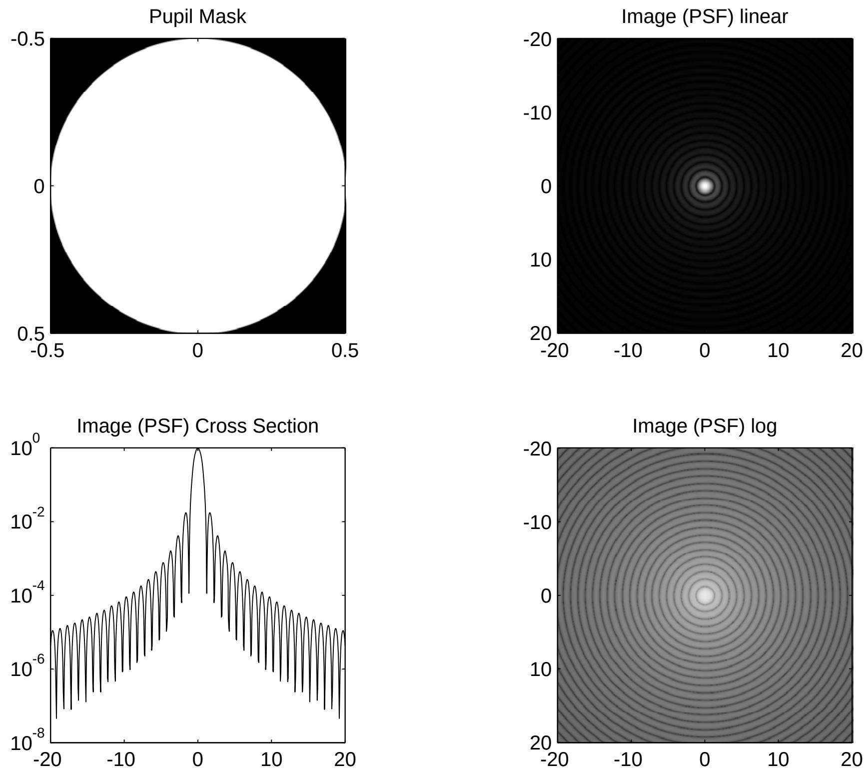
Useful Throughput:

$$\mathcal{T}_{\text{Useful}} = \int_0^{\rho_{\text{iwa}}} E^2(\rho)\rho d\rho.$$

Clear Aperture—Airy Pattern

$\rho_{\text{iwa}} = 1.24 \quad \mathcal{T}_{\text{Useful}} = 84.2\% \quad \text{Contrast} = 10^{-2}$

$\rho_{\text{iwa}} = 748 \quad \mathcal{T}_{\text{Useful}} = 100\% \quad \text{Contrast} = 10^{-10}$



Optimization

Simple First Case: Tinted Glass

Variably tinting glass is called **apodization**.

Find **apodization** function $\boldsymbol{A}()$ that solves:

$$
\begin{array}{lll}
\text{maximize} & \displaystyle\int_0^{1/2} A(r) 2\pi r dr & \\
\text{subject to} & -10^{-5}E(0) \leq E(\rho) \leq 10^{-5}E(0), & \rho_{\text{iwa}} \leq \rho \leq \rho_{\text{owa}}, \\
& 0 \leq A(r) \leq 1, & 0 \leq r \leq 1/2,
\end{array}
$$

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 & 0 \leq A(r) \leq 1, & 0 \leq r \leq 1/2, \\
 & -50 \leq A''(r) \leq 50, & 0 \leq r \leq 1/2
\end{array}
$$

An infinite dimensional **linear programming** problem.

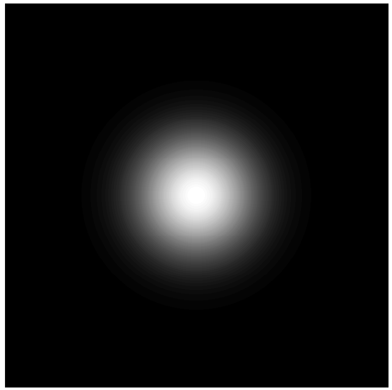
The AMPL Model for Apodization

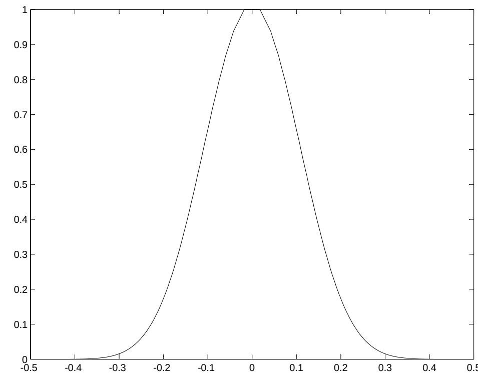
```
function J0;

param pi := 4*atan(1);
param N := 400; # discretization parameter
param rho0 := 4;
param rho1 := 60;

param dr := (1/2)/N;
set Rs ordered := setof {j in 0.5..N-0.5 by 1} (1/2)*j/N;

var A {Rs} >= 0, <= 1, := 1/2;

set Rhos ordered := setof {j in 0..N} j*rho1/N;
set PlanetBand := setof {rho in Rhos: rho>=rho0 && rho<=rho1} rho;

var E0 {rho in Rhos} =
    2*pi*sum {r in Rs} A[r]*J0(2*pi*r*rho)*r*dr;

maximize area: sum {r in Rs} 2*pi*A[r]*r*dr;
subject to sidelobe_pos {rho in PlanetBand}: E0[rho] <= 10^(-5)*E0[0];
subject to sidelobe_neg {rho in PlanetBand}: -10^(-5)*E0[0] <= E0[rho];

subject to smooth {r in Rs: r != first(Rs) && r != last(Rs)}:
    -50*dr^2 <= A[next(r)] - 2*A[r] + A[prev(r)] <= 50*dr^2;

solve;
```

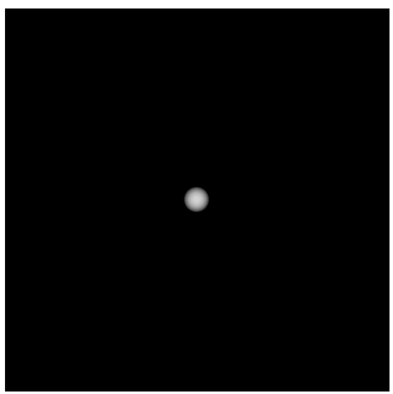
The Optimal Apodization

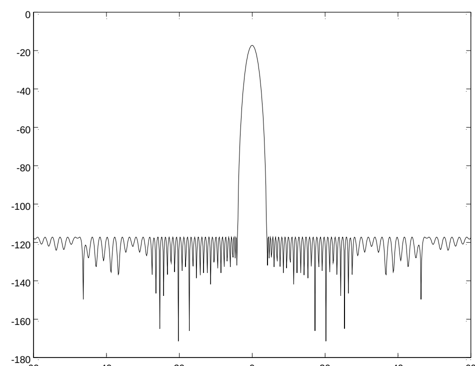
$$\rho_{\text{iwa}} = 4 \quad \mathcal{T}_{\text{Useful}} = 9\%$$

Excellent dark zone. Unmanufacturable.









Concentric Ring Masks

Recall that for circularly symmetric apodizations

$$E(\rho) = 2\pi \int_0^{1/2} J_0(r\rho) A(r) r dr,$$

where J_0 denotes the 0-th order Bessel function of the first kind.

Let

$$A(r) = \begin{cases} 1 & r_{2j} \leq r \leq r_{2j+1}, \qquad j = 0, 1, \ldots, m-1 \\ 0 & \text{otherwise,} \end{cases}$$

where

$$0 \leq r_0 \leq r_1 \leq \cdots \leq r_{2m-1} \leq 1/2.$$

The integral can now be written as a sum of integrals and each of these integrals can be explicitly integrated to get:

$$E(\rho) = \sum_{j=0}^{m-1} \frac{1}{\rho} \left(r_{2j+1} J_1 \left(\rho r_{2j+1} \right) - r_{2j} J_1 \left(\rho r_{2j} \right) \right).$$

Concentric Ring Optimization Problem

$$\text{maximize } \sum_{j=0}^{m-1} \pi(r_{2j+1}^2 - r_{2j}^2)$$

$$\text{subject to: } -10^{-5}E(0) \le E(\rho) \le 10^{-5}E(0), \quad \text{for } \rho_0 \le \rho \le \rho_1$$

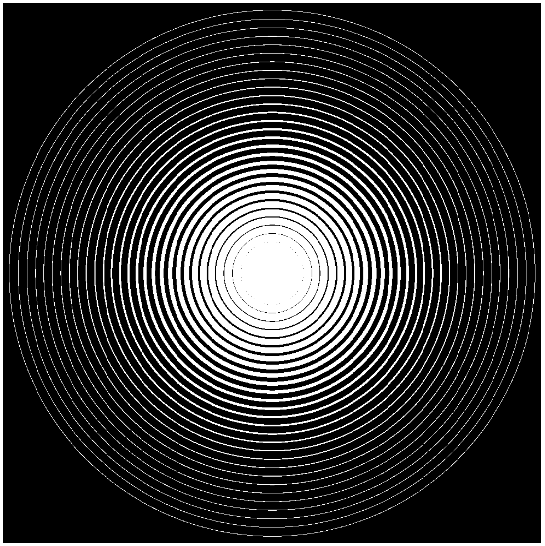
where $E(\rho)$ is the function of the r_j's given on the previous slide.

This problem is a semiinfinite nonconvex optimization problem.

The AMPL Model for Concentric Rings

```
function intrJ0;

param pi := 4*atan(1);
param N := 400; # discretization parameter
param rho0 := 4;
param rho1 := 60;

var r {j in 0..M} >= 0, <= 1/2, := r0[j];

set Rhos2 ordered := setof {j in 0..N} (j+0.5)*rho1/N;
set PlanetBand2 := setof {rho in Rhos2: rho>=rho0 && rho<=rho1} rho;

var E {rho in Rhos2} =
    (1/(2*pi*rho)^2)*sum {j in 0..M by 2}
(intrJ0(2*pi*rho*r[j+1]) - intrJ0(2*pi*rho*r[j]));

maximize area2: sum {j in 0..M by 2} (pi*r[j+1]^2 - pi*r[j]^2);
subject to sidelobe_pos2 {rho in PlanetBand2}: E[rho] <= 10^(-5)*E[first(rhos2)];
subject to sidelobe_neg2 {rho in PlanetBand2}: -10^(-5)*E[first(rhos2)] <= E[rho];

subject to order {j in 0..M-1}: r[j+1] >= r[j];

solve mask;
```

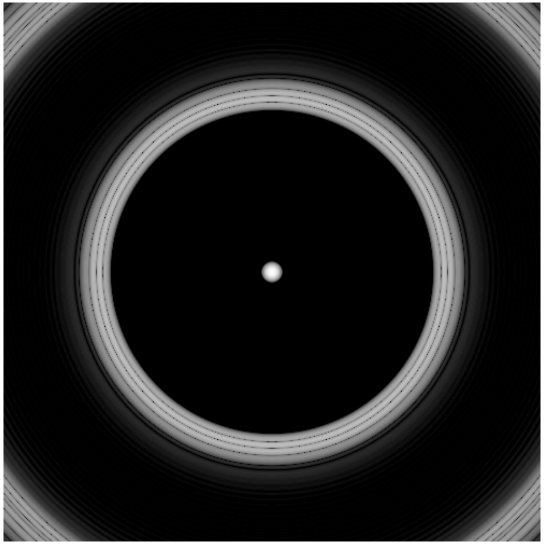
The Best Concentric Ring Mask

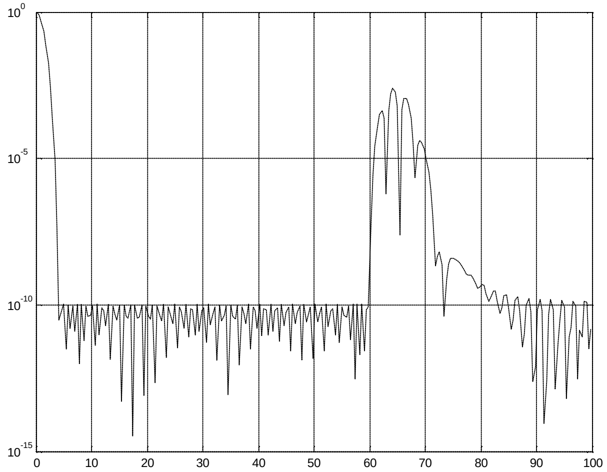


$$\rho_{\text{iwa}} = 4 \quad \rho_{\text{owa}} = 60$$

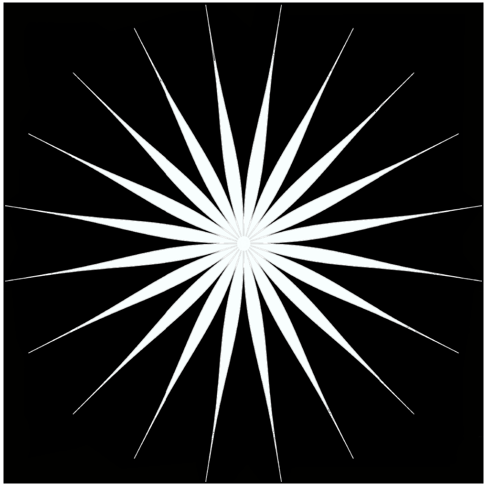
$$\mathcal{T}_{\text{Useful}} = 9\%$$

Lay it on glass?

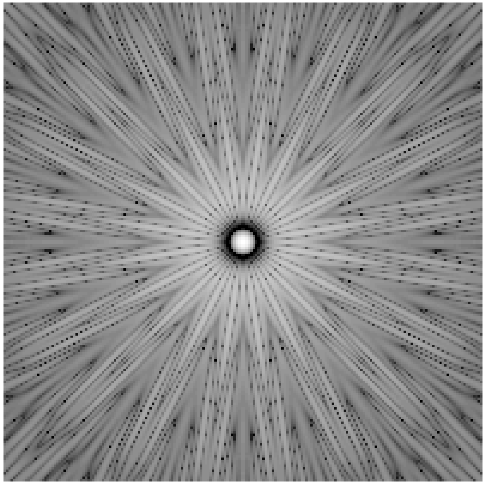




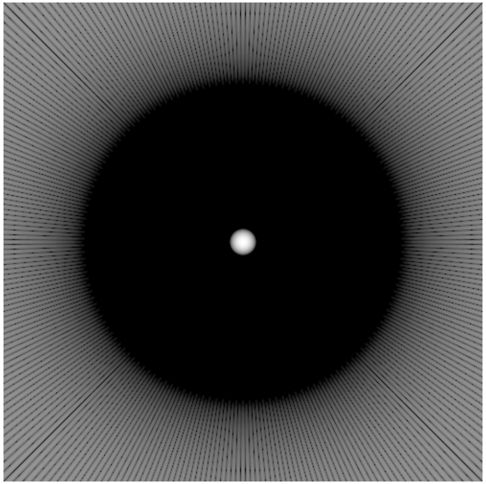
Petal-Shaped Mask



20 petals



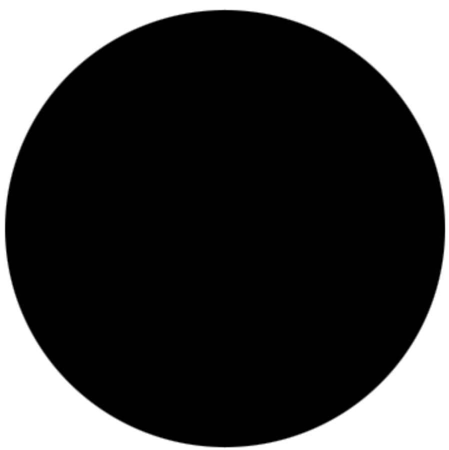
150 petals

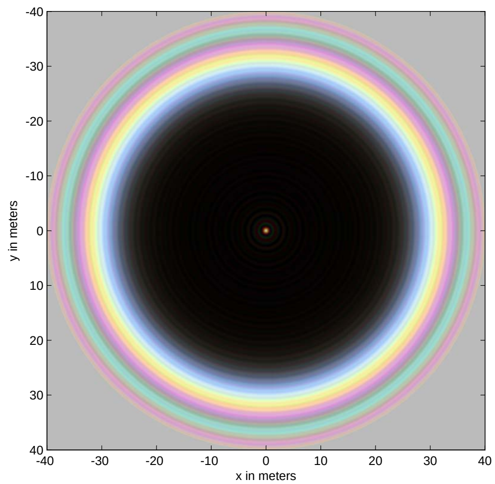


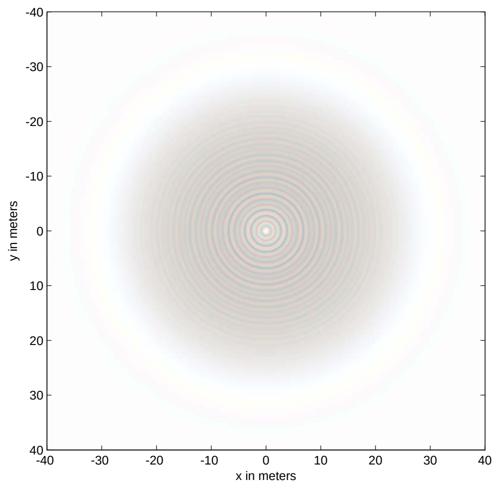
Space-based Occulter (TPF-O)

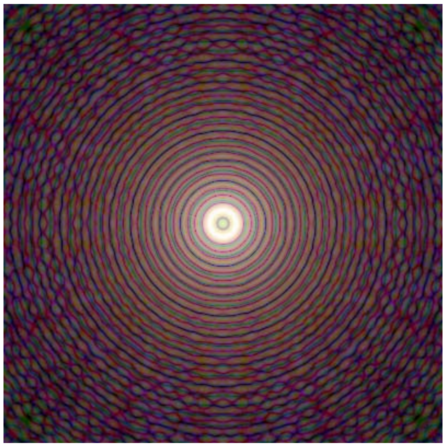
Telescope Aperture: 4m, Occulter Diameter: $\mathbf{50}$m, Occulter Distance: $\mathbf{72,000}$km

Plain External Occulter (Doesn't Work!)









Shaped Occulter