Princeton University
Department of Operations Research
and Financial Engineering

ORF 307
Optimization
Another Practice Second Midterm

Closed book. No computers. Calculators allowed (but not needed).

You are permitted to use a one-page two-sided cheat sheet.

Please return the exam questions and your cheat sheet with your exam booklet.

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(1) Consider the following linear programming problem:

$$\begin{array}{lrcrcrcr} \text{maximize} & 3x_1 & + & x_2 & + & 2x_3 & & \\ \text{subject to} & 2x_1 & + & x_2 & + & 2x_3 & \leq & 13 \\ & -x_1 & & & - & x_3 & \leq & -4 \\ & & & & x_1,\ x_2,\ & x_3 & \geq & 0 \quad . \end{array}$$

Let x_4 and x_5 denote the slack variables associated with the two inequality constraints. Consider the situation in which x_2 and x_3 are basic and all other variables are nonbasic. Write down:

(a) B,
(b) N,
(c) b,
(d) $c_{\mathcal{B}}$,
(e) $c_{\mathcal{N}}$,
(f) $B^{-1}N$,
(g) $x^*_{\mathcal{B}} = B^{-1}b$,
(h) $\zeta^* = c^T_{\mathcal{B}}B^{-1}b$,
(i) $z^*_{\mathcal{N}} = (B^{-1}N)^T c_{\mathcal{B}} - c_{\mathcal{N}}$,
(j) the dictionary corresponding to this basis.

(2) Use the parametric self-dual simplex method to solve this LP problem:

$$\begin{array}{lrcrcr} \text{maximize} & x_1 & + & 5x_2 & & \\ \text{subject to} & x_1 & + & x_2 & \leq & -4 \\ & -x_1 & - & 2x_2 & \leq & 5 \\ & & & x_1,\ x_2 & \geq & 0 \end{array} .$$

(3) Consider the following dictionary from a run of the parametric self-dual simplex method,

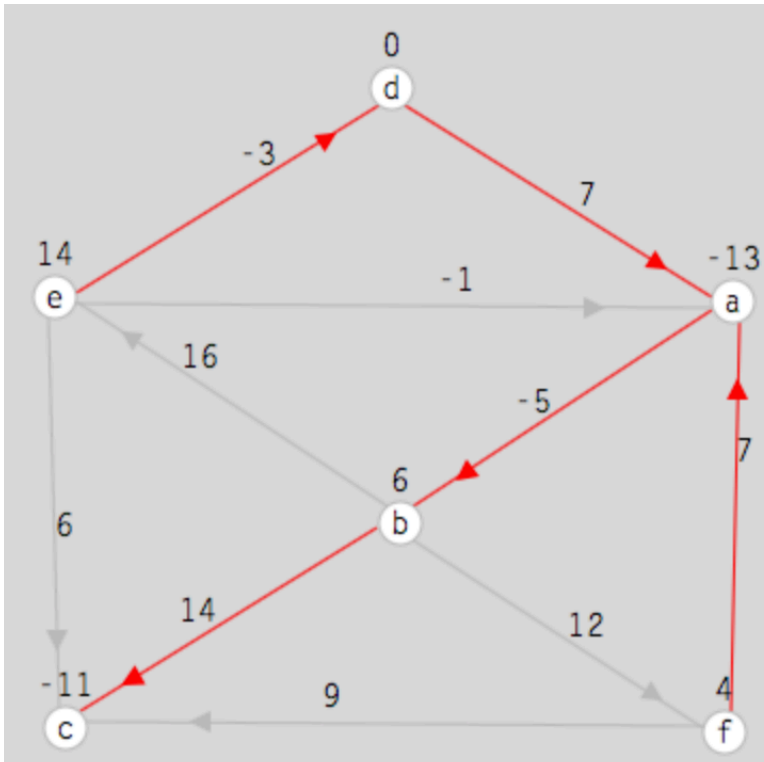
$$\begin{array}{rcccc} \zeta = & 1 & -(2+\mu)x_1 & -(3+\mu)x_2 \\ \\ x_3 = & -2+\mu & -x_1 & -x_2 \\ x_4 = & 1+\mu & +x_1 & -3x_2. \end{array}$$

(a) For what values of μ is this dictionary optimal?
(b) Does this dictionary give the optimal value to the original linear program? Explain your answer.

(4) Each of two players has an Ace, King, Queen and a Jack. They each simultaneously show a card. The "row" player wins if they both show an Ace or if neither shows an Ace and the cards do not match. Otherwise the "column" player wins. The winner receives a payment of $1 from the loser.
(a) List all of the possible actions for each player in this game.
(b) Write down the payoff matrix.
(c) Your friend (assuming you have a friend) says that the optimal strategy for both players is the same: show an Ace with probability $2/5$ and show any of the other three cards with probability $1/5$ each. Is this correct? Explain.

(5) A basis for a network flow problem is a spanning tree, which as we've seen, consists of $n-1$ arcs. But, there are n flow-balance constraints. This appears to be an inconsistancy. Explain why it's not.

(6) Consider the following network flow problem:



The numbers above the nodes are supplies (negative values represent demands) and numbers shown above the arcs are unit shipping costs. The emboldened (i.e., red) arcs form a spanning tree.

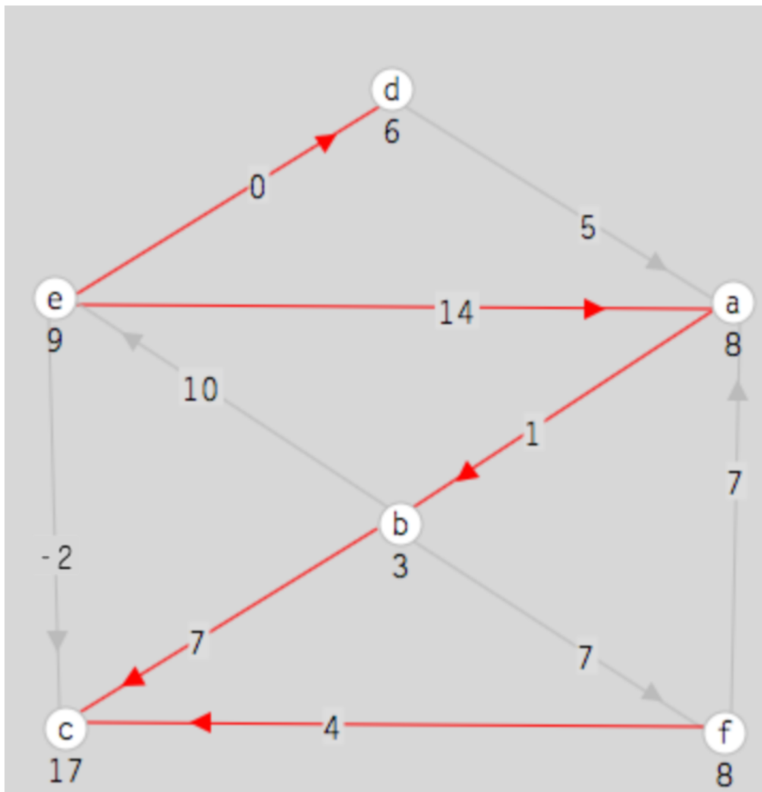
(a) Compute the primal flows for each tree arc (shown in red).

(b) Let b be the root node so that $y_b = 0$. Compute the dual variables for each node.

(c) Compute the dual slacks for each nontree arc (shown in gray).

NOTE: Use the blank network a few pages down to show your solution.

(7) Consider the following tree solution associated with a minimum cost network flow problem:

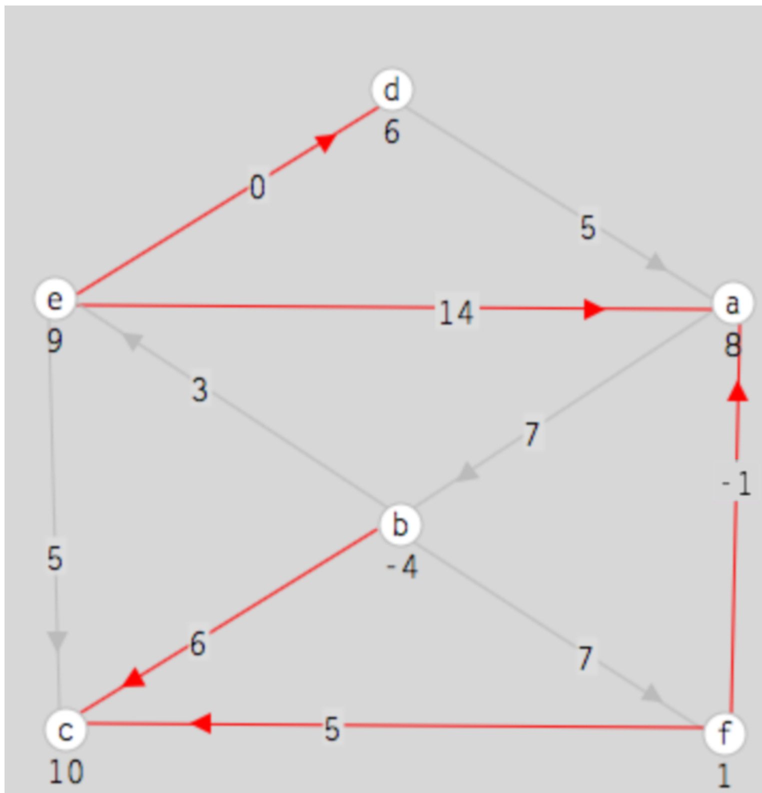


As usual, the numbers on the tree arcs represent primal flows while numbers on the nontree arcs are dual slacks. The numbers below the nodes are dual variables.

(a) Using the primal network simplex method, what is the entering arc?
(b) What is the leaving arc?
(c) After *one* pivot, what is the new tree?
(d) Compute the new primal flows.
(e) Compute the new dual slacks.

NOTE: Use the blank network a few pages down to show your solution.

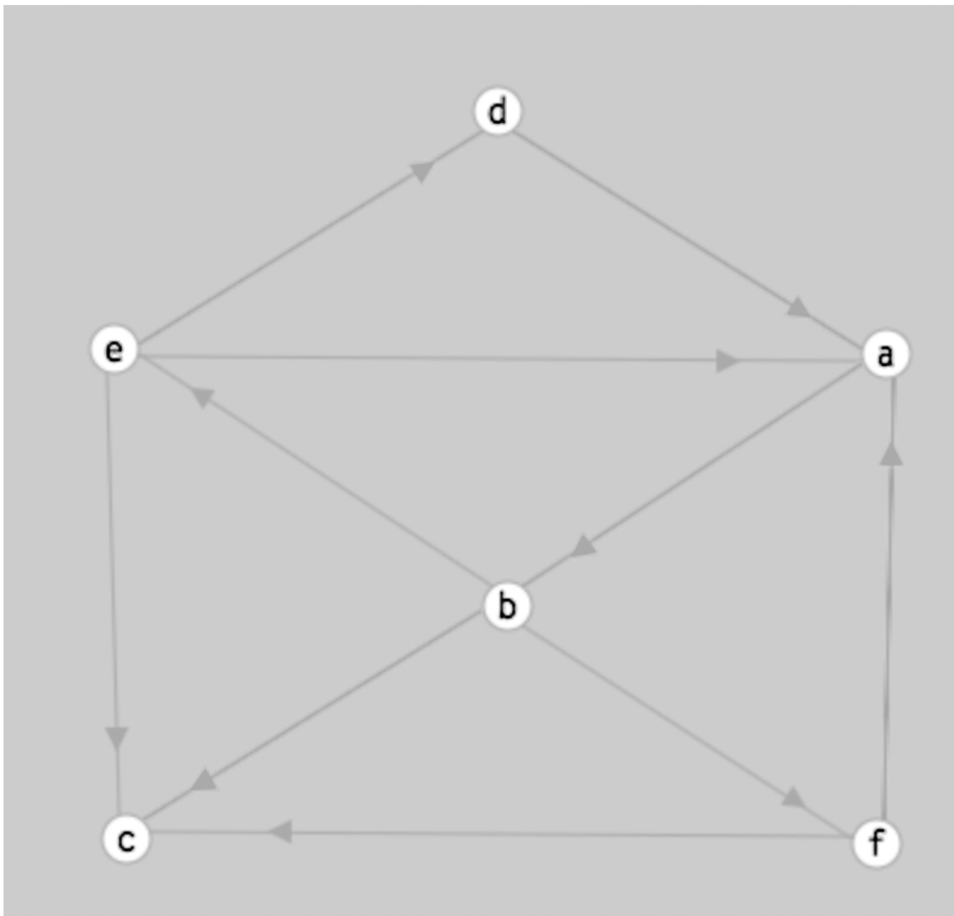
(8) Consider the following tree solution associated with a minimum cost network flow problem:



(a) Using the dual network simplex method, what is the leaving arc?
(b) What is the entering arc?
(c) After *one* pivot, what is the new tree?
(d) Compute the new primal flows.
(e) Compute the new dual slacks.

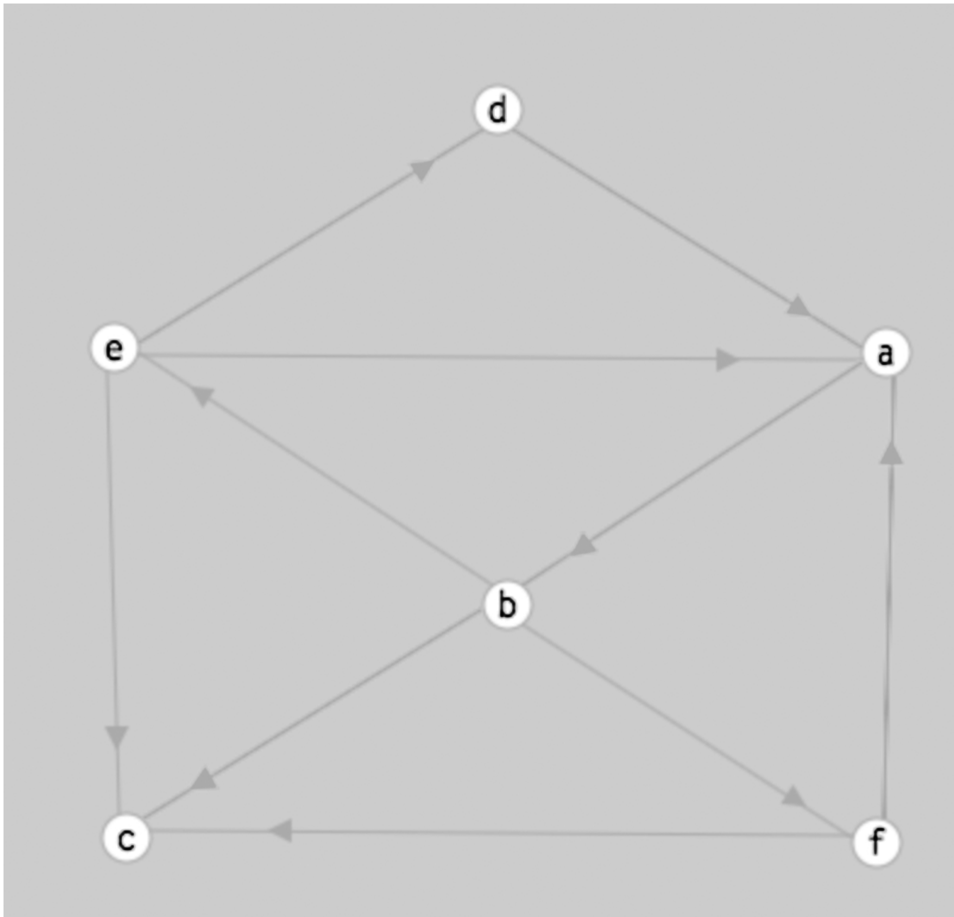
NOTE: Use the blank network a few pages down to show your solution.

Use these “blank” networks to “draw” your solutions to problems **??**, **??** and **??**.



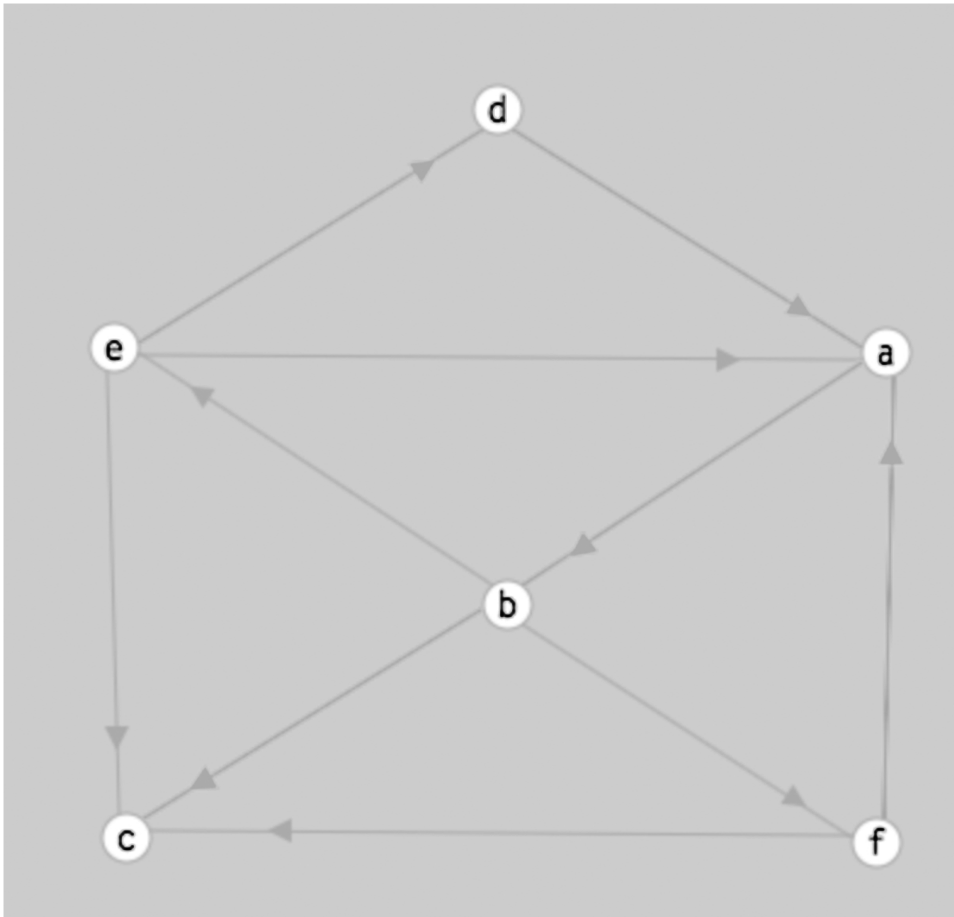
Problem **??**

WRITE YOUR NAME HERE: ____________________________________



Problem **??**

WRITE YOUR NAME HERE: ______________________________________



Problem **??**

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