

Q. 1 (Usual hypothesis testing with one-sided alternative:). The managers at Hunter Chemical Company claim that their major product contains on average $\mu = 4.0$ fluid ounces of caustic material per gallon. They further state that the distribution of caustic material is normal with a standard deviation of $\sigma = 1.3$ fluid ounces. If there is too much caustic material, the product will be dangerous. A government inspector is brought in to test the product. She randomly selects a sample of 100, gallon-size containers of the product and finds that their mean weight of caustic materials is $\bar{X} = 4.5$ fluid ounces per gallon.

- (a) What are the null and alternative hypotheses that the inspector is exploring?
- (b) Explain in English what the type I and type II errors for this problem would be?
- (c) What is the probability of getting a sample mean as high as or higher than 4.5, if the production process was working as the Company claims. Based on this, state your conclusion of the test. Reject at level 0.05.

Q. 2 (Find value of statistic for a given level of test). A tutoring program claims to increase IQ of children. They cite a study of 25 children randomly samples from all over Ontario that showed an average IQ of 104 after going through their program . We happen to know that In the general population of children, the average IQ is 100 and the population SD is 15. We want to test whether the program is effective or not.

- (a) What are the null and alternative hypotheses?
- (b) Perform a test based on the sample. Reject at level 0.05.
- (c) How high would the sample mean have to be for you to to reject the null hypothesis at the level 5%?

Q. 3 (CI and hypothesis testing). The MSCEIT is a measure of Emotional Intelligence. It has a mean of 100 and a standard deviation of 15 in the population. A sample of 49 people was collected. The sample mean was 99.

- (a) Calculate a 90% confidence interval for the population mean.
- (b) Only based on your answer to part (a), if you had tested the null hypothesis that the population mean was 100, would you have retained or rejected it? Use the significance level of 10%?

Q. 4 (Expectation). Suppose X has the probability density

$$f(x) = \frac{1}{2\sqrt{x}}$$

for $0 < x < 1$.

- (a) For what positive integers X does X^k have expectation?
- (b) Calculate $E(X^k)$ for the positive integers k such that expectation exists

Q. 5 (Expectation). Suppose X has the probability density

$$f(x) = \frac{1}{6}$$

for $0 < x < 2$ and

$$f(x) = \frac{1}{3}$$

for $2 < x < 4$.

- (a) Find the median
- (b) Find the median of the sum of 40 such random variables. (Use CLT)

Q. 6 (CI). How many questions need to be on a short answer test for us to be 95% sure that someone who knows 80% of the course material gets a grade between 75% and 85%?

Q. 7 (Probability). W, X, Y, Z independent standard normal. What is the probability that $W + X > Y + Z + 1$?